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Econometrics I

**WRITTEN EXAMINATION**
Friday June 2, 2017, 9 am - 2 pm

Allowed tools: Pocket calculator

Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description.

For the maximum number of points on each problem detailed and clear solutions are required.

If not indicated otherwise, the disturbance terms $u_i$ in the models are supposed to fulfill the usual requirements of normality, homoscedasticity and independence.

1. (25p) The simple linear regression model $Y_t = \beta_1 + \beta_2 t + u_t$ was used on the log US GDP quarterly series, 2000–2009 ($t = 1, \ldots, 40$). The following table shows the regression output, unfortunately with some values missing:

| ANOVA | | | |
|---|---|---|---|
| Source | Sum of Squares | df | Mean Square |
| Regression (SSE) | ? | ? | ? |
| Residuals (SSR) | 0.0361 | ? | ? |
| Total (SST) | 0.7032 | ? | ? |
| Coefficients Table | | | |
| Variable | Coefficient | SE($\hat{\beta}_i$) | $t_{obs}$ |
| Intercept | 9.2255 | ? | 931.87 |
| $t$ | ? | 0.0004 | 26.50 |

(a) Complete the table.

(10p)

(b) Compute $R^2$, $\bar{R}^2$, AIC and $\hat{\sigma}^2$.

(7p)

(c) Compute a 95% prediction interval for the GDP-value for the last quarter of 2010.

(8p)

2. (21p) Referring to the model and the results in the previous problem:

   (a) It was shown that $\widehat{Corr}(\hat{u}_t, \hat{u}_{t-1}) = 0.8$. Use a suitable test to decide whether the residuals are autocorrelated.

   (6p)

   (b) In the regression

   $$\hat{u}_t^2 = \alpha_1 + \alpha_2 t + \alpha_3 t^2 + v_t$$

   $R^2$ was found to be 0.62. Use a suitable (with a suitable approximation) test to decide whether the residuals are heteroscedastic. If so - what would you do to resolve this problem?

   (8p)

   (c) Another analyst included the Dow Jones Index and the Inflation as independent variables in the original model. $SSR$ and $SSE$ for this new model turned out to be 0.0344 and 0.6688 respectively. Use a suitable test to decide whether the inclusion of the Dow Jones Index and the Inflation significantly improves the model.

   (7p)

3. (17p) We want to study how income (X) affects consumption (Y) and if there are also seasonal variation (unknown units). The following estimated model was the result of a study of quarterly data for the years 2004-2009:

   $$\hat{Y}_t = 1.20 + 2.31 X_i - 0.71 D_{2i} - 0.77 D_{3i} - 0.03 D_{4i}. \qquad (1)$$

   where $D_{2i}$, $D_{3i}$ and $D_{4i}$ are dummy variables for quarters 2, 3 and 4, respectively.

   (a) Interpret the estimated intercept and the estimated parameter corresponding to $D_{2i}$. (Describe in words what they mean in this model.)

   (5p)

   (b) Rewrite equation (1) so that instead of $D_{2i}$, $D_{3i}$ and $D_{4i}$, you include $D_{1i}$, $D_{2i}$ and $D_{3i}$. (Hint: Use that $D_{1i} + D_{2i} + D_{3i} + D_{4i} = 1$.)

   (7p)

   (c) How do you now (after the rewriting) interpret the estimated parameters asked for in (a)?

   (5p)

4. (16p) The following model was used for a regression analysis:

$$Y_i = \beta_1 + \beta_2 X_i + u_i,$$

but it seems that the disturbance terms are functions of $X_i$ according to

$$u_i = e^{X_i} v_i, \tag{2}$$

where $v_i \sim N(0, \sigma^2)$.

(a) Which consequences do this have for the estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ and for the variances $V(\hat{\beta}_1)$ and $V(\hat{\beta}_2)$ using OLS?

(8p)

(b) What can be done here instead of using OLS? Try to be specific and use (2).

(8p)

5. (21p) True or false statements? Short motivation/comment also needed.

(a) The parameter estimators in a linear regession model are BLUE only if the disturbance variables $u_i$ are normally distributed.

(b) If both $Y_i$ and $X_i$ in the model $Y_i = \beta_1 + \beta_2 X_i + u_i$ are multiplied with 100, both $\hat{\beta}_1$ and $\hat{\beta}_2$ change their values.

(c) $R^2$ is a good "quality" measure when comparing model with varying number of regressors.

(d) The Durbin-Watson test, the Runs test and the White's test are all examples of tests for autocorrelation.

(e) If we have measurement error in $X$ using the model $Y_i = \beta_1 + \beta_2 X_i + u_i$, this implies that $Cov(X_i, u_i) \neq 0$.

(f) The exclusion of important variables in a linear model cannot lead to autocorrelated $u_i : s$.

(g) If you include dummy variables in your regression model, the intercept (constant) term must be excluded.

# Formula sheet, Econometrics I, Spring 2017

Under the simple linear model $Y_i = \beta_1 + \beta_2 X_i + u_i$, where $u_i \sim N(0, \sigma^2)$ and given independent pairs of observations $(Y_1, X_1), \ldots, (Y_n, X_n)$, the OLS (and ML) estimators are:

$$\hat{\beta}_1 = \overline{Y} - \hat{\beta}_2 \overline{X}$$

$$\hat{\beta}_2 = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sum (X_i - \overline{X})^2}$$

$$\hat{\sigma}^2 = \frac{RSS}{n-2} = \frac{\sum \left(Y_i - \hat{Y}_i\right)^2}{n-2}$$

where $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$ and where $E(\hat{\beta}_1) = \beta_1$, $E(\hat{\beta}_2) = \beta_2$ and $E(\hat{\sigma}^2) = \sigma^2$ and further

$$V(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum (X_i - \overline{X})^2} \sigma^2$$

$$V(\hat{\beta}_2) = \frac{\sigma^2}{\sum (X_i - \overline{X})^2}$$

$$V(\hat{Y}_0) = \sigma^2 \left( \frac{1}{n} + \frac{(X_0 - \overline{X})^2}{\sum (X_i - \overline{X})^2} \right)$$

$$V(Y_0 - \hat{Y}_0) = \sigma^2 \left( 1 + \frac{1}{n} + \frac{(X_0 - \overline{X})^2}{\sum (X_i - \overline{X})^2} \right)$$

Distributional results:

$$\frac{\hat{\beta}_i - \beta_i}{se(\hat{\beta}_i)} \sim t(n-2), \quad i = 1, 2$$

$$\frac{\hat{\sigma}^2 (n-2)}{\sigma^2} \sim \chi^2(n-2)$$

Coefficient of determination:

$$r^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum \left(Y_i - \hat{Y}_i\right)^2}{\sum (Y_i - \overline{Y})^2}$$

Coefficient of correlation:

$$r = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum (X_i - \overline{X})^2 \sum (Y_i - \overline{Y})^2}}$$

where $r = \pm\sqrt{r^2}$

If we let $Y_i^* = w_1 Y_i$ and $X_i^* = w_2 X_2$, then

$$\widehat{\beta}_1^* = w_1 \widehat{\beta}_1, \quad \widehat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\widehat{\beta}_2, \quad \widehat{\sigma}^{*2} = w_1^2 \widehat{\sigma}^2$$

Under the multiple linear regression model $Y_i = \beta_1 + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + u_i$, where $u_i \sim N(0, \sigma^2)$ and given independent vectors of observations $(Y_1, X_{21}, \ldots, X_{k1}), \ldots, (Y_n, X_{2n}, \ldots, X_{kn})$, the following holds for the OLS (ML) estimators:

$$\widehat{\sigma}^2 \;=\; \frac{RSS}{n-k} = \frac{\sum\left(Y_i - \widehat{Y}_i\right)^2}{n-k}$$

$$\frac{\widehat{\beta}_i - \beta_i}{se(\widehat{\beta}_i)} \;\sim\; t(n-k), \quad i = 1, \ldots, k$$

$$\frac{\widehat{\sigma}^2(n-k)}{\sigma^2} \;\sim\; \chi^2(n-k)$$

The multiple coefficient of determination:

$$R^2 \;=\; \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum\left(Y_i - \widehat{Y}_i\right)^2}{\sum\left(Y_i - \overline{Y}\right)^2}$$

Adjusted:

$$\overline{R}^2 \;=\; 1 - \frac{RSS/(n-k)}{TSS/(n-1)}$$

Testing $H_0 : \beta_2 = \ldots \beta_k = 0$:

$$F \;=\; \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{\sum\left(\widehat{Y}_i - \overline{Y}\right)^2/(k-1)}{\sum\left(Y_i - \widehat{Y}_i\right)^2/(n-k)}$$

Comparing an "old" model with a "new" (larger):

$$F \;=\; \frac{(ESS_{new} - ESS_{old})/\text{number of new regressors}}{RSS_{new}/(n - \text{number of parameters in the new model})}$$

$$=\; \frac{(R_{new}^2 - R_{old}^2)/\text{number of new regressors}}{(1 - R_{new}^2)/(n - \text{number of parameters in the new model})}$$

Comparing an "unrestricted" model with a "restricted":

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} = \frac{(R_{UR}^2 - R_R^2)/m}{(1 - R_{UR}^2)/(n-k)}$$

where $m$ is the number of linear constraints and $k$ is the number of parameters in the unrestricted model.

Variance inflation factor:

$$VIF_j = \frac{1}{1 - R_j^2}$$

Auxiliary regression:

$$F_j = \frac{R_j^2/(k-2)}{(1 - R_j^2)/(n-k+1)}$$

where $R_j^2 = R^2$ in the regression of the remaining $(k-2)$ regressors.

White's test for heteroscedasticity:

$$n R^2 \overset{appr}{\sim} \chi^2(df = \text{number of regressors in the auxiliary regression})$$

(Holds under $H_0$ : no heteroscedasticity.)

For $R$ = number of runs, where $N = N_1 + N_2$ total number of observations:

$$E(R) = \frac{2N_1 N_2}{N} + 1$$

$$V(R) = \frac{2N_1 N_2(2N_1 N_2 - N)}{N^2(N-1)}$$

The Durbin Watson $d$ statistic:

$$d = \frac{\sum_{t=2}^{n} \left(\widehat{u}_t - \widehat{u}_{t-1}\right)^2}{\sum_{t=1}^{n} \widehat{u}_t^2}$$

Akaike's information criterion:

$$AIC = \frac{e^{2k/n} RSS}{n}$$

Schwartz's information citerion:

$$SIC = \frac{n^{k/n} RSS}{n}$$

Mallow's $C_p$ criterion:
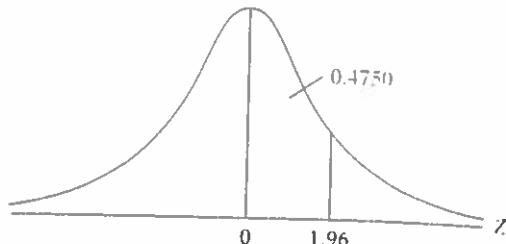
$$C_p = \frac{RSS_p}{\widehat{\sigma}^2} - (n - 2p)$$

**TABLE D.1**
**Areas Under the Standardized Normal Distribution**

**Example**

$Pr(0 \leq Z \leq 1.96) = 0.4750$

$Pr(Z \geq 1.96) = 0.5 - 0.4750 = 0.025$



0.4750

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4454 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |

*Note:* This table gives the area in the right-hand tail of the distribution (i.e., $Z \geq 0$). But since the normal distribution is symmetrical about $Z = 0$, the area in the left-hand tail is the same as the area in the corresponding right-hand tail. For example, $Pr(-1.96 \leq Z \leq 0) = 0.4750$. Therefore, $Pr(-1.96 \leq Z \leq 1.96) = 2(0.4750) = 0.95$.
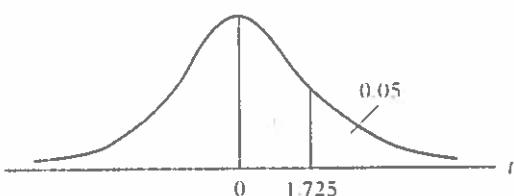
**Example**

$Pr(t > 2.086) = 0.025$

$Pr(t > 1.725) = 0.05$    for df = 20

$Pr(|t| > 1.725) = 0.10$
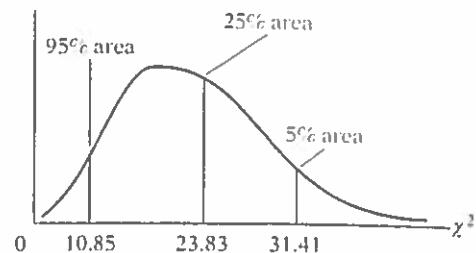


| Pr df | 0.25 0.50 | 0.10 0.20 | 0.05 0.10 | 0.025 0.05 | 0.01 0.02 | 0.005 0.010 | 0.001 0.002 |
|---|---|---|---|---|---|---|---|
| 1 | 1.000 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.31 |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.214 |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 |
| 120 | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 |
| ∞ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 |

*Note:* The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

**TABLE D.4**
**Upper Percentage Points of the $\chi^2$ Distribution**

**Example**

$Pr(\chi^2 > 10.85) = 0.95$

$Pr(\chi^2 > 23.83) = 0.25$    for df = 20

$Pr(\chi^2 > 31.41) = 0.05$



| Degrees Pr of freedom | .995 | .990 | .975 | .950 | .900 |
|---|---|---|---|---|---|
| 1 | $392704 \times 10^{-10}$ | $157088 \times 10^{-9}$ | $982069 \times 10^{-9}$ | $393214 \times 10^{-8}$ | .0157908 |
| 2 | .0100251 | .0201007 | .0506356 | .102587 | .210720 |
| 3 | .0717212 | .114832 | .215795 | .351846 | .584375 |
| 4 | .206990 | .297110 | .484419 | .710721 | 1.063623 |
| 5 | .411740 | .554300 | .831211 | 1.145476 | 1.61031 |
| 6 | .675727 | .872085 | 1.237347 | 1.63539 | 2.20413 |
| 7 | .989265 | 1.239043 | 1.68987 | 2.16735 | 2.83311 |
| 8 | 1.344419 | 1.646482 | 2.17973 | 2.73264 | 3.48954 |
| 9 | 1.734926 | 2.087912 | 2.70039 | 3.32511 | 4.16816 |
| 10 | 2.15585 | 2.55821 | 3.24697 | 3.94030 | 4.86518 |
| 11 | 2.60321 | 3.05347 | 3.81575 | 4.57481 | 5.57779 |
| 12 | 3.07382 | 3.57056 | 4.40379 | 5.22603 | 6.30380 |
| 13 | 3.56503 | 4.10691 | 5.00874 | 5.89186 | 7.04150 |
| 14 | 4.07468 | 4.66043 | 5.62872 | 6.57063 | 7.78953 |
| 15 | 4.60094 | 5.22935 | 6.26214 | 7.26094 | 8.54675 |
| 16 | 5.14224 | 5.81221 | 6.90766 | 7.96164 | 9.31223 |
| 17 | 5.69724 | 6.40776 | 7.56418 | 8.67176 | 10.0852 |
| 18 | 6.26481 | 7.01491 | 8.23075 | 9.39046 | 10.8649 |
| 19 | 6.84398 | 7.63273 | 8.90655 | 10.1170 | 11.6509 |
| 20 | 7.43386 | 8.26040 | 9.59083 | 10.8508 | 12.4426 |
| 21 | 8.03366 | 8.89720 | 10.28293 | 11.5913 | 13.2396 |
| 22 | 8.64272 | 9.54249 | 10.9823 | 12.3380 | 14.0415 |
| 23 | 9.26042 | 10.19567 | 11.6885 | 13.0905 | 14.8479 |
| 24 | 9.88623 | 10.8564 | 12.4011 | 13.8484 | 15.6587 |
| 25 | 10.5197 | 11.5240 | 13.1197 | 14.6114 | 16.4734 |
| 26 | 11.1603 | 12.1981 | 13.8439 | 15.3791 | 17.2919 |
| 27 | 11.8076 | 12.8786 | 14.5733 | 16.1513 | 18.1138 |
| 28 | 12.4613 | 13.5648 | 15.3079 | 16.9279 | 18.9392 |
| 29 | 13.1211 | 14.2565 | 16.0471 | 17.7083 | 19.7677 |
| 30 | 13.7867 | 14.9535 | 16.7908 | 18.4926 | 20.5992 |
| 40 | 20.7065 | 22.1643 | 24.4331 | 26.5093 | 29.0505 |
| 50 | 27.9907 | 29.7067 | 32.3574 | 34.7642 | 37.6886 |
| 60 | 35.5346 | 37.4848 | 40.4817 | 43.1879 | 46.4589 |
| 70 | 43.2752 | 45.4418 | 48.7576 | 51.7393 | 55.3290 |
| 80 | 51.1720 | 53.5400 | 57.1532 | 60.3915 | 64.2778 |
| 90 | 59.1963 | 61.7541 | 65.6466 | 69.1260 | 73.2912 |
| 100* | 67.3276 | 70.0648 | 74.2219 | 77.9295 | 82.3581 |

*For df greater than 100 the expression $\sqrt{2\chi^2} - \sqrt{(2k-1)} = Z$ follows the standardized normal distribution, where $k$ represents the degrees of freedom

# $\chi^2$-table continued

| .750 | .500 | .250 | .100 | .050 | .025 | .010 | .005 |
|---|---|---|---|---|---|---|---|
| .1015308 | .454937 | 1.32330 | 2.70554 | 3.84146 | 5.02389 | 6.63490 | 7.87944 |
| .575364 | 1.38629 | 2.77259 | 4.60517 | 5.99147 | 7.37776 | 9.21034 | 10.5966 |
| 1.212534 | 2.36597 | 4.10835 | 6.25139 | 7.81473 | 9.34840 | 11.3449 | 12.8381 |
| 1.92255 | 3.35670 | 5.38527 | 7.77944 | 9.48773 | 11.1433 | 13.2767 | 14.8602 |
| 2.67460 | 4.35146 | 6.62568 | 9.23635 | 11.0705 | 12.8325 | 15.0863 | 16.7496 |
| 3.45460 | 5.34812 | 7.84080 | 10.6446 | 12.5916 | 14.4494 | 16.8119 | 18.5476 |
| 4.25485 | 6.34581 | 9.03715 | 12.0170 | 14.0671 | 16.0128 | 18.4753 | 20.2777 |
| 5.07064 | 7.34412 | 10.2188 | 13.3616 | 15.5073 | 17.5346 | 20.0902 | 21.9550 |
| 5.89883 | 8.34283 | 11.3887 | 14.6837 | 16.9190 | 19.0228 | 21.6660 | 23.5893 |
| 6.73720 | 9.34182 | 12.5489 | 15.9871 | 18.3070 | 20.4831 | 23.2093 | 25.1882 |
| 7.58412 | 10.3410 | 13.7007 | 17.2750 | 19.6751 | 21.9200 | 24.7250 | 26.7569 |
| 8.43842 | 11.3403 | 14.8454 | 18.5494 | 21.0261 | 23.3367 | 26.2170 | 28.2995 |
| 9.29906 | 12.3398 | 15.9839 | 19.8119 | 22.3621 | 24.7356 | 27.6883 | 29.8194 |
| 10.1653 | 13.3393 | 17.1170 | 21.0642 | 23.6848 | 26.1190 | 29.1413 | 31.3193 |
| 11.0365 | 14.3389 | 18.2451 | 22.3072 | 24.9958 | 27.4884 | 30.5779 | 32.8013 |
| 11.9122 | 15.3385 | 19.3688 | 23.5418 | 26.2962 | 28.8454 | 31.9999 | 34.2672 |
| 12.7919 | 16.3381 | 20.4887 | 24.7690 | 27.5871 | 30.1910 | 33.4087 | 35.7185 |
| 13.6753 | 17.3379 | 21.6049 | 25.9894 | 28.8693 | 31.5264 | 34.8053 | 37.1564 |
| 14.5620 | 18.3376 | 22.7178 | 27.2036 | 30.1435 | 32.8523 | 36.1908 | 38.5822 |
| 15.4518 | 19.3374 | 23.8277 | 28.4120 | 31.4104 | 34.1696 | 37.5662 | 39.9968 |
| 16.3444 | 20.3372 | 24.9348 | 29.6151 | 32.6705 | 35.4789 | 38.9321 | 41.4010 |
| 17.2396 | 21.3370 | 26.0393 | 30.8133 | 33.9244 | 36.7807 | 40.2894 | 42.7956 |
| 18.1373 | 22.3369 | 27.1413 | 32.0069 | 35.1725 | 38.0757 | 41.6384 | 44.1813 |
| 19.0372 | 23.3367 | 28.2412 | 33.1963 | 36.4151 | 39.3641 | 42.9798 | 45.5585 |
| 19.9393 | 24.3366 | 29.3389 | 34.3816 | 37.6525 | 40.6465 | 44.3141 | 46.9278 |
| 20.8434 | 25.3364 | 30.4345 | 35.5631 | 38.8852 | 41.9232 | 45.6417 | 48.2899 |
| 21.7494 | 26.3363 | 31.5284 | 36.7412 | 40.1133 | 43.1944 | 46.9630 | 49.6449 |
| 22.6572 | 27.3363 | 32.6205 | 37.9159 | 41.3372 | 44.4607 | 48.2782 | 50.9933 |
| 23.5666 | 28.3362 | 33.7109 | 39.0875 | 42.5569 | 45.7222 | 49.5879 | 52.3356 |
| 24.4776 | 29.3360 | 34.7998 | 40.2560 | 43.7729 | 46.9792 | 50.8922 | 53.6720 |
| 33.6603 | 39.3354 | 45.6160 | 51.8050 | 55.7585 | 59.3417 | 63.6907 | 66.7659 |
| 42.9421 | 49.3349 | 56.3336 | 63.1671 | 67.5048 | 71.4202 | 76.1539 | 79.4900 |
| 52.2938 | 59.3347 | 66.9814 | 74.3970 | 79.0819 | 83.2976 | 88.3794 | 91.9517 |
| 61.6983 | 69.3344 | 77.5766 | 85.5271 | 90.5312 | 95.0231 | 100.425 | 104.215 |
| 71.1445 | 79.3343 | 88.1303 | 96.5782 | 101.879 | 106.629 | 112.329 | 116.321 |
| 80.6247 | 89.3342 | 98.6499 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 90.1332 | 99.3341 | 109.141 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |
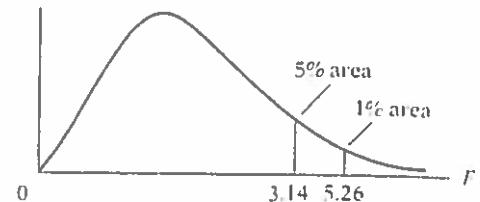
## TABLE D 3   Upper Percentage Points of the $F$ Distribution

**Example**

$Pr(F > 1.59) = 0.25$

$Pr(F > 2.42) = 0.10$   for df $N_1 = 10$

$Pr(F > 3.14) = 0.05$   and $N_2 = 9$

$Pr(F > 5.26) = 0.01$



| df for denom-inator $N_2$ | Pr | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | .25 | 5.83 | 7.50 | 8.20 | 8.58 | 8.82 | 8.98 | 9.10 | 9.19 | 9.26 | 9.32 | 9.36 | 9.41 |
| | .10 | 39.9 | 49.5 | 53.6 | 55.8 | 57.2 | 58.2 | 58.9 | 59.4 | 59.9 | 60.2 | 60.5 | 60.7 |
| | .05 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 | 242 | 243 | 244 |
| 2 | .25 | 2.57 | 3.00 | 3.15 | 3.23 | 3.28 | 3.31 | 3.34 | 3.35 | 3.37 | 3.38 | 3.39 | 3.39 |
| | .10 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 | 9.39 | 9.40 | 9.41 |
| | .05 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 |
| | .01 | 98.5 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 | 99.4 |
| 3 | .25 | 2.02 | 2.28 | 2.36 | 2.39 | 2.41 | 2.42 | 2.43 | 2.44 | 2.44 | 2.44 | 2.45 | 2.45 |
| | .10 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 | 5.23 | 5.22 | 5.22 |
| | .05 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.76 | 8.74 |
| | .01 | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 | 27.2 | 27.1 | 27.1 |
| 4 | .25 | 1.81 | 2.00 | 2.05 | 2.06 | 2.07 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 |
| | .10 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 | 3.92 | 3.91 | 3.90 |
| | .05 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.94 | 5.91 |
| | .01 | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 | 14.5 | 14.4 | 14.4 |
| 5 | .25 | 1.69 | 1.85 | 1.88 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 | 1.89 |
| | .10 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 | 3.30 | 3.28 | 3.27 |
| | .05 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.71 | 4.68 |
| | .01 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 | 10.1 | 9.96 | 9.89 |
| 6 | .25 | 1.62 | 1.76 | 1.78 | 1.79 | 1.79 | 1.78 | 1.78 | 1.78 | 1.77 | 1.77 | 1.77 | 1.77 |
| | .10 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 | 2.94 | 2.92 | 2.90 |
| | .05 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.03 | 4.00 |
| | .01 | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.79 | 7.72 |
| 7 | .25 | 1.57 | 1.70 | 1.72 | 1.72 | 1.71 | 1.71 | 1.70 | 1.70 | 1.69 | 1.69 | 1.69 | 1.68 |
| | .10 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 | 2.70 | 2.68 | 2.67 |
| | .05 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.60 | 3.57 |
| | .01 | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.54 | 6.47 |
| 8 | .25 | 1.54 | 1.66 | 1.67 | 1.66 | 1.66 | 1.65 | 1.64 | 1.64 | 1.63 | 1.63 | 1.63 | 1.62 |
| | .10 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 | 2.54 | 2.52 | 2.50 |
| | .05 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.31 | 3.28 |
| | .01 | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.73 | 5.67 |
| 9 | .25 | 1.51 | 1.62 | 1.63 | 1.63 | 1.62 | 1.61 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.58 |
| | .10 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 | 2.42 | 2.40 | 2.38 |
| | .05 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.10 | 3.07 |
| | .01 | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.18 | 5.11 |

df for numerator $N_1$

# F—table continued

| | | | | df for numerator $N_1$ | | | | | | | | | | df for denominator |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 20 | 24 | 30 | 40 | 50 | 60 | 100 | 120 | 200 | 500 | ∞ | Pr | $N_2$ |
| 9.49 | 9.58 | 9.63 | 9.67 | 9.71 | 9.74 | 9.76 | 9.78 | 9.80 | 9.82 | 9.84 | 9.85 | .25 | |
| 61.2 | 61.7 | 62.0 | 62.3 | 62.5 | 62.7 | 62.8 | 63.0 | 63.1 | 63.2 | 63.3 | 63.3 | .10 | 1 |
| 246 | 248 | 249 | 250 | 251 | 252 | 252 | 253 | 253 | 254 | 254 | 254 | .05 | |
| 3.41 | 3.43 | 3.43 | 3.44 | 3.45 | 3.45 | 3.46 | 3.47 | 3.47 | 3.48 | 3.48 | 3.48 | .25 | |
| 9.42 | 9.44 | 9.45 | 9.46 | 9.47 | 9.47 | 9.47 | 9.48 | 9.48 | 9.49 | 9.49 | 9.49 | .10 | 2 |
| 19.4 | 19.4 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 | .05 | |
| 99.4 | 99.4 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 | .01 | |
| 2.46 | 2.46 | 2.46 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | 2.47 | .25 | |
| 5.20 | 5.18 | 5.18 | 5.17 | 5.16 | 5.15 | 5.15 | 5.14 | 5.14 | 5.14 | 5.14 | 5.13 | .10 | 3 |
| 8.70 | 8.66 | 8.64 | 8.62 | 8.59 | 8.58 | 8.57 | 8.55 | 8.55 | 8.54 | 8.53 | 8.53 | .05 | |
| 26.9 | 26.7 | 26.6 | 26.5 | 26.4 | 26.4 | 26.3 | 26.2 | 26.2 | 26.2 | 26.1 | 26.1 | .01 | |
| 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | 2.08 | .25 | |
| 3.87 | 3.84 | 3.83 | 3.82 | 3.80 | 3.80 | 3.79 | 3.78 | 3.78 | 3.77 | 3.76 | 3.76 | .10 | 4 |
| 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.70 | 5.69 | 5.66 | 5.66 | 5.65 | 5.64 | 5.63 | .05 | |
| 14.2 | 14.0 | 13.9 | 13.8 | 13.7 | 13.7 | 13.7 | 13.6 | 13.6 | 13.5 | 13.5 | 13.5 | .01 | |
| 1.89 | 1.88 | 1.88 | 1.88 | 1.88 | 1.88 | 1.87 | 1.87 | 1.87 | 1.87 | 1.87 | 1.87 | .25 | |
| 3.24 | 3.21 | 3.19 | 3.17 | 3.16 | 3.15 | 3.14 | 3.13 | 3.12 | 3.12 | 3.11 | 3.10 | .10 | 5 |
| 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.44 | 4.43 | 4.41 | 4.40 | 4.39 | 4.37 | 4.36 | .05 | |
| 9.72 | 9.55 | 9.47 | 9.38 | 9.29 | 9.24 | 9.20 | 9.13 | 9.11 | 9.08 | 9.04 | 9.02 | .01 | |
| 1.76 | 1.76 | 1.75 | 1.75 | 1.75 | 1.75 | 1.74 | 1.74 | 1.74 | 1.74 | 1.74 | 1.74 | .25 | |
| 2.87 | 2.84 | 2.82 | 2.80 | 2.78 | 2.77 | 2.76 | 2.75 | 2.74 | 2.73 | 2.73 | 2.72 | .10 | 6 |
| 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.75 | 3.74 | 3.71 | 3.70 | 3.69 | 3.68 | 3.67 | .05 | |
| 7.56 | 7.40 | 7.31 | 7.23 | 7.14 | 7.09 | 7.06 | 6.99 | 6.97 | 6.93 | 6.90 | 6.88 | .01 | |
| 1.68 | 1.67 | 1.67 | 1.66 | 1.66 | 1.66 | 1.65 | 1.65 | 1.65 | 1.65 | 1.65 | 1.65 | .25 | |
| 2.63 | 2.59 | 2.58 | 2.56 | 2.54 | 2.52 | 2.51 | 2.50 | 2.49 | 2.48 | 2.48 | 2.47 | .10 | 7 |
| 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.32 | 3.30 | 3.27 | 3.27 | 3.25 | 3.24 | 3.23 | .05 | |
| 6.31 | 6.16 | 6.07 | 5.99 | 5.91 | 5.86 | 5.82 | 5.75 | 5.74 | 5.70 | 5.67 | 5.65 | .01 | |
| 1.62 | 1.61 | 1.60 | 1.60 | 1.59 | 1.59 | 1.59 | 1.58 | 1.58 | 1.58 | 1.58 | 1.58 | .25 | |
| 2.46 | 2.42 | 2.40 | 2.38 | 2.36 | 2.35 | 2.34 | 2.32 | 2.32 | 2.31 | 2.30 | 2.29 | .10 | 8 |
| 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 2.02 | 3.01 | 2.97 | 2.97 | 2.95 | 2.94 | 2.93 | .05 | |
| 5.52 | 5.36 | 5.28 | 5.20 | 5.12 | 5.07 | 5.03 | 4.96 | 4.95 | 4.91 | 4.88 | 4.86 | .01 | |
| 1.57 | 1.56 | 1.56 | 1.55 | 1.55 | 1.54 | 1.54 | 1.53 | 1.53 | 1.53 | 1.53 | 1.53 | .25 | |
| 2.34 | 2.30 | 2.28 | 2.25 | 2.23 | 2.22 | 2.21 | 2.19 | 2.18 | 2.17 | 2.17 | 2.16 | .10 | 9 |
| 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.80 | 2.79 | 2.76 | 2.75 | 2.73 | 2.72 | 2.71 | .05 | |
| 4.96 | 4.81 | 4.73 | 4.65 | 4.57 | 4.52 | 4.48 | 4.42 | 4.40 | 4.36 | 4.33 | 4.31 | .01 | |

(Continued)

TABLE D.3   Upper Percentage Points of the *F* Distribution   (*Continued*)

| df for denominator $N_2$ | Pr | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | df for numerator $N_1$ | | | | | | | |
| 10 | .25 | 1.49 | 1.60 | 1.60 | 1.59 | 1.59 | 1.58 | 1.57 | 1.56 | 1.56 | 1.55 | 1.55 | 1.54 |
| | .10 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 | 2.32 | 2.30 | 2.28 |
| | .05 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.94 | 2.91 |
| | .01 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.77 | 4.71 |
| 11 | .25 | 1.47 | 1.58 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 | 1.53 | 1.53 | 1.52 | 1.52 | 1.51 |
| | .10 | 3.23 | 2.86 | 2.66 | 2.54 | 2.45 | 2.39 | 2.34 | 2.30 | 2.27 | 2.25 | 2.23 | 2.21 |
| | .05 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.82 | 2.79 |
| | .01 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.46 | 4.40 |
| 12 | .25 | 1.46 | 1.56 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.51 | 1.51 | 1.50 | 1.50 | 1.49 |
| | .10 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 | 2.19 | 2.17 | 2.15 |
| | .05 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.72 | 2.69 |
| | .01 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.22 | 4.16 |
| 13 | .25 | 1.45 | 1.55 | 1.55 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.49 | 1.48 | 1.47 | 1.47 |
| | .10 | 3.14 | 2.76 | 2.56 | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 | 2.14 | 2.12 | 2.10 |
| | .05 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.63 | 2.60 |
| | .01 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 4.02 | 3.96 |
| 14 | .25 | 1.44 | 1.53 | 1.53 | 1.52 | 1.51 | 1.50 | 1.49 | 1.48 | 1.47 | 1.46 | 1.46 | 1.45 |
| | .10 | 3.10 | 2.73 | 2.52 | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 | 2.10 | 2.08 | 2.05 |
| | .05 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.57 | 2.53 |
| | .01 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.86 | 3.80 |
| 15 | .25 | 1.43 | 1.52 | 1.52 | 1.51 | 1.49 | 1.48 | 1.47 | 1.46 | 1.46 | 1.45 | 1.44 | 1.44 |
| | .10 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 | 2.06 | 2.04 | 2.02 |
| | .05 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.51 | 2.48 |
| | .01 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.73 | 3.67 |
| 16 | .25 | 1.42 | 1.51 | 1.51 | 1.50 | 1.48 | 1.47 | 1.46 | 1.45 | 1.44 | 1.44 | 1.44 | 1.43 |
| | .10 | 3.05 | 2.67 | 2.46 | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 | 2.03 | 2.01 | 1.99 |
| | .05 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.46 | 2.42 |
| | .01 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.62 | 3.55 |
| 17 | .25 | 1.42 | 1.51 | 1.50 | 1.49 | 1.47 | 1.46 | 1.45 | 1.44 | 1.43 | 1.43 | 1.42 | 1.41 |
| | .10 | 3.03 | 2.64 | 2.44 | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 | 2.00 | 1.98 | 1.96 |
| | .05 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.41 | 2.38 |
| | .01 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.52 | 3.46 |
| 18 | .25 | 1.41 | 1.50 | 1.49 | 1.48 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.42 | 1.41 | 1.40 |
| | .10 | 3.01 | 2.62 | 2.42 | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.98 | 1.96 | 1.93 |
| | .05 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.37 | 2.34 |
| | .01 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.43 | 3.37 |
| 19 | .25 | 1.41 | 1.49 | 1.49 | 1.47 | 1.46 | 1.44 | 1.43 | 1.42 | 1.41 | 1.41 | 1.40 | 1.40 |
| | .10 | 2.99 | 2.61 | 2.40 | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 | 1.96 | 1.94 | 1.91 |
| | .05 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.34 | 2.31 |
| | .01 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.36 | 3.30 |
| 20 | .25 | 1.40 | 1.49 | 1.48 | 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.41 | 1.40 | 1.39 | 1.39 |
| | .10 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 | 1.94 | 1.92 | 1.89 |
| | .05 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.31 | 2.28 |
| | .01 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.29 | 3.23 |

# F-table (continued)

| | | | | df for numerator $N_1$ | | | | | | | | | df for denominator $N_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 20 | 24 | 30 | 40 | 50 | 60 | 100 | 120 | 200 | 500 | ∞ | Pr | |
| | | | | | | | | 1.49 | 1.49 | 1.48 | 1.48 | .25 | |
| 1.53 | 1.52 | 1.52 | 1.51 | 1.51 | 1.50 | 1.50 | 1.49 | 2.08 | 2.07 | 2.06 | 2.06 | .10 | 10 |
| 2.24 | 2.20 | 2.18 | 2.16 | 2.13 | 2.12 | 2.11 | 2.09 | 2.58 | 2.56 | 2.55 | 2.54 | .05 | |
| 2.85 | 2.77 | 2.74 | 2.70 | 2.66 | 2.64 | 2.62 | 2.59 | 4.00 | 3.96 | 3.93 | 3.91 | .01 | |
| 4.56 | 4.41 | 4.33 | 4.25 | 4.17 | 4.12 | 4.08 | 4.01 | | | | | | |
| | | | | | | | | 1.46 | 1.46 | 1.45 | 1.45 | .25 | |
| 1.50 | 1.49 | 1.49 | 1.48 | 1.47 | 1.47 | 1.47 | 1.46 | 2.00 | 1.99 | 1.98 | 1.97 | .10 | 11 |
| 2.17 | 2.12 | 2.10 | 2.08 | 2.05 | 2.04 | 2.03 | 2.00 | 2.45 | 2.43 | 2.42 | 2.40 | .05 | |
| 2.72 | 2.65 | 2.61 | 2.57 | 2.53 | 2.51 | 2.49 | 2.46 | 3.69 | 3.66 | 3.62 | 3.60 | .01 | |
| 4.25 | 4.10 | 4.02 | 3.94 | 3.86 | 3.81 | 3.78 | 3.71 | | | | | | |
| | | | | | | | | 1.43 | 1.43 | 1.42 | 1.42 | .25 | |
| 1.48 | 1.47 | 1.46 | 1.45 | 1.45 | 1.44 | 1.44 | 1.43 | 1.93 | 1.92 | 1.91 | 1.90 | .10 | 12 |
| 2.10 | 2.06 | 2.04 | 2.01 | 1.99 | 1.97 | 1.96 | 1.94 | 2.34 | 2.32 | 2.31 | 2.30 | .05 | |
| 2.62 | 2.54 | 2.51 | 2.47 | 2.43 | 2.40 | 2.38 | 2.35 | 3.45 | 3.41 | 3.38 | 3.36 | .01 | |
| 4.01 | 3.86 | 3.78 | 3.70 | 3.62 | 3.57 | 3.54 | 3.47 | | | | | | |
| | | | | | | | | 1.41 | 1.40 | 1.40 | 1.40 | .25 | |
| 1.46 | 1.45 | 1.44 | 1.43 | 1.42 | 1.42 | 1.42 | 1.41 | 1.88 | 1.86 | 1.85 | 1.85 | .10 | 13 |
| 2.05 | 2.01 | 1.98 | 1.96 | 1.93 | 1.92 | 1.90 | 1.88 | 2.25 | 2.23 | 2.22 | 2.21 | .05 | |
| 2.53 | 2.46 | 2.42 | 2.38 | 2.34 | 2.31 | 2.30 | 2.26 | 3.25 | 3.22 | 3.19 | 3.17 | .01 | |
| 3.82 | 3.66 | 3.59 | 3.51 | 3.43 | 3.38 | 3.34 | 3.27 | | | | | | |
| | | | | | | | | 1.39 | 1.39 | 1.38 | 1.38 | .25 | |
| 1.44 | 1.43 | 1.42 | 1.41 | 1.41 | 1.40 | 1.40 | 1.39 | 1.83 | 1.82 | 1.80 | 1.80 | .10 | 14 |
| 2.01 | 1.96 | 1.94 | 1.91 | 1.89 | 1.87 | 1.86 | 1.83 | 2.18 | 2.16 | 2.14 | 2.13 | .05 | |
| 2.46 | 2.39 | 2.35 | 2.31 | 2.27 | 2.24 | 2.22 | 2.19 | 3.09 | 3.06 | 3.03 | 3.00 | .01 | |
| 3.66 | 3.51 | 3.43 | 3.35 | 3.27 | 3.22 | 3.18 | 3.11 | | | | | | |
| | | | | | | | | 1.37 | 1.37 | 1.36 | 1.36 | .25 | |
| 1.43 | 1.41 | 1.41 | 1.40 | 1.39 | 1.39 | 1.38 | 1.38 | 1.79 | 1.77 | 1.76 | 1.76 | .10 | 15 |
| 1.97 | 1.92 | 1.90 | 1.87 | 1.85 | 1.83 | 1.82 | 1.79 | 2.11 | 2.10 | 2.08 | 2.07 | .05 | |
| 2.40 | 2.33 | 2.29 | 2.25 | 2.20 | 2.18 | 2.16 | 2.12 | 2.96 | 2.92 | 2.89 | 2.87 | .01 | |
| 3.52 | 3.37 | 3.29 | 3.21 | 3.13 | 3.08 | 3.05 | 2.98 | | | | | | |
| | | | | | | | | 1.35 | 1.35 | 1.34 | 1.34 | .25 | |
| 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.37 | 1.36 | 1.36 | 1.75 | 1.74 | 1.73 | 1.72 | .10 | 16 |
| 1.94 | 1.89 | 1.87 | 1.84 | 1.81 | 1.79 | 1.78 | 1.76 | 2.06 | 2.04 | 2.02 | 2.01 | .05 | |
| 2.35 | 2.28 | 2.24 | 2.19 | 2.15 | 2.12 | 2.11 | 2.07 | 2.84 | 2.81 | 2.78 | 2.75 | .01 | |
| 3.41 | 3.26 | 3.18 | 3.10 | 3.02 | 2.97 | 2.93 | 2.86 | | | | | | |
| | | | | | | | | 1.34 | 1.34 | 1.33 | 1.33 | .25 | |
| 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.35 | 1.34 | 1.72 | 1.71 | 1.69 | 1.69 | .10 | 17 |
| 1.91 | 1.86 | 1.84 | 1.81 | 1.78 | 1.76 | 1.75 | 1.73 | 2.01 | 1.99 | 1.97 | 1.96 | .05 | |
| 2.31 | 2.23 | 2.19 | 2.15 | 2.10 | 2.08 | 2.06 | 2.02 | 2.75 | 2.71 | 2.68 | 2.65 | .01 | |
| 3.31 | 3.16 | 3.08 | 3.00 | 2.92 | 2.87 | 2.83 | 2.76 | | | | | | |
| | | | | | | | | 1.33 | 1.32 | 1.32 | 1.32 | .25 | |
| 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.34 | 1.33 | 1.69 | 1.68 | 1.67 | 1.66 | .10 | 18 |
| 1.89 | 1.84 | 1.81 | 1.78 | 1.75 | 1.74 | 1.72 | 1.70 | 1.97 | 1.95 | 1.93 | 1.92 | .05 | |
| 2.27 | 2.19 | 2.15 | 2.11 | 2.06 | 2.04 | 2.02 | 1.98 | 2.66 | 2.62 | 2.59 | 2.57 | .01 | |
| 3.23 | 3.08 | 3.00 | 2.92 | 2.84 | 2.78 | 2.75 | 2.68 | | | | | | |
| | | | | | | | | 1.32 | 1.31 | 1.31 | 1.30 | .25 | |
| 1.38 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.33 | 1.32 | 1.67 | 1.65 | 1.64 | 1.63 | .10 | 19 |
| 1.86 | 1.81 | 1.79 | 1.76 | 1.73 | 1.71 | 1.70 | 1.67 | 1.93 | 1.91 | 1.89 | 1.88 | .05 | |
| 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 2.00 | 1.98 | 1.94 | 2.58 | 2.55 | 2.51 | 2.49 | .01 | |
| 3.15 | 3.00 | 2.92 | 2.84 | 2.76 | 2.71 | 2.67 | 2.60 | | | | | | |
| | | | | | | | | 1.31 | 1.30 | 1.30 | 1.29 | .25 | |
| 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.33 | 1.32 | 1.31 | 1.64 | 1.63 | 1.62 | 1.61 | .10 | 20 |
| 1.84 | 1.79 | 1.77 | 1.74 | 1.71 | 1.69 | 1.68 | 1.65 | 1.90 | 1.88 | 1.86 | 1.84 | .05 | |
| 2.20 | 2.12 | 2.08 | 2.04 | 1.99 | 1.97 | 1.95 | 1.91 | 2.52 | 2.48 | 2.44 | 2.42 | .01 | |
| 3.09 | 2.94 | 2.86 | 2.78 | 2.69 | 2.64 | 2.61 | 2.54 | | | | | | |

**TABLE D.3**  Upper Percentage Points of the *F* Distribution    (*Continued*)

| df for denominator $N_2$ | Pr | df for numerator $N_1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 22 | .25 | 1.40 | 1.48 | 1.47 | 1.45 | 1.44 | 1.42 | 1.41 | 1.40 | 1.39 | 1.39 | 1.38 | 1.37 |
| | .10 | 2.95 | 2.56 | 2.35 | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 | 1.90 | 1.88 | 1.86 |
| | .05 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.26 | 2.23 |
| | .01 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.18 | 3.12 |
| 24 | .25 | 1.39 | 1.47 | 1.46 | 1.44 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.38 | 1.37 | 1.36 |
| | .10 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 | 1.88 | 1.85 | 1.83 |
| | .05 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.21 | 2.18 |
| | .01 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.09 | 3.03 |
| 26 | .25 | 1.38 | 1.46 | 1.45 | 1.44 | 1.42 | 1.41 | 1.39 | 1.38 | 1.37 | 1.37 | 1.36 | 1.35 |
| | .10 | 2.91 | 2.52 | 2.31 | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 | 1.86 | 1.84 | 1.81 |
| | .05 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.18 | 2.15 |
| | .01 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 3.02 | 2.96 |
| 28 | .25 | 1.38 | 1.46 | 1.45 | 1.43 | 1.41 | 1.40 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.34 |
| | .10 | 2.89 | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 | 1.84 | 1.81 | 1.79 |
| | .05 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.15 | 2.12 |
| | .01 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.96 | 2.90 |
| 30 | .25 | 1.38 | 1.45 | 1.44 | 1.42 | 1.41 | 1.39 | 1.38 | 1.37 | 1.36 | 1.35 | 1.35 | 1.34 |
| | .10 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 | 1.82 | 1.79 | 1.77 |
| | .05 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.13 | 2.09 |
| | .01 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.91 | 2.84 |
| 40 | .25 | 1.36 | 1.44 | 1.42 | 1.40 | 1.39 | 1.37 | 1.36 | 1.35 | 1.34 | 1.33 | 1.32 | 1.31 |
| | .10 | 2.84 | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 | 1.76 | 1.73 | 1.71 |
| | .05 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.04 | 2.00 |
| | .01 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.73 | 2.66 |
| 60 | .25 | 1.35 | 1.42 | 1.41 | 1.38 | 1.37 | 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.29 |
| | .10 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 | 1.71 | 1.68 | 1.66 |
| | .05 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.95 | 1.92 |
| | .01 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.56 | 2.50 |
| 120 | .25 | 1.34 | 1.40 | 1.39 | 1.37 | 1.35 | 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.27 | 1.26 |
| | .10 | 2.75 | 2.35 | 2.13 | 1.99 | 1.90 | 1.82 | 1.77 | 1.72 | 1.68 | 1.65 | 1.62 | 1.60 |
| | .05 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.17 | 2.09 | 2.02 | 1.96 | 1.91 | 1.87 | 1.83 |
| | .01 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.40 | 2.34 |
| 200 | .25 | 1.33 | 1.39 | 1.38 | 1.36 | 1.34 | 1.32 | 1.31 | 1.29 | 1.28 | 1.27 | 1.26 | 1.25 |
| | .10 | 2.73 | 2.33 | 2.11 | 1.97 | 1.88 | 1.80 | 1.75 | 1.70 | 1.66 | 1.63 | 1.60 | 1.57 |
| | .05 | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.84 | 1.80 |
| | .01 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 | 2.41 | 2.34 | 2.27 |
| ∞ | .25 | 1.32 | 1.39 | 1.37 | 1.35 | 1.33 | 1.31 | 1.29 | 1.28 | 1.27 | 1.25 | 1.24 | 1.24 |
| | .10 | 2.71 | 2.30 | 2.08 | 1.94 | 1.85 | 1.77 | 1.72 | 1.67 | 1.63 | 1.60 | 1.57 | 1.55 |
| | .05 | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.79 | 1.75 |
| | .01 | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.25 | 2.18 |

# F-table continued

| 15 | 20 | 24 | 30 | 40 | 50 | 60 | 100 | 120 | 200 | 500 | ∞ | Pr | N₂ |
|----|----|----|----|----|----|----|-----|-----|-----|-----|---|----|----|
| 1.36 | 1.34 | 1.33 | 1.32 | 1.31 | 1.31 | 1.30 | 1.30 | 1.30 | 1.29 | 1.29 | 1.28 | .25 | 22 |
| 1.81 | 1.76 | 1.73 | 1.70 | 1.67 | 1.65 | 1.64 | 1.61 | 1.60 | 1.59 | 1.58 | 1.57 | .10 | |
| 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.91 | 1.89 | 1.85 | 1.84 | 1.82 | 1.80 | 1.78 | .05 | |
| 2.98 | 2.83 | 2.75 | 2.67 | 2.58 | 2.53 | 2.50 | 2.42 | 2.40 | 2.36 | 2.33 | 2.31 | .01 | |
| 1.35 | 1.33 | 1.32 | 1.31 | 1.30 | 1.29 | 1.29 | 1.28 | 1.28 | 1.27 | 1.27 | 1.26 | .25 | 24 |
| 1.78 | 1.73 | 1.70 | 1.67 | 1.64 | 1.62 | 1.61 | 1.58 | 1.57 | 1.56 | 1.54 | 1.53 | .10 | |
| 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.86 | 1.84 | 1.80 | 1.79 | 1.77 | 1.75 | 1.73 | .05 | |
| 2.89 | 2.74 | 2.66 | 2.58 | 2.49 | 2.44 | 2.40 | 2.33 | 2.31 | 2.27 | 2.24 | 2.21 | .01 | |
| 1.34 | 1.32 | 1.31 | 1.30 | 1.29 | 1.28 | 1.28 | 1.26 | 1.26 | 1.26 | 1.25 | 1.25 | .25 | 26 |
| 1.76 | 1.71 | 1.68 | 1.65 | 1.61 | 1.59 | 1.58 | 1.55 | 1.54 | 1.53 | 1.51 | 1.50 | .10 | |
| 2.07 | 1.99 | 1.95 | 1.90 | 1.85 | 1.82 | 1.80 | 1.76 | 1.75 | 1.73 | 1.71 | 1.69 | .05 | |
| 2.81 | 2.66 | 2.58 | 2.50 | 2.42 | 2.36 | 2.33 | 2.25 | 2.23 | 2.19 | 2.16 | 2.13 | .01 | |
| 1.33 | 1.31 | 1.30 | 1.29 | 1.28 | 1.27 | 1.27 | 1.26 | 1.25 | 1.25 | 1.24 | 1.24 | .25 | 28 |
| 1.74 | 1.69 | 1.66 | 1.63 | 1.59 | 1.57 | 1.56 | 1.53 | 1.52 | 1.50 | 1.49 | 1.48 | .10 | |
| 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.79 | 1.77 | 1.73 | 1.71 | 1.69 | 1.67 | 1.65 | .05 | |
| 2.75 | 2.60 | 2.52 | 2.44 | 2.35 | 2.30 | 2.26 | 2.19 | 2.17 | 2.13 | 2.09 | 2.06 | .01 | |
| 1.32 | 1.30 | 1.29 | 1.28 | 1.27 | 1.26 | 1.26 | 1.25 | 1.24 | 1.24 | 1.23 | 1.23 | .25 | 30 |
| 1.72 | 1.67 | 1.64 | 1.61 | 1.57 | 1.55 | 1.54 | 1.51 | 1.50 | 1.48 | 1.47 | 1.46 | .10 | |
| 2.01 | 1.93 | 1.89 | 1.84 | 1.79 | 1.76 | 1.74 | 1.70 | 1.68 | 1.66 | 1.64 | 1.62 | .05 | |
| 2.70 | 2.55 | 2.47 | 2.39 | 2.30 | 2.25 | 2.21 | 2.13 | 2.11 | 2.07 | 2.03 | 2.01 | .01 | |
| 1.30 | 1.28 | 1.26 | 1.25 | 1.24 | 1.23 | 1.22 | 1.21 | 1.21 | 1.20 | 1.19 | 1.19 | .25 | 40 |
| 1.66 | 1.61 | 1.57 | 1.54 | 1.51 | 1.48 | 1.47 | 1.43 | 1.42 | 1.41 | 1.39 | 1.38 | .10 | |
| 1.92 | 1.84 | 1.79 | 1.74 | 1.69 | 1.66 | 1.64 | 1.59 | 1.58 | 1.55 | 1.53 | 1.51 | .05 | |
| 2.52 | 2.37 | 2.29 | 2.20 | 2.11 | 2.06 | 2.02 | 1.94 | 1.92 | 1.87 | 1.83 | 1.80 | .01 | |
| 1.27 | 1.25 | 1.24 | 1.22 | 1.21 | 1.20 | 1.19 | 1.17 | 1.17 | 1.16 | 1.15 | 1.15 | .25 | 60 |
| 1.60 | 1.54 | 1.51 | 1.48 | 1.44 | 1.41 | 1.40 | 1.36 | 1.35 | 1.33 | 1.31 | 1.29 | .10 | |
| 1.84 | 1.75 | 1.70 | 1.65 | 1.59 | 1.56 | 1.53 | 1.48 | 1.47 | 1.44 | 1.41 | 1.39 | .05 | |
| 2.35 | 2.20 | 2.12 | 2.03 | 1.94 | 1.88 | 1.84 | 1.75 | 1.73 | 1.68 | 1.63 | 1.60 | .01 | |
| 1.24 | 1.22 | 1.21 | 1.19 | 1.18 | 1.17 | 1.16 | 1.14 | 1.13 | 1.12 | 1.11 | 1.10 | .25 | 120 |
| 1.55 | 1.48 | 1.45 | 1.41 | 1.37 | 1.34 | 1.32 | 1.27 | 1.26 | 1.24 | 1.21 | 1.19 | .10 | |
| 1.75 | 1.66 | 1.61 | 1.55 | 1.50 | 1.46 | 1.43 | 1.37 | 1.35 | 1.32 | 1.28 | 1.25 | .05 | |
| 2.19 | 2.03 | 1.95 | 1.86 | 1.76 | 1.70 | 1.66 | 1.56 | 1.53 | 1.48 | 1.42 | 1.38 | .01 | |
| 1.23 | 1.21 | 1.20 | 1.18 | 1.16 | 1.14 | 1.12 | 1.11 | 1.10 | 1.09 | 1.08 | 1.06 | .25 | 200 |
| 1.52 | 1.46 | 1.42 | 1.38 | 1.34 | 1.31 | 1.28 | 1.24 | 1.22 | 1.20 | 1.17 | 1.14 | .10 | |
| 1.72 | 1.62 | 1.57 | 1.52 | 1.46 | 1.41 | 1.39 | 1.32 | 1.29 | 1.26 | 1.22 | 1.19 | .05 | |
| 2.13 | 1.97 | 1.89 | 1.79 | 1.69 | 1.63 | 1.58 | 1.48 | 1.44 | 1.39 | 1.33 | 1.28 | .01 | |
| 1.22 | 1.19 | 1.18 | 1.16 | 1.14 | 1.13 | 1.12 | 1.09 | 1.08 | 1.07 | 1.04 | 1.00 | .25 | ∞ |
| 1.49 | 1.42 | 1.38 | 1.34 | 1.30 | 1.26 | 1.24 | 1.18 | 1.17 | 1.13 | 1.08 | 1.00 | .10 | |
| 1.67 | 1.57 | 1.52 | 1.46 | 1.39 | 1.35 | 1.32 | 1.24 | 1.22 | 1.17 | 1.11 | 1.00 | .05 | |
| 2.04 | 1.88 | 1.79 | 1.70 | 1.59 | 1.52 | 1.47 | 1.36 | 1.32 | 1.25 | 1.15 | 1.00 | .01 | |

df for numerator N₁ — df for denominator N₂

## TABLE D.5A   Durbin–Watson $d$ Statistic: Significance Points of $d_L$ and $d_U$ at 0.05 Level of Significance

| | $k'=1$ | | $k'=2$ | | $k'=3$ | | $k'=4$ | | $k'=5$ | | $k'=6$ | | $k'=7$ | | $k'=8$ | | $k'=9$ | | $k'=10$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ |
| 6 | 0.610 | 1.400 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 7 | 0.700 | 1.356 | 0.467 | 1.896 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 8 | 0.763 | 1.332 | 0.559 | 1.777 | 0.368 | 2.287 | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 9 | 0.824 | 1.320 | 0.629 | 1.699 | 0.455 | 2.128 | 0.296 | 2.588 | — | — | — | — | — | — | — | — | — | — | — | — |
| 10 | 0.879 | 1.320 | 0.697 | 1.641 | 0.525 | 2.016 | 0.376 | 2.414 | 0.243 | 2.822 | — | — | — | — | — | — | — | — | — | — |
| 11 | 0.927 | 1.324 | 0.658 | 1.604 | 0.595 | 1.928 | 0.444 | 2.283 | 0.316 | 2.645 | 0.203 | 3.005 | — | — | — | — | — | — | — | — |
| 12 | 0.971 | 1.331 | 0.812 | 1.579 | 0.658 | 1.864 | 0.512 | 2.177 | 0.379 | 2.506 | 0.268 | 2.832 | 0.171 | 3.149 | — | — | — | — | — | — |
| 13 | 1.010 | 1.340 | 0.861 | 1.562 | 0.715 | 1.816 | 0.574 | 2.094 | 0.445 | 2.390 | 0.328 | 2.692 | 0.230 | 2.985 | 0.147 | 3.266 | — | — | — | — |
| 14 | 1.045 | 1.350 | 0.905 | 1.551 | 0.767 | 1.779 | 0.632 | 2.030 | 0.505 | 2.296 | 0.389 | 2.572 | 0.286 | 2.848 | 0.200 | 3.111 | 0.127 | 3.360 | — | — |
| 15 | 1.077 | 1.361 | 0.946 | 1.543 | 0.814 | 1.750 | 0.685 | 1.977 | 0.562 | 2.220 | 0.447 | 2.472 | 0.343 | 2.727 | 0.251 | 2.979 | 0.175 | 3.216 | 0.111 | 3.438 |
| 16 | 1.106 | 1.371 | 0.982 | 1.539 | 0.857 | 1.728 | 0.734 | 1.935 | 0.615 | 2.157 | 0.502 | 2.388 | 0.398 | 2.624 | 0.304 | 2.860 | 0.222 | 3.090 | 0.155 | 3.304 |
| 17 | 1.133 | 1.381 | 1.015 | 1.536 | 0.897 | 1.710 | 0.779 | 1.900 | 0.664 | 2.104 | 0.554 | 2.318 | 0.451 | 2.537 | 0.356 | 2.757 | 0.272 | 2.975 | 0.198 | 3.184 |
| 18 | 1.158 | 1.391 | 1.046 | 1.535 | 0.933 | 1.696 | 0.820 | 1.872 | 0.710 | 2.060 | 0.603 | 2.257 | 0.502 | 2.461 | 0.407 | 2.667 | 0.321 | 2.873 | 0.244 | 3.073 |
| 19 | 1.180 | 1.401 | 1.074 | 1.536 | 0.967 | 1.685 | 0.859 | 1.848 | 0.752 | 2.023 | 0.649 | 2.206 | 0.549 | 2.396 | 0.456 | 2.589 | 0.369 | 2.783 | 0.290 | 2.974 |
| 20 | 1.201 | 1.411 | 1.100 | 1.537 | 0.998 | 1.676 | 0.894 | 1.828 | 0.792 | 1.991 | 0.692 | 2.162 | 0.595 | 2.339 | 0.502 | 2.521 | 0.416 | 2.704 | 0.336 | 2.885 |
| 21 | 1.221 | 1.420 | 1.125 | 1.538 | 1.026 | 1.669 | 0.927 | 1.812 | 0.829 | 1.964 | 0.732 | 2.124 | 0.637 | 2.290 | 0.547 | 2.460 | 0.461 | 2.633 | 0.380 | 2.806 |
| 22 | 1.239 | 1.429 | 1.147 | 1.541 | 1.053 | 1.664 | 0.958 | 1.797 | 0.863 | 1.940 | 0.769 | 2.090 | 0.677 | 2.246 | 0.588 | 2.407 | 0.504 | 2.571 | 0.424 | 2.734 |
| 23 | 1.257 | 1.437 | 1.168 | 1.543 | 1.078 | 1.660 | 0.986 | 1.785 | 0.895 | 1.920 | 0.804 | 2.061 | 0.715 | 2.208 | 0.628 | 2.360 | 0.545 | 2.514 | 0.465 | 2.670 |
| 24 | 1.273 | 1.446 | 1.188 | 1.546 | 1.101 | 1.656 | 1.013 | 1.775 | 0.925 | 1.902 | 0.837 | 2.035 | 0.751 | 2.174 | 0.666 | 2.318 | 0.584 | 2.464 | 0.506 | 2.613 |
| 25 | 1.288 | 1.454 | 1.206 | 1.550 | 1.123 | 1.654 | 1.038 | 1.767 | 0.953 | 1.886 | 0.868 | 2.012 | 0.784 | 2.144 | 0.702 | 2.280 | 0.621 | 2.419 | 0.544 | 2.560 |
| 26 | 1.302 | 1.461 | 1.224 | 1.553 | 1.143 | 1.652 | 1.062 | 1.759 | 0.979 | 1.873 | 0.897 | 1.992 | 0.816 | 2.117 | 0.735 | 2.246 | 0.657 | 2.379 | 0.581 | 2.513 |
| 27 | 1.316 | 1.469 | 1.240 | 1.556 | 1.162 | 1.651 | 1.084 | 1.753 | 1.004 | 1.861 | 0.925 | 1.974 | 0.845 | 2.093 | 0.767 | 2.216 | 0.691 | 2.342 | 0.616 | 2.470 |
| 28 | 1.328 | 1.476 | 1.255 | 1.560 | 1.181 | 1.650 | 1.104 | 1.747 | 1.028 | 1.850 | 0.951 | 1.958 | 0.874 | 2.071 | 0.798 | 2.188 | 0.723 | 2.309 | 0.650 | 2.431 |
| 29 | 1.341 | 1.483 | 1.270 | 1.563 | 1.198 | 1.650 | 1.124 | 1.743 | 1.050 | 1.841 | 0.975 | 1.944 | 0.900 | 2.052 | 0.826 | 2.164 | 0.753 | 2.278 | 0.682 | 2.396 |
| 30 | 1.352 | 1.489 | 1.284 | 1.567 | 1.214 | 1.650 | 1.143 | 1.739 | 1.071 | 1.833 | 0.998 | 1.931 | 0.926 | 2.034 | 0.854 | 2.141 | 0.782 | 2.251 | 0.712 | 2.363 |
| 31 | 1.363 | 1.496 | 1.297 | 1.570 | 1.229 | 1.650 | 1.160 | 1.735 | 1.090 | 1.825 | 1.020 | 1.920 | 0.950 | 2.018 | 0.879 | 2.120 | 0.810 | 2.226 | 0.741 | 2.333 |
| 32 | 1.373 | 1.502 | 1.309 | 1.574 | 1.244 | 1.650 | 1.177 | 1.732 | 1.109 | 1.819 | 1.041 | 1.909 | 0.972 | 2.004 | 0.904 | 2.102 | 0.836 | 2.203 | 0.769 | 2.306 |
| 33 | 1.383 | 1.508 | 1.321 | 1.577 | 1.258 | 1.651 | 1.193 | 1.730 | 1.127 | 1.813 | 1.061 | 1.900 | 0.994 | 1.991 | 0.927 | 2.085 | 0.861 | 2.181 | 0.795 | 2.281 |
| 34 | 1.393 | 1.514 | 1.333 | 1.580 | 1.271 | 1.652 | 1.208 | 1.728 | 1.144 | 1.808 | 1.080 | 1.891 | 1.015 | 1.979 | 0.950 | 2.069 | 0.885 | 2.162 | 0.821 | 2.257 |
| 35 | 1.402 | 1.519 | 1.343 | 1.584 | 1.283 | 1.653 | 1.222 | 1.726 | 1.160 | 1.803 | 1.097 | 1.884 | 1.034 | 1.967 | 0.971 | 2.054 | 0.908 | 2.144 | 0.845 | 2.236 |
| 36 | 1.411 | 1.525 | 1.354 | 1.587 | 1.295 | 1.654 | 1.236 | 1.724 | 1.175 | 1.799 | 1.114 | 1.877 | 1.053 | 1.957 | 0.991 | 2.041 | 0.930 | 2.127 | 0.868 | 2.216 |
| 37 | 1.419 | 1.530 | 1.364 | 1.590 | 1.307 | 1.655 | 1.249 | 1.723 | 1.190 | 1.795 | 1.131 | 1.870 | 1.071 | 1.948 | 1.011 | 2.029 | 0.951 | 2.112 | 0.891 | 2.198 |
| 38 | 1.427 | 1.535 | 1.373 | 1.594 | 1.318 | 1.656 | 1.261 | 1.722 | 1.204 | 1.792 | 1.146 | 1.864 | 1.088 | 1.939 | 1.029 | 2.017 | 0.970 | 2.098 | 0.912 | 2.180 |
| 39 | 1.435 | 1.540 | 1.382 | 1.597 | 1.328 | 1.658 | 1.273 | 1.722 | 1.218 | 1.789 | 1.161 | 1.859 | 1.104 | 1.932 | 1.047 | 2.007 | 0.990 | 2.085 | 0.932 | 2.164 |
| 40 | 1.442 | 1.544 | 1.391 | 1.600 | 1.338 | 1.659 | 1.285 | 1.721 | 1.230 | 1.786 | 1.175 | 1.854 | 1.120 | 1.924 | 1.064 | 1.997 | 1.008 | 2.072 | 0.952 | 2.149 |
| 45 | 1.475 | 1.566 | 1.430 | 1.615 | 1.383 | 1.666 | 1.336 | 1.720 | 1.287 | 1.776 | 1.238 | 1.835 | 1.189 | 1.895 | 1.139 | 1.958 | 1.089 | 2.022 | 1.038 | 2.088 |
| 50 | 1.503 | 1.585 | 1.462 | 1.628 | 1.421 | 1.674 | 1.378 | 1.721 | 1.335 | 1.771 | 1.291 | 1.822 | 1.246 | 1.875 | 1.201 | 1.930 | 1.156 | 1.986 | 1.110 | 2.044 |
| 55 | 1.528 | 1.601 | 1.490 | 1.641 | 1.452 | 1.681 | 1.414 | 1.724 | 1.374 | 1.768 | 1.334 | 1.814 | 1.294 | 1.861 | 1.253 | 1.909 | 1.212 | 1.959 | 1.170 | 2.010 |
| 60 | 1.549 | 1.616 | 1.514 | 1.652 | 1.480 | 1.689 | 1.444 | 1.727 | 1.408 | 1.767 | 1.372 | 1.808 | 1.335 | 1.850 | 1.298 | 1.894 | 1.260 | 1.939 | 1.222 | 1.984 |
| 65 | 1.567 | 1.629 | 1.536 | 1.662 | 1.503 | 1.696 | 1.471 | 1.731 | 1.438 | 1.767 | 1.404 | 1.805 | 1.370 | 1.843 | 1.336 | 1.882 | 1.301 | 1.923 | 1.266 | 1.964 |
| 70 | 1.583 | 1.641 | 1.554 | 1.672 | 1.525 | 1.703 | 1.494 | 1.735 | 1.464 | 1.768 | 1.433 | 1.802 | 1.401 | 1.837 | 1.369 | 1.873 | 1.337 | 1.910 | 1.305 | 1.948 |
| 75 | 1.598 | 1.652 | 1.571 | 1.680 | 1.543 | 1.709 | 1.515 | 1.739 | 1.487 | 1.770 | 1.458 | 1.801 | 1.428 | 1.834 | 1.399 | 1.867 | 1.369 | 1.901 | 1.339 | 1.935 |
| 80 | 1.611 | 1.662 | 1.586 | 1.688 | 1.560 | 1.715 | 1.534 | 1.743 | 1.507 | 1.772 | 1.480 | 1.801 | 1.453 | 1.831 | 1.425 | 1.861 | 1.397 | 1.893 | 1.369 | 1.925 |
| 85 | 1.624 | 1.671 | 1.600 | 1.696 | 1.575 | 1.721 | 1.550 | 1.747 | 1.525 | 1.774 | 1.500 | 1.801 | 1.474 | 1.829 | 1.448 | 1.857 | 1.422 | 1.886 | 1.396 | 1.916 |
| 90 | 1.635 | 1.679 | 1.612 | 1.703 | 1.589 | 1.726 | 1.566 | 1.751 | 1.542 | 1.776 | 1.518 | 1.801 | 1.494 | 1.827 | 1.469 | 1.854 | 1.445 | 1.881 | 1.420 | 1.909 |
| 95 | 1.645 | 1.687 | 1.623 | 1.709 | 1.602 | 1.732 | 1.579 | 1.755 | 1.557 | 1.778 | 1.535 | 1.802 | 1.512 | 1.827 | 1.489 | 1.852 | 1.465 | 1.877 | 1.442 | 1.903 |
| 100 | 1.654 | 1.694 | 1.634 | 1.715 | 1.613 | 1.736 | 1.592 | 1.758 | 1.571 | 1.780 | 1.550 | 1.803 | 1.528 | 1.826 | 1.506 | 1.850 | 1.484 | 1.874 | 1.462 | 1.898 |
| 150 | 1.720 | 1.746 | 1.706 | 1.760 | 1.693 | 1.774 | 1.679 | 1.788 | 1.665 | 1.802 | 1.651 | 1.817 | 1.637 | 1.832 | 1.622 | 1.847 | 1.608 | 1.862 | 1.594 | 1.877 |
| 200 | 1.758 | 1.778 | 1.748 | 1.789 | 1.738 | 1.799 | 1.728 | 1.810 | 1.718 | 1.820 | 1.707 | 1.831 | 1.697 | 1.841 | 1.686 | 1.852 | 1.675 | 1.863 | 1.665 | 1.874 |

| n | k'=11 d_L | d_U | k'=12 d_L | d_U | k'=13 d_L | d_U | k'=14 d_L | d_U | k'=15 d_L | d_U | k'=16 d_L | d_U | k'=17 d_L | d_U | k'=18 d_L | d_U | k'=19 d_L | d_U | k'=20 d_L | d_U |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 16 | 0.098 | 3.503 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 17 | 0.138 | 3.378 | 0.087 | 3.557 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 18 | 0.177 | 3.265 | 0.123 | 3.441 | 0.078 | 3.603 | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 19 | 0.220 | 3.159 | 0.160 | 3.335 | 0.111 | 3.496 | 0.070 | 3.642 | — | — | — | — | — | — | — | — | — | — | — | — |
| 20 | 0.263 | 3.063 | 0.200 | 3.234 | 0.145 | 3.395 | 0.100 | 3.542 | 0.063 | 3.676 | — | — | — | — | — | — | — | — | — | — |
| 21 | 0.307 | 2.976 | 0.240 | 3.141 | 0.182 | 3.300 | 0.132 | 3.448 | 0.091 | 3.583 | 0.058 | 3.705 | — | — | — | — | — | — | — | — |
| 22 | 0.349 | 2.897 | 0.281 | 3.057 | 0.220 | 3.211 | 0.166 | 3.358 | 0.120 | 3.495 | 0.083 | 3.619 | 0.052 | 3.731 | — | — | — | — | — | — |
| 23 | 0.391 | 2.826 | 0.322 | 2.979 | 0.259 | 3.128 | 0.202 | 3.272 | 0.153 | 3.409 | 0.110 | 3.535 | 0.076 | 3.650 | 0.048 | 3.753 | — | — | — | — |
| 24 | 0.431 | 2.761 | 0.362 | 2.908 | 0.297 | 3.053 | 0.239 | 3.193 | 0.186 | 3.327 | 0.141 | 3.454 | 0.101 | 3.572 | 0.070 | 3.678 | 0.044 | 3.773 | — | — |
| 25 | 0.470 | 2.702 | 0.400 | 2.844 | 0.335 | 2.983 | 0.275 | 3.119 | 0.221 | 3.251 | 0.172 | 3.376 | 0.130 | 3.494 | 0.094 | 3.604 | 0.065 | 3.702 | 0.041 | 3.790 |
| 26 | 0.508 | 2.649 | 0.438 | 2.784 | 0.373 | 2.919 | 0.312 | 3.051 | 0.256 | 3.179 | 0.205 | 3.303 | 0.160 | 3.420 | 0.120 | 3.531 | 0.087 | 3.632 | 0.060 | 3.724 |
| 27 | 0.544 | 2.600 | 0.475 | 2.730 | 0.409 | 2.859 | 0.348 | 2.987 | 0.291 | 3.112 | 0.238 | 3.233 | 0.191 | 3.349 | 0.149 | 3.460 | 0.112 | 3.563 | 0.081 | 3.658 |
| 28 | 0.578 | 2.555 | 0.510 | 2.680 | 0.445 | 2.805 | 0.383 | 2.928 | 0.325 | 3.050 | 0.271 | 3.168 | 0.222 | 3.283 | 0.178 | 3.392 | 0.138 | 3.495 | 0.104 | 3.592 |
| 29 | 0.612 | 2.515 | 0.544 | 2.634 | 0.479 | 2.755 | 0.418 | 2.874 | 0.359 | 2.992 | 0.305 | 3.107 | 0.254 | 3.219 | 0.208 | 3.327 | 0.166 | 3.431 | 0.129 | 3.528 |
| 30 | 0.643 | 2.477 | 0.577 | 2.592 | 0.512 | 2.708 | 0.451 | 2.823 | 0.392 | 2.937 | 0.337 | 3.050 | 0.286 | 3.160 | 0.238 | 3.266 | 0.195 | 3.368 | 0.156 | 3.465 |
| 31 | 0.674 | 2.443 | 0.608 | 2.553 | 0.545 | 2.665 | 0.484 | 2.776 | 0.425 | 2.887 | 0.370 | 2.996 | 0.317 | 3.103 | 0.269 | 3.208 | 0.224 | 3.309 | 0.183 | 3.406 |
| 32 | 0.703 | 2.411 | 0.638 | 2.517 | 0.576 | 2.625 | 0.515 | 2.733 | 0.457 | 2.840 | 0.401 | 2.946 | 0.349 | 3.050 | 0.299 | 3.153 | 0.253 | 3.252 | 0.211 | 3.348 |
| 33 | 0.731 | 2.382 | 0.668 | 2.484 | 0.606 | 2.588 | 0.546 | 2.692 | 0.488 | 2.796 | 0.432 | 2.899 | 0.379 | 3.000 | 0.329 | 3.100 | 0.283 | 3.198 | 0.239 | 3.293 |
| 34 | 0.758 | 2.355 | 0.695 | 2.454 | 0.634 | 2.554 | 0.575 | 2.654 | 0.518 | 2.754 | 0.462 | 2.854 | 0.409 | 2.954 | 0.359 | 3.051 | 0.312 | 3.147 | 0.267 | 3.240 |
| 35 | 0.783 | 2.330 | 0.722 | 2.425 | 0.662 | 2.521 | 0.604 | 2.619 | 0.547 | 2.716 | 0.492 | 2.813 | 0.439 | 2.910 | 0.388 | 3.005 | 0.340 | 3.099 | 0.295 | 3.190 |
| 36 | 0.808 | 2.306 | 0.748 | 2.398 | 0.689 | 2.492 | 0.631 | 2.586 | 0.575 | 2.680 | 0.520 | 2.774 | 0.467 | 2.868 | 0.417 | 2.961 | 0.369 | 3.053 | 0.323 | 3.142 |
| 37 | 0.831 | 2.285 | 0.772 | 2.374 | 0.714 | 2.464 | 0.657 | 2.555 | 0.602 | 2.646 | 0.548 | 2.738 | 0.495 | 2.829 | 0.445 | 2.920 | 0.397 | 3.009 | 0.351 | 3.097 |
| 38 | 0.854 | 2.265 | 0.796 | 2.351 | 0.739 | 2.438 | 0.683 | 2.526 | 0.628 | 2.614 | 0.575 | 2.703 | 0.522 | 2.792 | 0.472 | 2.880 | 0.424 | 2.968 | 0.378 | 3.054 |
| 39 | 0.875 | 2.246 | 0.819 | 2.329 | 0.763 | 2.413 | 0.707 | 2.499 | 0.653 | 2.585 | 0.600 | 2.671 | 0.549 | 2.757 | 0.499 | 2.843 | 0.451 | 2.929 | 0.404 | 3.013 |
| 40 | 0.896 | 2.228 | 0.840 | 2.309 | 0.785 | 2.391 | 0.731 | 2.473 | 0.678 | 2.557 | 0.626 | 2.641 | 0.575 | 2.724 | 0.525 | 2.808 | 0.477 | 2.892 | 0.430 | 2.974 |
| 45 | 0.988 | 2.156 | 0.938 | 2.225 | 0.887 | 2.296 | 0.838 | 2.367 | 0.788 | 2.439 | 0.740 | 2.512 | 0.692 | 2.586 | 0.644 | 2.659 | 0.598 | 2.733 | 0.553 | 2.807 |
| 50 | 1.064 | 2.103 | 1.019 | 2.163 | 0.973 | 2.225 | 0.927 | 2.287 | 0.882 | 2.350 | 0.836 | 2.414 | 0.792 | 2.479 | 0.747 | 2.544 | 0.703 | 2.610 | 0.660 | 2.675 |
| 55 | 1.129 | 2.062 | 1.087 | 2.116 | 1.045 | 2.170 | 1.003 | 2.225 | 0.961 | 2.281 | 0.919 | 2.338 | 0.877 | 2.396 | 0.836 | 2.454 | 0.795 | 2.512 | 0.754 | 2.571 |
| 60 | 1.184 | 2.031 | 1.145 | 2.079 | 1.106 | 2.127 | 1.068 | 2.177 | 1.029 | 2.227 | 0.990 | 2.278 | 0.951 | 2.330 | 0.913 | 2.382 | 0.874 | 2.434 | 0.836 | 2.487 |
| 65 | 1.231 | 2.006 | 1.195 | 2.049 | 1.160 | 2.093 | 1.124 | 2.138 | 1.088 | 2.183 | 1.052 | 2.229 | 1.016 | 2.276 | 0.980 | 2.323 | 0.944 | 2.371 | 0.908 | 2.419 |
| 70 | 1.272 | 1.986 | 1.239 | 2.026 | 1.206 | 2.066 | 1.172 | 2.106 | 1.139 | 2.148 | 1.105 | 2.189 | 1.072 | 2.232 | 1.038 | 2.275 | 1.005 | 2.318 | 0.971 | 2.362 |
| 75 | 1.308 | 1.970 | 1.277 | 2.006 | 1.247 | 2.043 | 1.215 | 2.080 | 1.184 | 2.118 | 1.153 | 2.156 | 1.121 | 2.195 | 1.090 | 2.235 | 1.058 | 2.275 | 1.027 | 2.315 |
| 80 | 1.340 | 1.957 | 1.311 | 1.991 | 1.283 | 2.024 | 1.253 | 2.059 | 1.224 | 2.093 | 1.195 | 2.129 | 1.165 | 2.165 | 1.136 | 2.201 | 1.106 | 2.238 | 1.076 | 2.275 |
| 85 | 1.369 | 1.946 | 1.342 | 1.977 | 1.315 | 2.009 | 1.287 | 2.040 | 1.260 | 2.073 | 1.232 | 2.105 | 1.205 | 2.139 | 1.177 | 2.172 | 1.149 | 2.206 | 1.121 | 2.241 |
| 90 | 1.395 | 1.937 | 1.369 | 1.966 | 1.344 | 1.995 | 1.318 | 2.025 | 1.292 | 2.055 | 1.266 | 2.085 | 1.240 | 2.116 | 1.213 | 2.148 | 1.187 | 2.179 | 1.160 | 2.211 |
| 95 | 1.418 | 1.929 | 1.394 | 1.956 | 1.370 | 1.984 | 1.345 | 2.012 | 1.321 | 2.040 | 1.296 | 2.068 | 1.271 | 2.097 | 1.247 | 2.126 | 1.222 | 2.156 | 1.197 | 2.186 |
| 100 | 1.439 | 1.923 | 1.416 | 1.948 | 1.393 | 1.974 | 1.371 | 2.000 | 1.347 | 2.026 | 1.324 | 2.053 | 1.301 | 2.080 | 1.277 | 2.108 | 1.253 | 2.135 | 1.229 | 2.164 |
| 150 | 1.579 | 1.892 | 1.564 | 1.908 | 1.550 | 1.924 | 1.535 | 1.940 | 1.519 | 1.956 | 1.504 | 1.972 | 1.489 | 1.989 | 1.474 | 2.006 | 1.458 | 2.023 | 1.443 | 2.040 |
| 200 | 1.654 | 1.885 | 1.643 | 1.896 | 1.632 | 1.908 | 1.621 | 1.919 | 1.610 | 1.931 | 1.599 | 1.943 | 1.588 | 1.955 | 1.576 | 1.967 | 1.565 | 1.979 | 1.554 | 1.991 |

*Note: $n$ = number of observations, $k'$ = number of explanatory variables excluding the constant term.*

Source: This table is an extension of the original Durbin–Watson table and is reproduced from N. E. Savin and K. J. White, "The Durbin–Watson Test for Serial Correlation with Extreme Small Samples or Many Regressors," *Econometrica*, vol. 45, November 1977, pp. 1989–96 and as corrected by R. W. Farebrother, *Econometrica*, vol. 48, September 1980, p. 1554. Reprinted by permission of the Econometric Society.

**EXAMPLE 1**   If $n = 40$ and $k' = 4$, $d_L = 1.285$ and $d_U = 1.721$. If a computed $d$ value is less than 1.285, there is evidence of positive first-order serial correlation; if it is greater than 1.721, there is no evidence of positive first-order serial correlation; but if $d$ lies between the lower and the upper limit, there is inconclusive evidence regarding the presence or absence of positive first-order serial correlation.

TABLE D.5E   **Durbin–Watson d Statistic: Significance Points of $d_L$ and $d_U$ at 0.01 Level of Significance**

| n | k'=1 | | k'=2 | | k'=3 | | k'=4 | | k'=5 | | k'=6 | | k'=7 | | k'=8 | | k'=9 | | k'=10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ |
| 6 | 0.390 | 1.142 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 7 | 0.435 | 1.036 | 0.294 | 1.676 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 8 | 0.497 | 1.003 | 0.345 | 1.489 | 0.229 | 2.102 | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 9 | 0.554 | 0.998 | 0.408 | 1.389 | 0.279 | 1.875 | 0.183 | 2.433 | — | — | — | — | — | — | — | — | — | — | — | — |
| 10 | 0.604 | 1.001 | 0.466 | 1.333 | 0.340 | 1.733 | 0.230 | 2.193 | 0.150 | 2.690 | — | — | — | — | — | — | — | — | — | — |
| 11 | 0.653 | 1.010 | 0.519 | 1.297 | 0.396 | 1.640 | 0.286 | 2.030 | 0.193 | 2.453 | 0.124 | 2.892 | — | — | — | — | — | — | — | — |
| 12 | 0.697 | 1.023 | 0.569 | 1.274 | 0.449 | 1.575 | 0.339 | 1.913 | 0.244 | 2.280 | 0.164 | 2.665 | 0.105 | 3.053 | — | — | — | — | — | — |
| 13 | 0.738 | 1.038 | 0.616 | 1.261 | 0.499 | 1.526 | 0.391 | 1.826 | 0.294 | 2.150 | 0.211 | 2.490 | 0.140 | 2.838 | 0.090 | 3.182 | — | — | — | — |
| 14 | 0.776 | 1.054 | 0.660 | 1.254 | 0.547 | 1.490 | 0.441 | 1.757 | 0.343 | 2.049 | 0.257 | 2.354 | 0.183 | 2.667 | 0.122 | 2.981 | 0.078 | 3.287 | — | — |
| 15 | 0.811 | 1.070 | 0.700 | 1.252 | 0.591 | 1.464 | 0.488 | 1.704 | 0.391 | 1.967 | 0.303 | 2.244 | 0.226 | 2.530 | 0.161 | 2.817 | 0.107 | 3.101 | 0.068 | 3.374 |
| 16 | 0.844 | 1.086 | 0.737 | 1.252 | 0.633 | 1.446 | 0.532 | 1.663 | 0.437 | 1.900 | 0.349 | 2.153 | 0.269 | 2.416 | 0.200 | 2.681 | 0.142 | 2.944 | 0.094 | 3.201 |
| 17 | 0.874 | 1.102 | 0.772 | 1.255 | 0.672 | 1.432 | 0.574 | 1.630 | 0.480 | 1.847 | 0.393 | 2.078 | 0.313 | 2.319 | 0.241 | 2.566 | 0.179 | 2.811 | 0.127 | 3.053 |
| 18 | 0.902 | 1.118 | 0.805 | 1.259 | 0.708 | 1.422 | 0.613 | 1.604 | 0.522 | 1.803 | 0.435 | 2.015 | 0.355 | 2.238 | 0.282 | 2.467 | 0.216 | 2.697 | 0.160 | 2.925 |
| 19 | 0.928 | 1.132 | 0.835 | 1.265 | 0.742 | 1.415 | 0.650 | 1.584 | 0.561 | 1.767 | 0.476 | 1.963 | 0.396 | 2.169 | 0.322 | 2.381 | 0.255 | 2.597 | 0.196 | 2.813 |
| 20 | 0.952 | 1.147 | 0.863 | 1.271 | 0.773 | 1.411 | 0.685 | 1.567 | 0.598 | 1.737 | 0.515 | 1.918 | 0.436 | 2.110 | 0.362 | 2.308 | 0.294 | 2.510 | 0.232 | 2.714 |
| 21 | 0.975 | 1.161 | 0.890 | 1.277 | 0.803 | 1.408 | 0.718 | 1.554 | 0.633 | 1.712 | 0.552 | 1.881 | 0.474 | 2.059 | 0.400 | 2.244 | 0.331 | 2.434 | 0.268 | 2.625 |
| 22 | 0.997 | 1.174 | 0.914 | 1.284 | 0.831 | 1.407 | 0.748 | 1.543 | 0.667 | 1.691 | 0.587 | 1.849 | 0.510 | 2.015 | 0.437 | 2.188 | 0.368 | 2.367 | 0.304 | 2.548 |
| 23 | 1.018 | 1.187 | 0.938 | 1.291 | 0.858 | 1.407 | 0.777 | 1.534 | 0.698 | 1.673 | 0.620 | 1.821 | 0.545 | 1.977 | 0.473 | 2.140 | 0.404 | 2.308 | 0.340 | 2.479 |
| 24 | 1.037 | 1.199 | 0.960 | 1.298 | 0.882 | 1.407 | 0.805 | 1.528 | 0.728 | 1.658 | 0.652 | 1.797 | 0.578 | 1.944 | 0.507 | 2.097 | 0.439 | 2.255 | 0.375 | 2.417 |
| 25 | 1.055 | 1.211 | 0.981 | 1.305 | 0.906 | 1.409 | 0.831 | 1.523 | 0.756 | 1.645 | 0.682 | 1.776 | 0.610 | 1.915 | 0.540 | 2.059 | 0.473 | 2.209 | 0.409 | 2.362 |
| 26 | 1.072 | 1.222 | 1.001 | 1.312 | 0.928 | 1.411 | 0.855 | 1.518 | 0.783 | 1.635 | 0.711 | 1.759 | 0.640 | 1.889 | 0.572 | 2.026 | 0.505 | 2.168 | 0.441 | 2.313 |
| 27 | 1.089 | 1.233 | 1.019 | 1.319 | 0.949 | 1.413 | 0.878 | 1.515 | 0.808 | 1.626 | 0.738 | 1.743 | 0.669 | 1.867 | 0.602 | 1.997 | 0.536 | 2.131 | 0.473 | 2.269 |
| 28 | 1.104 | 1.244 | 1.037 | 1.325 | 0.969 | 1.415 | 0.900 | 1.513 | 0.832 | 1.618 | 0.764 | 1.729 | 0.696 | 1.847 | 0.630 | 1.970 | 0.566 | 2.098 | 0.504 | 2.229 |
| 29 | 1.119 | 1.254 | 1.054 | 1.332 | 0.988 | 1.418 | 0.921 | 1.512 | 0.855 | 1.611 | 0.788 | 1.718 | 0.723 | 1.830 | 0.658 | 1.947 | 0.595 | 2.068 | 0.533 | 2.193 |
| 30 | 1.133 | 1.263 | 1.070 | 1.339 | 1.006 | 1.421 | 0.941 | 1.511 | 0.877 | 1.606 | 0.812 | 1.707 | 0.748 | 1.814 | 0.684 | 1.925 | 0.622 | 2.041 | 0.562 | 2.160 |
| 31 | 1.147 | 1.273 | 1.085 | 1.345 | 1.023 | 1.425 | 0.960 | 1.510 | 0.897 | 1.601 | 0.834 | 1.698 | 0.772 | 1.800 | 0.710 | 1.906 | 0.649 | 2.017 | 0.589 | 2.131 |
| 32 | 1.160 | 1.282 | 1.100 | 1.352 | 1.040 | 1.428 | 0.979 | 1.510 | 0.917 | 1.597 | 0.856 | 1.690 | 0.794 | 1.788 | 0.734 | 1.889 | 0.674 | 1.995 | 0.615 | 2.104 |
| 33 | 1.172 | 1.291 | 1.114 | 1.358 | 1.055 | 1.432 | 0.996 | 1.510 | 0.936 | 1.594 | 0.876 | 1.683 | 0.816 | 1.776 | 0.757 | 1.874 | 0.698 | 1.975 | 0.641 | 2.080 |
| 34 | 1.184 | 1.299 | 1.128 | 1.364 | 1.070 | 1.435 | 1.012 | 1.511 | 0.954 | 1.591 | 0.896 | 1.677 | 0.837 | 1.766 | 0.779 | 1.860 | 0.722 | 1.957 | 0.665 | 2.057 |
| 35 | 1.195 | 1.307 | 1.140 | 1.370 | 1.085 | 1.439 | 1.028 | 1.512 | 0.971 | 1.589 | 0.914 | 1.671 | 0.857 | 1.757 | 0.800 | 1.847 | 0.744 | 1.940 | 0.689 | 2.037 |
| 36 | 1.206 | 1.315 | 1.153 | 1.376 | 1.098 | 1.442 | 1.043 | 1.513 | 0.988 | 1.588 | 0.932 | 1.666 | 0.877 | 1.749 | 0.821 | 1.836 | 0.766 | 1.925 | 0.711 | 2.018 |
| 37 | 1.217 | 1.323 | 1.165 | 1.382 | 1.112 | 1.446 | 1.058 | 1.514 | 1.004 | 1.586 | 0.950 | 1.662 | 0.895 | 1.742 | 0.841 | 1.825 | 0.787 | 1.911 | 0.733 | 2.001 |
| 38 | 1.227 | 1.330 | 1.176 | 1.388 | 1.124 | 1.449 | 1.072 | 1.515 | 1.019 | 1.585 | 0.966 | 1.658 | 0.913 | 1.735 | 0.860 | 1.816 | 0.807 | 1.899 | 0.754 | 1.985 |
| 39 | 1.237 | 1.337 | 1.187 | 1.393 | 1.137 | 1.453 | 1.085 | 1.517 | 1.034 | 1.584 | 0.982 | 1.655 | 0.930 | 1.729 | 0.878 | 1.807 | 0.826 | 1.887 | 0.774 | 1.970 |
| 40 | 1.246 | 1.344 | 1.198 | 1.398 | 1.148 | 1.457 | 1.098 | 1.518 | 1.048 | 1.584 | 0.997 | 1.652 | 0.946 | 1.724 | 0.895 | 1.799 | 0.844 | 1.876 | 0.749 | 1.956 |
| 45 | 1.288 | 1.376 | 1.245 | 1.423 | 1.201 | 1.474 | 1.156 | 1.528 | 1.111 | 1.584 | 1.065 | 1.643 | 1.019 | 1.704 | 0.974 | 1.768 | 0.927 | 1.834 | 0.881 | 1.902 |
| 50 | 1.324 | 1.403 | 1.285 | 1.446 | 1.245 | 1.491 | 1.205 | 1.538 | 1.164 | 1.587 | 1.123 | 1.639 | 1.081 | 1.692 | 1.039 | 1.748 | 0.997 | 1.805 | 0.955 | 1.864 |
| 55 | 1.356 | 1.427 | 1.320 | 1.466 | 1.284 | 1.506 | 1.247 | 1.548 | 1.209 | 1.592 | 1.172 | 1.638 | 1.134 | 1.685 | 1.095 | 1.734 | 1.057 | 1.785 | 1.018 | 1.837 |
| 60 | 1.383 | 1.449 | 1.350 | 1.484 | 1.317 | 1.520 | 1.283 | 1.558 | 1.249 | 1.598 | 1.214 | 1.639 | 1.179 | 1.682 | 1.144 | 1.726 | 1.108 | 1.771 | 1.072 | 1.817 |
| 65 | 1.407 | 1.468 | 1.377 | 1.500 | 1.346 | 1.534 | 1.315 | 1.568 | 1.283 | 1.604 | 1.251 | 1.642 | 1.218 | 1.680 | 1.186 | 1.720 | 1.153 | 1.761 | 1.120 | 1.802 |
| 70 | 1.429 | 1.485 | 1.400 | 1.515 | 1.372 | 1.546 | 1.343 | 1.578 | 1.313 | 1.611 | 1.283 | 1.645 | 1.253 | 1.680 | 1.223 | 1.716 | 1.192 | 1.754 | 1.162 | 1.792 |
| 75 | 1.448 | 1.501 | 1.422 | 1.529 | 1.395 | 1.557 | 1.368 | 1.587 | 1.340 | 1.617 | 1.313 | 1.649 | 1.284 | 1.682 | 1.256 | 1.714 | 1.227 | 1.748 | 1.199 | 1.783 |
| 80 | 1.466 | 1.515 | 1.441 | 1.541 | 1.416 | 1.568 | 1.390 | 1.595 | 1.364 | 1.624 | 1.338 | 1.653 | 1.312 | 1.683 | 1.285 | 1.714 | 1.259 | 1.745 | 1.232 | 1.777 |
| 85 | 1.482 | 1.528 | 1.458 | 1.553 | 1.435 | 1.578 | 1.411 | 1.603 | 1.386 | 1.630 | 1.362 | 1.657 | 1.337 | 1.685 | 1.312 | 1.714 | 1.287 | 1.743 | 1.262 | 1.773 |
| 90 | 1.496 | 1.540 | 1.474 | 1.563 | 1.452 | 1.587 | 1.429 | 1.611 | 1.406 | 1.636 | 1.383 | 1.661 | 1.360 | 1.687 | 1.336 | 1.714 | 1.312 | 1.741 | 1.288 | 1.769 |
| 95 | 1.510 | 1.552 | 1.489 | 1.573 | 1.468 | 1.596 | 1.446 | 1.618 | 1.425 | 1.642 | 1.403 | 1.666 | 1.381 | 1.690 | 1.358 | 1.715 | 1.336 | 1.741 | 1.313 | 1.767 |
| 100 | 1.522 | 1.562 | 1.503 | 1.583 | 1.482 | 1.604 | 1.462 | 1.625 | 1.441 | 1.647 | 1.421 | 1.670 | 1.400 | 1.693 | 1.378 | 1.717 | 1.357 | 1.741 | 1.335 | 1.765 |
| 150 | 1.611 | 1.637 | 1.598 | 1.651 | 1.584 | 1.665 | 1.571 | 1.679 | 1.557 | 1.693 | 1.543 | 1.708 | 1.530 | 1.722 | 1.515 | 1.737 | 1.501 | 1.752 | 1.486 | 1.767 |
| 200 | 1.664 | 1.684 | 1.653 | 1.693 | 1.643 | 1.704 | 1.633 | 1.715 | 1.623 | 1.725 | 1.613 | 1.735 | 1.603 | 1.746 | 1.592 | 1.757 | 1.582 | 1.768 | 1.571 | 1.779 |

| n | $k'=11$ $d_L$ | $d_U$ | $k'=12$ $d_L$ | $d_U$ | $k'=13$ $d_L$ | $d_U$ | $k'=14$ $d_L$ | $d_U$ | $k'=15$ $d_L$ | $d_U$ | $k'=16$ $d_L$ | $d_U$ | $k'=17$ $d_L$ | $d_U$ | $k'=18$ $d_L$ | $d_U$ | $k'=19$ $d_L$ | $d_U$ | $k'=20$ $d_L$ | $d_U$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 16 | 0.060 | 3.446 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 17 | 0.084 | 3.286 | 0.053 | 3.506 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 18 | 0.113 | 3.146 | 0.075 | 3.358 | 0.047 | 3.357 | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 19 | 0.145 | 3.023 | 0.102 | 3.227 | 0.067 | 3.420 | 0.043 | 3.601 | — | — | — | — | — | — | — | — | — | — | — | — |
| 20 | 0.178 | 2.914 | 0.131 | 3.109 | 0.092 | 3.297 | 0.061 | 3.474 | 0.038 | 3.639 | — | — | — | — | — | — | — | — | — | — |
| 21 | 0.212 | 2.817 | 0.162 | 3.004 | 0.119 | 3.185 | 0.084 | 3.358 | 0.055 | 3.521 | 0.035 | 3.671 | — | — | — | — | — | — | — | — |
| 22 | 0.246 | 2.729 | 0.194 | 2.909 | 0.148 | 3.084 | 0.109 | 3.252 | 0.077 | 3.412 | 0.050 | 3.562 | 0.032 | 3.700 | — | — | — | — | — | — |
| 23 | 0.281 | 2.651 | 0.227 | 2.822 | 0.178 | 2.991 | 0.136 | 3.155 | 0.100 | 3.311 | 0.070 | 3.459 | 0.046 | 3.597 | 0.029 | 3.725 | — | — | — | — |
| 24 | 0.315 | 2.580 | 0.260 | 2.744 | 0.209 | 2.906 | 0.165 | 3.065 | 0.125 | 3.218 | 0.092 | 3.363 | 0.065 | 3.501 | 0.043 | 3.629 | 0.027 | 3.747 | — | — |
| 25 | 0.348 | 2.517 | 0.292 | 2.674 | 0.240 | 2.829 | 0.194 | 2.982 | 0.152 | 3.131 | 0.116 | 3.274 | 0.085 | 3.410 | 0.060 | 3.538 | 0.039 | 3.657 | 0.025 | 3.766 |
| 26 | 0.381 | 2.460 | 0.324 | 2.610 | 0.272 | 2.758 | 0.224 | 2.906 | 0.180 | 3.050 | 0.141 | 3.191 | 0.107 | 3.325 | 0.079 | 3.452 | 0.055 | 3.572 | 0.036 | 3.682 |
| 27 | 0.413 | 2.409 | 0.356 | 2.552 | 0.303 | 2.694 | 0.253 | 2.836 | 0.208 | 2.976 | 0.167 | 3.113 | 0.131 | 3.245 | 0.100 | 3.371 | 0.073 | 3.490 | 0.051 | 3.602 |
| 28 | 0.444 | 2.363 | 0.387 | 2.499 | 0.333 | 2.635 | 0.283 | 2.772 | 0.237 | 2.907 | 0.194 | 3.040 | 0.156 | 3.169 | 0.122 | 3.294 | 0.093 | 3.412 | 0.068 | 3.524 |
| 29 | 0.474 | 2.321 | 0.417 | 2.451 | 0.363 | 2.582 | 0.313 | 2.713 | 0.266 | 2.843 | 0.222 | 2.972 | 0.182 | 3.098 | 0.146 | 3.220 | 0.114 | 3.338 | 0.087 | 3.450 |
| 30 | 0.503 | 2.283 | 0.447 | 2.407 | 0.393 | 2.533 | 0.342 | 2.659 | 0.294 | 2.785 | 0.249 | 2.909 | 0.208 | 3.032 | 0.171 | 3.152 | 0.137 | 3.267 | 0.107 | 3.379 |
| 31 | 0.531 | 2.248 | 0.475 | 2.367 | 0.422 | 2.487 | 0.371 | 2.609 | 0.322 | 2.730 | 0.277 | 2.851 | 0.234 | 2.970 | 0.196 | 3.087 | 0.160 | 3.201 | 0.128 | 3.311 |
| 32 | 0.558 | 2.216 | 0.503 | 2.330 | 0.450 | 2.446 | 0.399 | 2.563 | 0.350 | 2.680 | 0.304 | 2.797 | 0.261 | 2.912 | 0.221 | 3.026 | 0.184 | 3.137 | 0.151 | 3.246 |
| 33 | 0.585 | 2.187 | 0.530 | 2.296 | 0.477 | 2.408 | 0.426 | 2.520 | 0.377 | 2.633 | 0.331 | 2.746 | 0.287 | 2.858 | 0.246 | 2.969 | 0.209 | 3.078 | 0.174 | 3.184 |
| 34 | 0.610 | 2.160 | 0.556 | 2.266 | 0.503 | 2.373 | 0.452 | 2.481 | 0.404 | 2.590 | 0.357 | 2.699 | 0.313 | 2.808 | 0.272 | 2.915 | 0.233 | 3.022 | 0.197 | 3.126 |
| 35 | 0.634 | 2.136 | 0.581 | 2.237 | 0.529 | 2.340 | 0.478 | 2.444 | 0.430 | 2.550 | 0.383 | 2.655 | 0.339 | 2.761 | 0.297 | 2.865 | 0.257 | 2.969 | 0.221 | 3.071 |
| 36 | 0.658 | 2.113 | 0.605 | 2.210 | 0.554 | 2.310 | 0.504 | 2.410 | 0.455 | 2.512 | 0.409 | 2.614 | 0.364 | 2.717 | 0.322 | 2.818 | 0.282 | 2.919 | 0.244 | 3.019 |
| 37 | 0.680 | 2.092 | 0.628 | 2.186 | 0.578 | 2.282 | 0.528 | 2.379 | 0.480 | 2.477 | 0.434 | 2.576 | 0.389 | 2.675 | 0.347 | 2.774 | 0.306 | 2.872 | 0.268 | 2.969 |
| 38 | 0.702 | 2.073 | 0.651 | 2.164 | 0.601 | 2.256 | 0.552 | 2.350 | 0.504 | 2.445 | 0.458 | 2.540 | 0.414 | 2.637 | 0.371 | 2.733 | 0.330 | 2.828 | 0.291 | 2.923 |
| 39 | 0.723 | 2.055 | 0.673 | 2.143 | 0.623 | 2.232 | 0.575 | 2.323 | 0.528 | 2.414 | 0.482 | 2.507 | 0.438 | 2.600 | 0.395 | 2.694 | 0.354 | 2.787 | 0.315 | 2.879 |
| 40 | 0.744 | 2.039 | 0.694 | 2.123 | 0.645 | 2.210 | 0.597 | 2.297 | 0.551 | 2.386 | 0.505 | 2.476 | 0.461 | 2.566 | 0.418 | 2.657 | 0.377 | 2.748 | 0.338 | 2.838 |
| 45 | 0.835 | 1.972 | 0.790 | 2.044 | 0.744 | 2.118 | 0.700 | 2.193 | 0.655 | 2.269 | 0.612 | 2.346 | 0.570 | 2.424 | 0.528 | 2.503 | 0.488 | 2.582 | 0.448 | 2.661 |
| 50 | 0.913 | 1.925 | 0.871 | 1.987 | 0.829 | 2.051 | 0.787 | 2.116 | 0.746 | 2.182 | 0.705 | 2.250 | 0.665 | 2.318 | 0.625 | 2.387 | 0.586 | 2.456 | 0.548 | 2.526 |
| 55 | 0.979 | 1.891 | 0.940 | 1.945 | 0.902 | 2.002 | 0.863 | 2.059 | 0.825 | 2.117 | 0.786 | 2.176 | 0.748 | 2.237 | 0.711 | 2.298 | 0.674 | 2.359 | 0.637 | 2.421 |
| 60 | 1.037 | 1.865 | 1.001 | 1.914 | 0.965 | 1.964 | 0.929 | 2.015 | 0.893 | 2.067 | 0.857 | 2.120 | 0.822 | 2.173 | 0.786 | 2.227 | 0.751 | 2.283 | 0.716 | 2.338 |
| 65 | 1.087 | 1.845 | 1.053 | 1.889 | 1.020 | 1.934 | 0.986 | 1.980 | 0.953 | 2.027 | 0.919 | 2.075 | 0.886 | 2.123 | 0.852 | 2.172 | 0.819 | 2.221 | 0.786 | 2.272 |
| 70 | 1.131 | 1.831 | 1.099 | 1.870 | 1.068 | 1.911 | 1.037 | 1.953 | 1.005 | 1.995 | 0.974 | 2.038 | 0.943 | 2.082 | 0.911 | 2.127 | 0.880 | 2.172 | 0.849 | 2.217 |
| 75 | 1.170 | 1.819 | 1.141 | 1.856 | 1.111 | 1.893 | 1.082 | 1.931 | 1.052 | 1.970 | 1.023 | 2.009 | 0.993 | 2.049 | 0.964 | 2.090 | 0.934 | 2.131 | 0.905 | 2.172 |
| 80 | 1.205 | 1.810 | 1.177 | 1.844 | 1.150 | 1.878 | 1.122 | 1.913 | 1.094 | 1.949 | 1.066 | 1.984 | 1.039 | 2.022 | 1.011 | 2.059 | 0.983 | 2.097 | 0.955 | 2.135 |
| 85 | 1.236 | 1.803 | 1.210 | 1.834 | 1.184 | 1.866 | 1.158 | 1.898 | 1.132 | 1.931 | 1.106 | 1.965 | 1.080 | 1.999 | 1.053 | 2.033 | 1.027 | 2.068 | 1.000 | 2.104 |
| 90 | 1.264 | 1.798 | 1.240 | 1.827 | 1.215 | 1.856 | 1.191 | 1.886 | 1.166 | 1.917 | 1.141 | 1.948 | 1.116 | 1.979 | 1.091 | 2.012 | 1.066 | 2.044 | 1.041 | 2.077 |
| 95 | 1.290 | 1.793 | 1.267 | 1.821 | 1.244 | 1.848 | 1.221 | 1.876 | 1.197 | 1.905 | 1.174 | 1.934 | 1.150 | 1.963 | 1.126 | 1.993 | 1.102 | 2.023 | 1.079 | 2.054 |
| 100 | 1.314 | 1.790 | 1.292 | 1.816 | 1.270 | 1.841 | 1.248 | 1.868 | 1.225 | 1.895 | 1.203 | 1.922 | 1.181 | 1.949 | 1.158 | 1.977 | 1.136 | 2.006 | 1.113 | 2.034 |
| 150 | 1.473 | 1.783 | 1.458 | 1.799 | 1.444 | 1.814 | 1.429 | 1.830 | 1.414 | 1.847 | 1.400 | 1.863 | 1.385 | 1.880 | 1.370 | 1.897 | 1.355 | 1.913 | 1.340 | 1.931 |
| 200 | 1.561 | 1.791 | 1.550 | 1.801 | 1.539 | 1.813 | 1.528 | 1.824 | 1.518 | 1.836 | 1.507 | 1.847 | 1.495 | 1.860 | 1.484 | 1.871 | 1.474 | 1.883 | 1.462 | 1.896 |

Note: $n$ = number of observations.
$k'$ = number of explanatory variables excluding the constant term.
Source: Savin and White, op. cit., by permission of the Econometric Society.

Statistiska institutionen

Stockholms
universitet

# Rättningsblad

**Datum:** 2/6-2017

**Sal:** Ugglevikssalen

**Tenta:** Regressionsanalys

**Kurs:** Ekonometri

**ANONYMKOD:** ERU-0024

☒ Jag godkänner att min tenta får läggas ut anonymt på hemsidan som studentsvar.

## OBS! SKRIV ÄVEN PÅ BAKSIDAN AV SKRIVBLADEN

Markera besvarade uppgifter med kryss

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Antal inl. blad |
|---|---|---|---|---|---|---|---|---|-----------------|
| X | X | X | X | X |   |   |   |   | 5 |
| **Lär.ant.** 13 | 21 | 15 | 14 | 18 |   |   |   |   | |

| POÄNG | BETYG | Lärarens sign. |
|-------|-------|----------------|
| 81 | B | |

① a) ① $SSE = SST - SSR = 0,6671$

② $df$ för $SSE = k-1 = 1$

③ $df$ för $SSR = n-k = 38$

④ $df$ för $SST = n-1 = 39$

⑤ Mean Square för $SSE = \dfrac{0,6671}{1} = 0,6671$

⑥ Mean Square för $SSR = \dfrac{0,0361}{38} = 0,00095$

⑦ Mean Square för $SST = \dfrac{0,7032}{39} = 0,01803076923$

⑧

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b) $R^2 = \dfrac{SSE}{SST} = \dfrac{0,6671}{0,7032} = 0,9486632537$ OK

$\overline{R^2} = 1 - \left(\dfrac{\frac{SSR}{n-k}}{\frac{SST}{n-1}}\right) = 1 - \left(\dfrac{0,00095}{0,01803076923}\right) = 0,9473122867$ OK

$AIC = \dfrac{e^{\frac{2k}{n}} RSS}{n} = \dfrac{e^{0,1} \times 0,0361}{40} = 0,00099741675$ OK

$\hat{\sigma}^2 = \dfrac{SSR}{n-2} = \dfrac{0,0361}{38} = 0,00095$ OK

/7

c) —

/13
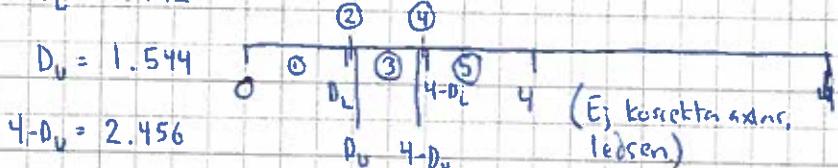
②  a)  Vi har här inte data nog för att använda runs-test eller den
fullständiga versionen av Durbin-Watsons d-test. Men vi kan använda
oss av att

$$d = \frac{\sum_{t=2}^{n}\left(\hat{u}_t - \hat{u}_{t-1}\right)^2}{\sum_{t=1}^{n}\hat{u}_t^2} \approx 2\left(1 - \hat{\rho}\right)$$

Vi har att $\hat{\rho} = 0,8$, och $d \approx 0,4$. Vi har även att $k' = k - 1 = 1$. Med hjälp
av formelbladet får vi då dessa beslutsregler:

$D_L = 1.442$

$D_U = 1.544$

$4 - D_U = 2.456$

$4 - D_L = 2.558$

① Positiv autokorrelation (förkasta $H_0$)

② Osäkerhetszon

③ Ej bevis för autokorrelation (förkasta ej $H_0$)

④ Osäkerhetszon

⑤ Negativ autokorrelation (förkasta $H_0$)

0,4 ligger i ①, och vi förkastar därför $H_0$ till förmån för alternativhypotesen $H_A$,
att det föreligger positiv autokorrelation.

(2) b) Ett White-test för heteroskedasticitet är redan genomfört och gav $R^2 = 0,62$, vilket betyder att variablerna förklarar cirka 62% av variationen i de kvadrerade residualerna. För att undersöka om detta är statistiskt signifikant (dvs. för att testa antagandet om heteroskedasticitet) kan man utnyttja att

$$nR^2 \sim \chi^2\text{-fördelad (här med } df = 2) \text{ under } H_0 \text{ (ingen heteroskedasticitet)}$$

$$\Rightarrow \quad nR^2 = 40 \times 0,62 = 24,8$$

Kritiskt värde hämtar vi från formelblad och är för 5% signifikansnivå (med $df = 2$) lika med 55,7585.

$24,8 < 55,7585$, så vi förkastar ej $H_0$ på 5% signifikansnivå. Vi saknar statistiskt stöd för antagandet om heteroskedasticitet.

Det finns en mängd sätt att försöka bli av med heteroskedasticitet.

① Undersöka om modellen är underanpassad eller har fel funktionell form och korrigera detta.

② Försöka vikta bort korrelationen men hjälp av delning av alla termer med någon term som ser ut att korrelera med residualerna.

③ Använda Whites generella metod för komprimering av ~~standar~~ varianserna.

④ Tänka igenom den underliggande ~~dataen~~ teorin och datainsamlingen. Det kan till exempel finnas en störfaktor som påverkar såväl regressorernas och residualernas varians.

$/8$

c) Den ursprungliga modellen är nästad i den nya, så vi kan jämföra dem med ett F-test.

$H_0$: ingen skillnad mellan modellerna i förklaringsgrad

$H_A$: nya modellen har högre förklaringsgrad

$$F = \left( \frac{ESS_{nya} - ESS_{gamla}}{\frac{nya \ regressorer}{}} \middle/ \frac{RSS_{nya}}{n - antalet\ parametrar\ i\ nya\ modellen} \right) \Rightarrow F = \frac{\frac{(0,6688 - 0,6671)}{2}}{\frac{(0,0344)}{36}} = 0,8895348889$$

$F_{(2,36)} \approx 3,32$ (för $N_2 = 30$ och 5% signifikansnivå — 36 finns ej i tabellen)

$0,89 < 3,32$, så vi förkastar ej $H_0$. Det finns inget statistiskt stöd för för antagandet att den nya modellen är bättre.

$/7$

$/21$

③ a)  Interceptet = Konsumtion i första kvartalet givet noll inkomst

$D_{2i}$ = Skillnaden i nivå mellan konsumtion i första och andra kvartalet givet ~~noll i inkomst~~ **övrigt hållet konstant**

b)  $D_{4i} = 1 - D_{1i} - D_{2i} - D_3$

$$\Rightarrow \hat{Q}_i = 1,20 + 2,31 X_i - 0,71 D_{2i} - 0,77 D_{3i} - 0,03 (1 - D_{1i} - D_{2i} - D_{3i}) \Leftrightarrow$$

$$\Leftrightarrow \hat{Q}_i = 1,17 + 2,31 X_i + 0,03 D_{1i} - 0,68 D_{2i} - 0,74 D_{3i}$$

c)  Interceptet = Konsumtion i fjärde kvartalet givet noll i inkomst

$D_{2i}$ = Skillnaden i nivå av konsumtion mellan andra och fjärde kvartalet givet noll i inkomst.

**/15**

④ a)  Vi kommer att få heteroskedasticitet, vilket kommer att öka varianserna för såväl $\hat{\beta}_1$ som $\hat{\beta}_2$. OLS-estimatorerna kommer dock fortfarande vara vvr och konsistenta, men nu inte BLUE.

**/6**

b)  Vi kan åtgärda problemet genom att använda WLS istället för OLS. Här med $\dfrac{1}{\sqrt{e^{x_i}}}$ som vikt.

Då får vi

$$\frac{Y_i}{\sqrt{e^{x_i}}} = \frac{\beta_1}{\sqrt{e^{x_i}}} + \frac{\beta_2}{\sqrt{e^{x_i}}} + \frac{u_i}{\sqrt{e^{x_i}}} \Rightarrow V\left(\frac{u_i}{\sqrt{e^{x_i}}}\right) = \frac{1}{e^{x_i}} V(u_i) = \frac{e^{x_i} v_i}{e^{x_i}} = v_i$$

$v_i$ är alltså nu homoskedastisk.

**/8**

**/14**

⑤ a) Sant ✓ Annars kommer de var biased.

b) Falskt, se min.

Om $Y_i^* = w_1 Y_i$ och $X_i^* = w_2 X_2$ så är

$\hat{\beta}_1^* = w_1 \hat{\beta}_1$ , $\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right) \hat{\beta}_2$

här är $w_1 = w_2 = 100$ , och $\hat{\beta}_2$'s värde ändras inte. OK

c) Falskt. $R^2$ ökar alltid när man lägger till regressorer $\bar{R}^2$, AIC, BIC är bättre i så fall. OK

d) Falskt. White-testet testar för heteroskedasticitet OK

e) Ja $X_i^* = X_i + w_i \Rightarrow Y_i = \beta_1 + B_2 (X_i + w_i) + u_i \Rightarrow$

$\Rightarrow Y_i = \beta_1 + \beta_2 X_i + \left[ B_2 w_i + u_i \right]$

← Ny störningsvariabel som korrelerar med $\beta_2$  OK

f) Falskt. Om variabler utesluts kommer feltermen innehålla dem och den kommer då förmodligen bli autokorrelerad. OK

g) Falskt. Om de förklarande variablerna BARA består av dummyvariabler gäller det (för att undvika dummyfällan) men om vi väljer m−1 dummyvariabler med intercept. OK

/18

# Rättningsblad

**Datum:** 2/6-2017

**Sal:** Ugglevikssalen

**Tenta:** Regressionsanalys

**Kurs:** Ekonometri

**ANONYMKOD:** ERO-0022

☒ Jag godkänner att min tenta får läggas ut anonymt på hemsidan som studentsvar.

## OBS! SKRIV ÄVEN PÅ BAKSIDAN AV SKRIVBLADEN

**Markera besvarade uppgifter med kryss**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Antal inl. blad |
|---|---|---|---|---|---|---|---|---|---|
| ✗ | ✗ | ✗ | ✗ | ✗ | | | | | 5 |
| 9 | 16 | 9 | 8 | 19 | | | | | |

| POÄNG | BETYG | Lärarens sign. |
|---|---|---|
| 61 | D | |

1. a) $SSE = 0,7032 - 0,0361 = 0,6671$ OK

| | sum of squares | df | mean square |
|---|---|---|---|
| Regression: | 0,6671 | 38 | 0,01756 |
| Residuals : | 0,0361 | 2 | 0,01805 ✓ |
| Total : | 0,7032 | 40 39 | 0,01758 |

/2

b) $R^2 = 1 - \dfrac{RSS}{TSS} \Rightarrow 1 - \dfrac{0,0361}{0,7032} = 0,9487$ OK

$n = 40$
$k = 2$

$\bar{R}^2 = 1 - \dfrac{RSS/(n-k)}{TSS/(n-1)} \Rightarrow 1 - \dfrac{0,0361/38}{0,7032/39} = 0,9473$ OK

$AIC = \dfrac{e^{2k/n} \, RSS}{n} \Rightarrow \dfrac{e^{4/40} \cdot 0,0361}{40} = 0,0009974$ OK

$\hat{\sigma}^2 = \dfrac{RSS}{n-2} \Rightarrow \dfrac{0,0361}{38} = 0,00095$ OK

/7

/9

2. a) Durbin-watson test! $d = \dfrac{\sum_{t=2}^{n} (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^{n} \hat{u}_t^2}$  $k = 1$

$n = 40$

$\sum_{t=2}^{n} (\hat{u}_t - \hat{u}_{t+1})^2 = 0,0361 \cdot 0,8 = 0,02888$ ✓

$\sum_{t=1}^{n} \hat{u}_t^2 = 0,0361$

$H_0$: Ingen autocorr. förekommer.

$H_1$: Autocorr. förekommer. 5% sig. nivå

$d = \dfrac{0,02888}{0,0361} = 0,8$ ✓

$D(1, 40) \Rightarrow d_v = 1,544$

$d_L = 1,442$

| Förk. $H_0$ | Grå-zon | $H_0$ fört. ej | Grå-zon | Förk. $H_0$ |
|---|---|---|---|---|
| $d_L$ | $d_v$ | $4-d_v$ | $4-d_L$ |
| 1,442 | 1,544 | 2,456 | 2,558 |

∴ $H_0$ förk. på 5% nivå eftersom $0,8 < d_v$

Det förekommer positiv autocorrelation!  **/3**

b) White's test: $nR^2 \overset{appr.}{\sim} \chi^2 (df = \# \text{ af regressors})$  $n = 40$

$R^2 = 0,62$

$H_0$: Det föreligger ingen heterosked.

$H_1$: " " — heterosked.  , på 5% signifikansnivå.

$df = 2$

$H_0$ förkastas om $\chi^2_{obs} > \chi^2_{krit}$
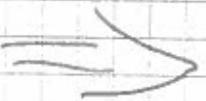
$\chi^2_{obs} = 40 \cdot 0,62 = 24,8$

$\chi^2_{krit} = 5,99147 \ [\text{enl. tabell}]$

∴ $H_0$ förkastas på 5% signifikansnivå, eftersom $\chi^2_{obs} > \chi^2_{krit}$ $(24,8 > 5.99147)$. Det förekommer alltså heteroskedasticitet!

**/6**

2. c) Jag använder ett F-test:

$$F = \frac{(ESS_{new} - ESS_{old}) / \text{\# of new regressors}}{RSS_{new} / (n - \text{\# parameters in new model})}$$

$ESS_{new} = 0,6688$    $RSS_{new} = 0,0344$

$ESS_{old} = 0,6671$    \# new reg: 2

$n = 40$    \# param in new: 4

$$F = \frac{(0,6688 - 0,6671)/2}{0,0344/36} = 0,8895$$

$H_0$: Båda modellerna är lika bra
$H_1$: Nya modellen är bättre    , på 5% signifikansnivå

$F(2,36) \approx F(2,40) = 3,23$

$H_0$ förkastas om $F_{obs} > F_{krit}(2,36)$

∴ $H_0$ förkastas inte eftersom $F_{obs} < F_{krit}$ $(0,8895 < 3,23)$
Den nya modellen är inte signifikant bättre än den
gamla, så vi behåller den gamla!

På 5% signifikansnivå

/7

/16

*vilket värde?*

3. a) Interceptet 1,20 är *värdet* på dummyvariabeln för $Q_1$ $(1,20 \, D_{4i})$
eftersom det används som benchmark. Denna dummyvariabel
är exkluderad för att undvika "the dummy trap".
Om vi skal inkludera dummyvariabeln för $Q_1$ ska vi plocka
bort interceptet.
Parametern för $D_{2i}$, som är -0,71, betyder att om vi
befinner oss i det andra kvartalet blir $D_{2i} = 1$. Detta
innebär att konsumptionen kommer minska med faktor 0,71.
Resterande dummyvariabler blir då 0, eftersom dem bara
blir 1 om vi befinner oss i just det kvartalet.

/3

b) $Q_1 = 1,20$

$Q_2 = -0,71$

$Q_3 = -0,77$

$Q_4 = -0,03$

$\hat{Y}_i = -0,03 + 2,31 X_i + 1,\checkmark D_{1i} - 0,68 D_{2i} - 0,74 D_{3i}$

$D_{4i}$ är nu benchmark istället för $D_{1i}$.

/3

c) Interceptet är nu istället parametern för kvartal 4 $(Q_4)$. *Tolkning?*
Parametern för $Q_2$ $(-0,68 D_{2i})$ har minskat något eftersom
$D_{4i}$ istället används som benchmark!

/3

/9

4. a) Eftersom $u_i$ ej är normalfördelad är OLS-skattningarna förmodligen inte BLUE (Best linear unbiased estimator). Skattningarna kan förvisso vara ~~konsekventa~~ men ej effektiva.

*väntenta*

/4

b) Istället för att använda OLS kan vi använda oss utav GLS (generalized linear square) eller WLS (weighted linear square).

Dessa modeller passar bättre då förutsättningarna för OLS ej är uppfyllda.

/4

/8
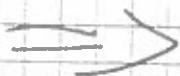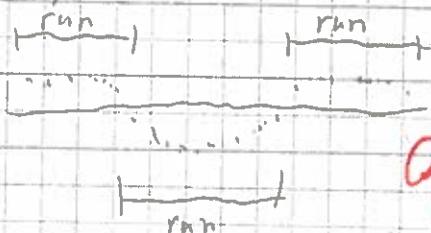
5. a) Fel. Detta gäller bara för OLS-skattningar, WLS eller GLS kan istället användas om detta är fallet. Då blir dessa istället BLUE (Best linear unbiased estimator).

b) Fel. $\beta_1$ ändrar värde men inte $\beta_2$. Proportionerna mellan $Y_i$ och $X_i$ förblir desamma trots ökningen av $Y_i$ och $X_i$. ok

c) Fel. För att $R^2$ skall vara ett bra mått skall båda modellerna innehålla samma antal regressorer. Om modellerna har olika många regressorer kan vi istället använda oss utav ett F-test.

$$F = \frac{(ESS_{new} - ESS_{old}) / \text{number of new regressors}}{RSS_{new} / (n - \text{number of parameters in new model})}$$ ok

d) Fel, white's test används för att identifiera hetroskedasticitet. Durbin-watson anv. för att upptäcka autokorrelation. Run-test kan visa tecken på autokorrelation om feltermerna följer tydliga Runs.



ok

5. e) Sant. Om vi har mätfel i X, betyder det att vi missar data som kommer fångas upp i feltermerna istället.

OK

f) Fel. Om vi exkluderar viktiga variabler kommer detta leda till autokorrelation. Datan vi skulle fått med den exkluderade variabeln hamnar istället i feltermerna.

OK

g) Fel. Detta gäller endast om dummyvariablerna är beroende av varandra t.ex. vid perioder eller säsonger. Om vi har dummyvariabler för t.ex. 4 kvartal ska vi utesluta en för att undvika "the dummy trap". Om vi inkluderar en dummy som t.ex. säger vilket kön en person har behöver vi inte ta bort interceptet.

OK

/19