Stockholm University
Department of Statistics
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Econometrics II

WRITTEN EXAMINATION
Wednesday May 31, 2017, 10 pm - 3 pm

Tools allowed: Pocket calculator

Passing rate: 50% of overall total, which is 90 points. For detailed grading criteria, see the course description.

For the maximum number of points on each problem detailed and clear solutions are required.

Observe:: If not indicated otherwise, the error terms ϵ_t in the models are assumed independent and $N(0, \sigma^2)$.

No Swedish version this time, but you may answer in Swedish.

1. (18p) The athlete Usain Bolt performed the following yearly best results for the 100 meter event.

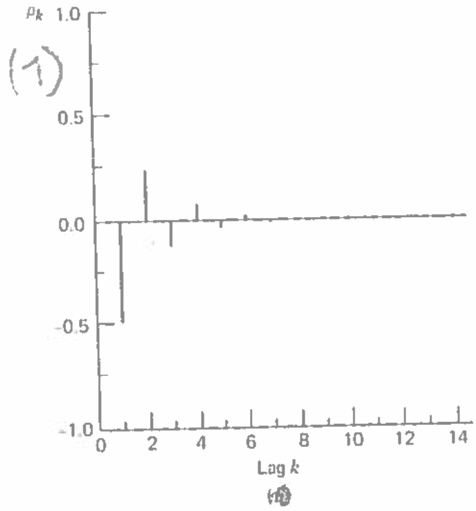
Year	Best result (seconds)
2005	10.18
2006	9.96
2007	9.85
2008	9.69

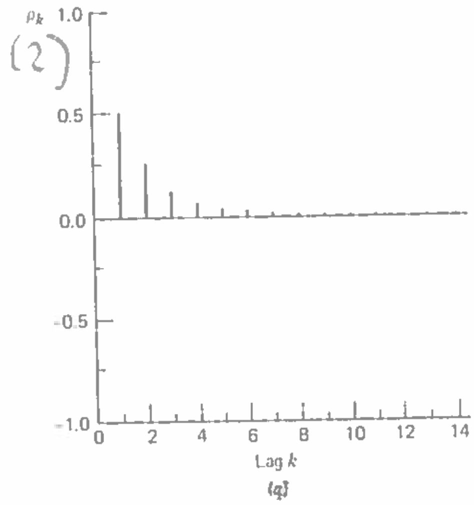
   (a) Use an appropriate smoothing method for this time series. Choose the value of the discount factor/factors equal to 0.3 and use the whole series to determine the starting value(s).

   (b) Use the smoothing to predict the best result for Usain Bolt in 2009. (The actual value (time) for 2009 was 9.58 and a world record which still holds.)

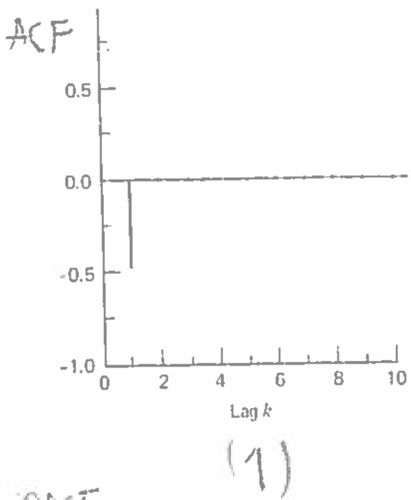
2. (15p)

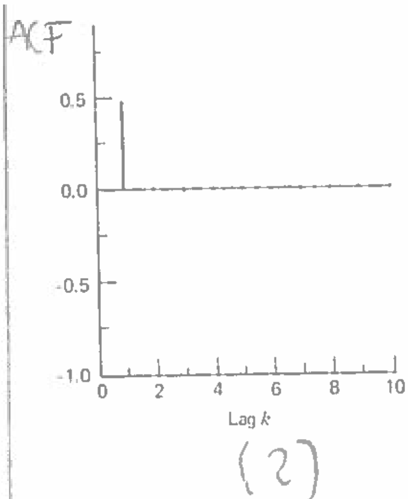
(a) Below we have ACF:s for two stationary AR(1) processes. Determine the values of ϕ in both cases.

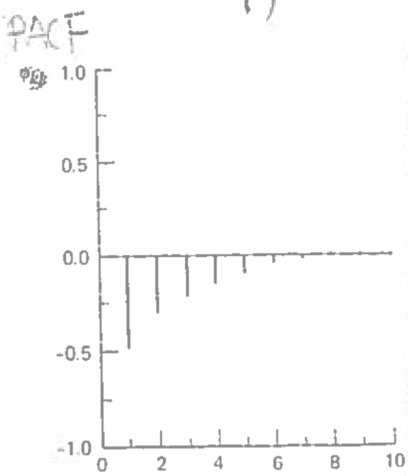


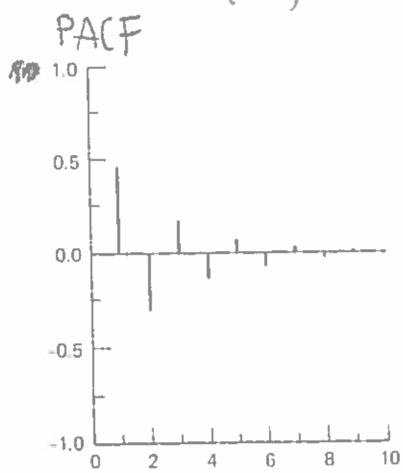


(b) Below we have ACF:s and PACF:s for two stationary processes. Which type of model seems to fit in both cases? Can you say something about any of the parameters for these two cases? How do they differ?









3. (18p) Below we have plots corresponding to $n = 369$ daily closing IBM stock prices.

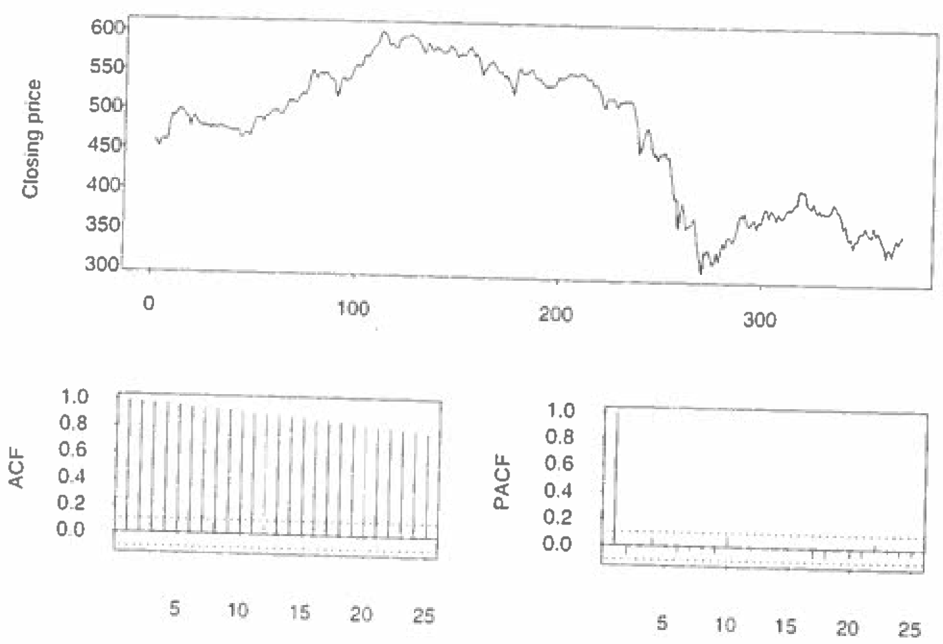
(a) Why does this time series not seem stationary?

(b) Which model do you suggest?

(c) Based on your answer in (b), which transformation would you use in order to obtain stationarity? What do you call the model for the transformed time series? What do you call the model for the original time series?

(d) According to your modelling, what would be the forecasted value of the stock price in time point 400? (This has to be done approximately.)

(e) Can we use some particular test here to decide analytically whether we have stationarity or not? If so, shortly describe the construction of this test.



4. (18p) Below and on the next page we have STATA results for data, where for seven countries we have the dependent variable Y and one independent variable $X1$, both measured in 10 time points. The models are the pooled OLS model and the FEM using dummy variables.

(a) Write down the OLS model using appropriate notation.

(b) Looking at the OLS results, how is $F(1,68)$ computed? How is the corresponding F-test formulated? What is the result here of this test?

(c) Write down the FEM using dummy variables, where country A (or 1) is used as "benchmark". Use appropriate notation.

(d) Looking at the FEM results, what is the main improvement compared with the OLS results?

(e) Perform a suitable test which compares the two models. Result?

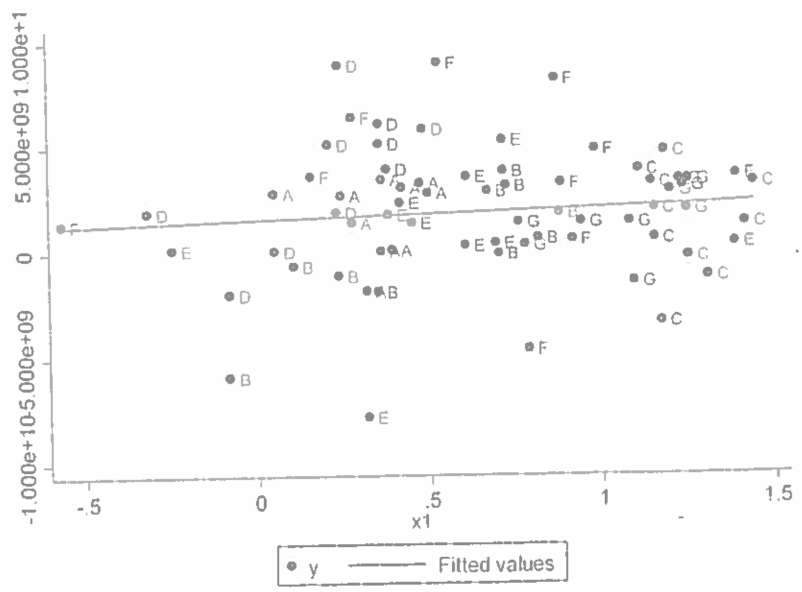
OLS regression

```
. regress y x1

      Source |       SS       df       MS              Number of obs =      70
-------------+------------------------------           F(  1,    68) =    0.40
       Model |  3.7039e+18     1  3.7039e+18           Prob > F      =  0.5272
    Residual |  6.2359e+20    68  9.1705e+18           R-squared     =  0.0059
-------------+------------------------------           Adj R-squared = -0.0087
       Total |  6.2729e+20    69  9.0912e+18           Root MSE      =  3.0e+09

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   4.95e+08   7.79e+08     0.64   0.527    -1.06e+09    2.05e+09
       _cons |   1.52e+09   6.21e+08     2.45   0.017     2.85e+08    2.76e+09
------------------------------------------------------------------------------
```

```
twoway scatter y x1,
mlabel(country) || lfit y x1,
clstyle(p2)
```



```
. xi: regress y x1 i.country
i.country         _Icountry_1-7         (naturally coded; _Icountry_1 omitted)
```

Source	SS	df	MS
Model	1.4276e+20	7	2.0394e+19
Residual	4.8454e+20	62	7.8151e+18
Total	6.2729e+20	69	9.0912e+18

Number of obs	= 70
F(7, 62)	= 2.61
Prob > F	= 0.0199
R-squared	= 0.2276
Adj R-squared	= 0.1404
Root MSE	= 2.8e+09

Fixed Effects using least squares dummy variable model (LSDV)

y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
x1	2.48e+09	1.11e+09	2.24	0.029	2.63e+08	4.69e+09
_Icountry_2	-1.94e+09	1.26e+09	-1.53	0.130	-4.47e+09	5.89e+08
_Icountry_3	-2.60e+09	1.60e+09	-1.63	0.108	-5.79e+09	5.87e+08
_Icountry_4	2.28e+09	1.26e+09	1.81	0.075	-2.39e+08	4.80e+09
_Icountry_5	-1.48e+09	1.27e+09	-1.17	0.247	-4.02e+09	1.05e+09
_Icountry_6	1.13e+09	1.29e+09	0.88	0.384	-1.45e+09	3.71e+09
_Icountry_7	-1.87e+09	1.50e+09	-1.25	0.218	-4.86e+09	1.13e+09
_cons	8.81e+08	9.62e+08	0.92	0.363	-1.04e+09	2.80e+09

5. (18p) True or false? Short motivation/comment also needed.

   (a) Autocorrelation in residuals can be tested for using the Ljung-Box test.

   (b) The expectation of a stationary AR-model without the constant term δ is always 0.

   (c) The Durbin h-test can be used to detect autocorrelation in AR-models.

   (d) Rejection of the null hypothesis in the Ljung-Box test means that we must have identified a nonstationary process.

   (e) In the Koyck-model, $Cor(v_t, v_{t-1}) = 0$, where v_t is the disturbance variable.

   (f) In first-order smoothing, an increased value of the discount factor λ means more smoothing, not less.

6. (13p) To decribe yearly GNP (Gross National Product) values for Sweden, the following model was used:

$$y_t = \delta + \frac{\sum_{k=1}^{m+1} z_{t-k}}{m} + z_t,$$

   where z_t is Gaussian white noise with variance 1 and m is some positive integer.

   (a) Identify the model.

   (b) Derive expressions for $E(y_t), V(y_t)$ and ρ_1.

Formula sheet, Econometrics II, Spring 2017

Under the simple linear model $y_t = \beta_1 + \beta_2 x_t + u_t$, where $u_t \sim N(0, \sigma^2)$ and given independent pairs of observations $(y_1, x_1), \ldots, (y_n, x_n)$, the OLS (and ML) estimators are:

$$\widehat{\beta}_1 = \overline{y} - \widehat{\beta}_2 \overline{x}$$

$$\widehat{\beta}_2 = \frac{\sum (x_t - \overline{x})(y_t - \overline{y})}{\sum (x_t - \overline{x})^2}$$

$$\widehat{\sigma}^2 = \frac{RSS}{n-2} = \frac{\sum (y_t - \widehat{y}_t)^2}{n-2}$$

where $\widehat{y}_t = \widehat{\beta}_1 + \widehat{\beta}_2 x_t$ and where $E(\widehat{\beta_1}) = \beta_1$, $E(\widehat{\beta_2}) = \beta_2$ and $E(\widehat{\sigma}^2) = \sigma^2$

Comparing an "old" model with a "new" (larger):

$$F = \frac{(ESS_{new} - ESS_{old})/\text{number of new regressors}}{RSS_{new}/(n - \text{number of parameters in the new model})}$$

$$= \frac{(R^2_{new} - R^2_{old})/\text{number of new regressors}}{(1 - R^2_{new})/(n - \text{number of parameters in the new model})}$$

Comparing an "unrestricted" model with a "restricted":

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} = \frac{(R^2_{UR} - R^2_R)/m}{(1 - R^2_{UR})/(n-k)}$$

where m is the number of linear constraints and k is the number of parameters in the unrestricted model.

Dynamic models: $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_{t-1} + v_t$

Koyck: $y_t = \alpha(1-\lambda) + \beta_0 x_t + \lambda y_{t-1} + v_t$
Adaptive expectations: $y_t = \gamma\beta_0 + \gamma\beta_1 x_t + (1-\gamma)y_{t-1} + (u_t - (1-\gamma)u_{t-1})$
Partial adjustment: $y_t = \delta\beta_0 + \delta\beta_1 x_t + (1-\delta)y_{t-1} + \delta u_t$

The Durbin Watson d statistic:

$$d = \frac{\sum_{t=2}^{n} (\widehat{u}_t - \widehat{u}_{t-1})^2}{\sum_{t=1}^{n} \widehat{u}_t^2}$$

The Durbin h statistic:

$$h = \widehat{\rho} \sqrt{\frac{n}{1 - n\left[\hat{V}(\widehat{\alpha}_2)\right]}} \approx N(0,1), \text{ if } \rho = 0$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = \frac{1}{n}\sum_{t=1}^{n}[y_t - \widehat{y}_t(t-1)]^2$$

Autocorrelation function:

$$\rho_k = \frac{Cov(y_t, y_{t+k})}{V(y_t)}, \quad k = 0, 1, 2, \ldots$$

Sample correlation function:

$$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n-k}(y_t - \bar{y})^2}, \quad k = 0, 1, 2, \ldots$$

Simple moving average:

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} y_t$$

First-order exponential smoothing:

$$\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$$

Second-order exponential smoothing:

$$\tilde{y}_T^{(2)} = \lambda\tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)},$$

where $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$

Holt's method:

$$\begin{aligned} L_t &= \alpha y_t + (1-\alpha)(L_{t-1} + T_{t-1}) \\ T_t &= \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1} \end{aligned}$$

$$\widehat{y}_{T+\tau}(T) = L_T + \tau T_T, \quad \tau = 1, 2, \ldots$$

Forecast under a constant process:

$$\widehat{y}_{T+\tau}(T) = \widetilde{y}_T \quad \tau = 1, 2, ...$$

Forecast under a linear trend:

$$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T}\tau,$$

where $\hat{y}_T = \hat{\beta}_{0,T} + \hat{\beta}_{1,T}T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$

For white noise:

$$\hat{\rho}_k \approx N(0, 1/n)\ , k = 1, 2, \ldots$$

The Q statistic:

$$Q = n\sum_{k=1}^{K}\hat{\rho}_k^2 \approx \chi^2(K)$$

The Ljung-Box statistic:

$$Q_{LB} = n\,(n+2)\sum_{k=1}^{K}\left(\frac{\hat{\rho}_k^2}{n-k}\right) \approx \chi^2(K)$$

ARMA(p,q):

$$y_t = \delta + \sum_{i=1}^{p}\varphi_i y_{t-i} + \varepsilon_t - \sum_{i=1}^{q}\theta_i\varepsilon_{t-i}$$

Stationarity and invertibility conditions for some time series models:

Model	Stationarity conditions	Invertibility conditions
AR(1)	$\|\phi_1\| < 1$	None
AR(2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $\|\phi_2\| < 1$	None
MA(1)	None	$\|\theta_1\| < 1$
MA(2)	None	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\|\theta_2\| < 1$
ARMA(1,1)	$\|\phi_1\| < 1$	$\|\theta_1\| < 1$
ARMA(2,2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $\|\phi_2\| < 1$	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\|\theta_2\| < 1$

The Yule-Walker equations for AR(p):

$$\rho_k = \sum_{i=1}^{p} \phi_i \rho_{k-i}, \quad k = 1, 2, \ldots$$

TABLE D.3 Upper Percentage Points of the F Distribution

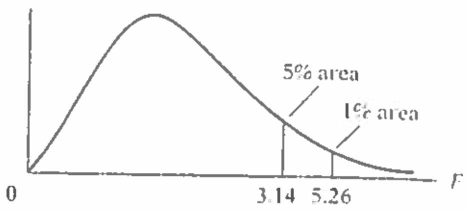
Example

$\Pr(F > 1.59) = 0.25$

$\Pr(F > 2.42) = 0.10$ for df $N_1 = 10$

$\Pr(F > 3.14) = 0.05$ and $N_2 = 9$

$\Pr(F > 5.26) = 0.01$



df for denominator N_2		df for numerator N_1											
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.05	161	200	216	225	230	234	237	239	241	242	243	244
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, Vol. 1, 3d ed., table 18, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

F-table continued

df for numerator N_1													df for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	
9.49	9.58	9.63	9.67	9.71	9.74	9.76	9.78	9.80	9.82	9.84	9.85	.25	1
61.2	61.7	62.0	62.3	62.5	62.7	62.8	63.0	63.1	63.2	63.3	63.3	.10	
246	248	249	250	251	252	252	253	253	254	254	254	.05	
3.41	3.43	3.43	3.44	3.45	3.45	3.46	3.47	3.47	3.48	3.48	3.48	.25	2
9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.48	9.49	9.49	9.49	.10	
19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	.05	
99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	.01	
2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	.25	3
5.20	5.18	5.18	5.17	5.16	5.15	5.15	5.14	5.14	5.14	5.14	5.13	.10	
8.70	8.66	8.64	8.62	8.59	8.58	8.57	8.55	8.55	8.54	8.53	8.53	.05	
26.9	26.7	26.6	26.5	26.4	26.4	26.3	26.2	26.2	26.2	26.1	26.1	.01	
2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	.25	4
3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.78	3.77	3.76	3.76	.10	
5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.66	5.65	5.64	5.63	.05	
14.2	14.0	13.9	13.8	13.7	13.7	13.7	13.6	13.6	13.5	13.5	13.5	.01	
1.89	1.88	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.87	1.87	.25	5
3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.10	.10	
4.62	4.56	4.53	4.50	4.46	4.44	4.43	4.41	4.40	4.39	4.37	4.36	.05	
9.72	9.55	9.47	9.38	9.29	9.24	9.20	9.13	9.11	9.08	9.04	9.02	.01	
1.76	1.76	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	.25	6
2.87	2.84	2.82	2.80	2.78	2.77	2.76	2.75	2.74	2.73	2.73	2.72	.10	
3.94	3.87	3.84	3.81	3.77	3.75	3.74	3.71	3.70	3.69	3.68	3.67	.05	
7.56	7.40	7.31	7.23	7.14	7.09	7.06	6.99	6.97	6.93	6.90	6.88	.01	
1.68	1.67	1.67	1.66	1.66	1.66	1.65	1.65	1.65	1.65	1.65	1.65	.25	7
2.63	2.59	2.58	2.56	2.54	2.52	2.51	2.50	2.49	2.48	2.48	2.47	.10	
3.51	3.44	3.41	3.38	3.34	3.32	3.30	3.27	3.27	3.25	3.24	3.23	.05	
6.31	6.16	6.07	5.99	5.91	5.86	5.82	5.75	5.74	5.70	5.67	5.65	.01	
1.62	1.61	1.60	1.60	1.59	1.59	1.59	1.58	1.58	1.58	1.58	1.58	.25	8
2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.32	2.31	2.30	2.29	.10	
3.22	3.15	3.12	3.08	3.04	2.02	3.01	2.97	2.97	2.95	2.94	2.93	.05	
5.52	5.36	5.28	5.20	5.12	5.07	5.03	4.96	4.95	4.91	4.88	4.86	.01	
1.57	1.56	1.56	1.55	1.55	1.54	1.54	1.53	1.53	1.53	1.53	1.53	.25	9
2.34	2.30	2.28	2.25	2.23	2.22	2.21	2.19	2.18	2.17	2.17	2.16	.10	
3.01	2.94	2.90	2.86	2.83	2.80	2.79	2.76	2.75	2.73	2.72	2.71	.05	
4.96	4.81	4.73	4.65	4.57	4.52	4.48	4.42	4.40	4.36	4.33	4.31	.01	

(Continued)

TABLE D.3 Upper Percentage Points of the F Distribution *(Continued)*

df for denominator N_2	Pr	df for numerator N_1 1	2	3	4	5	6	7	8	9	10	11	12
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
11	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
13	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

F-table continued

df for numerator N_1

df for denominator N_2	Pr	15	20	24	30	40	50	60	100	120	200	500	∞
10	.25	1.53	1.52	1.52	1.51	1.51	1.50	1.50	1.49	1.49	1.49	1.48	1.48
	.10	2.24	2.20	2.18	2.16	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.06
	.05	2.85	2.77	2.74	2.70	2.66	2.64	2.62	2.59	2.58	2.56	2.55	2.54
	.01	4.56	4.41	4.33	4.25	4.17	4.12	4.08	4.01	4.00	3.96	3.93	3.91
11	.25	1.50	1.49	1.49	1.48	1.47	1.47	1.47	1.46	1.46	1.46	1.45	1.45
	.10	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	2.00	1.99	1.98	1.97
	.05	2.72	2.65	2.61	2.57	2.53	2.51	2.49	2.46	2.45	2.43	2.42	2.40
	.01	4.25	4.10	4.02	3.94	3.86	3.81	3.78	3.71	3.69	3.66	3.62	3.60
12	.25	1.48	1.47	1.46	1.45	1.45	1.44	1.44	1.43	1.43	1.43	1.42	1.42
	.10	2.10	2.06	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90
	.05	2.62	2.54	2.51	2.47	2.43	2.40	2.38	2.35	2.34	2.32	2.31	2.30
	.01	4.01	3.86	3.78	3.70	3.62	3.57	3.54	3.47	3.45	3.41	3.38	3.36
13	.25	1.46	1.45	1.44	1.43	1.42	1.42	1.42	1.41	1.41	1.40	1.40	1.40
	.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.88	1.86	1.85	1.85
	.05	2.53	2.46	2.42	2.38	2.34	2.31	2.30	2.26	2.25	2.23	2.22	2.21
	.01	3.82	3.66	3.59	3.51	3.43	3.38	3.34	3.27	3.25	3.22	3.19	3.17
14	.25	1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.39	1.39	1.39	1.38	1.38
	.10	2.01	1.96	1.94	1.91	1.89	1.87	1.86	1.83	1.83	1.82	1.80	1.80
	.05	2.46	2.39	2.35	2.31	2.27	2.24	2.22	2.19	2.18	2.16	2.14	2.13
	.01	3.66	3.51	3.43	3.35	3.27	3.22	3.18	3.11	3.09	3.06	3.03	3.00
15	.25	1.43	1.41	1.41	1.40	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.36
	.10	1.97	1.92	1.90	1.87	1.85	1.83	1.82	1.79	1.79	1.77	1.76	1.76
	.05	2.40	2.33	2.29	2.25	2.20	2.18	2.16	2.12	2.11	2.10	2.08	2.07
	.01	3.52	3.37	3.29	3.21	3.13	3.08	3.05	2.98	2.96	2.92	2.89	2.87
16	.25	1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.36	1.35	1.35	1.34	1.34
	.10	1.94	1.89	1.87	1.84	1.81	1.79	1.78	1.76	1.75	1.74	1.73	1.72
	.05	2.35	2.28	2.24	2.19	2.15	2.12	2.11	2.07	2.06	2.04	2.02	2.01
	.01	3.41	3.26	3.18	3.10	3.02	2.97	2.93	2.86	2.84	2.81	2.78	2.75
17	.25	1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34	1.34	1.34	1.33	1.33
	.10	1.91	1.86	1.84	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.69	1.69
	.05	2.31	2.23	2.19	2.15	2.10	2.08	2.06	2.02	2.01	1.99	1.97	1.96
	.01	3.31	3.16	3.08	3.00	2.92	2.87	2.83	2.76	2.75	2.71	2.68	2.65
18	.25	1.39	1.38	1.37	1.36	1.35	1.34	1.34	1.33	1.33	1.32	1.32	1.32
	.10	1.89	1.84	1.81	1.78	1.75	1.74	1.72	1.70	1.69	1.68	1.67	1.66
	.05	2.27	2.19	2.15	2.11	2.06	2.04	2.02	1.98	1.97	1.95	1.93	1.92
	.01	3.23	3.08	3.00	2.92	2.84	2.78	2.75	2.68	2.66	2.62	2.59	2.57
19	.25	1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31	1.30
	.10	1.86	1.81	1.79	1.76	1.73	1.71	1.70	1.67	1.67	1.65	1.64	1.63
	.05	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.94	1.93	1.91	1.89	1.88
	.01	3.15	3.00	2.92	2.84	2.76	2.71	2.67	2.60	2.58	2.55	2.51	2.49
20	.25	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31	1.30	1.30	1.29
	.10	1.84	1.79	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61
	.05	2.20	2.12	2.08	2.04	1.99	1.97	1.95	1.91	1.90	1.88	1.86	1.84
	.01	3.09	2.94	2.86	2.78	2.69	2.64	2.61	2.54	2.52	2.48	2.44	2.42

(Continued)

TABLE D.3 Upper Percentage Points of the F Distribution *(Continued)*

df for denominator N_2	df for numerator N_1												
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

F-table continued

	df for numerator N_1												df for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	
1.36	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.30	1.29	1.29	1.28	.25	22
1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.61	1.60	1.59	1.58	1.57	.10	
2.15	2.07	2.03	1.98	1.94	1.91	1.89	1.85	1.84	1.82	1.80	1.78	.05	
2.98	2.83	2.75	2.67	2.58	2.53	2.50	2.42	2.40	2.36	2.33	2.31	.01	
1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.28	1.28	1.27	1.27	1.26	.25	24
1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.58	1.57	1.56	1.54	1.53	.10	
2.11	2.03	1.98	1.94	1.89	1.86	1.84	1.80	1.79	1.77	1.75	1.73	.05	
2.89	2.74	2.66	2.58	2.49	2.44	2.40	2.33	2.31	2.27	2.24	2.21	.01	
1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.26	1.26	1.26	1.25	1.25	.25	26
1.76	1.71	1.68	1.65	1.61	1.59	1.58	1.55	1.54	1.53	1.51	1.50	.10	
2.07	1.99	1.95	1.90	1.85	1.82	1.80	1.76	1.75	1.73	1.71	1.69	.05	
2.81	2.66	2.58	2.50	2.42	2.36	2.33	2.25	2.23	2.19	2.16	2.13	.01	
1.33	1.31	1.30	1.29	1.28	1.27	1.27	1.26	1.25	1.25	1.24	1.24	.25	28
1.74	1.69	1.66	1.63	1.59	1.57	1.56	1.53	1.52	1.50	1.49	1.48	.10	
2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.73	1.71	1.69	1.67	1.65	.05	
2.75	2.60	2.52	2.44	2.35	2.30	2.26	2.19	2.17	2.13	2.09	2.06	.01	
1.32	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	.25	30
1.72	1.67	1.64	1.61	1.57	1.55	1.54	1.51	1.50	1.48	1.47	1.46	.10	
2.01	1.93	1.89	1.84	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62	.05	
2.70	2.55	2.47	2.39	2.30	2.25	2.21	2.13	2.11	2.07	2.03	2.01	.01	
1.30	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.21	1.20	1.19	1.19	.25	40
1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.43	1.42	1.41	1.39	1.38	.10	
1.92	1.84	1.79	1.74	1.69	1.66	1.64	1.59	1.58	1.55	1.53	1.51	.05	
2.52	2.37	2.29	2.20	2.11	2.06	2.02	1.94	1.92	1.87	1.83	1.80	.01	
1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.16	1.15	1.15	.25	60
1.60	1.54	1.51	1.48	1.44	1.41	1.40	1.36	1.35	1.33	1.31	1.29	.10	
1.84	1.75	1.70	1.65	1.59	1.56	1.53	1.48	1.47	1.44	1.41	1.39	.05	
2.35	2.20	2.12	2.03	1.94	1.88	1.84	1.75	1.73	1.68	1.63	1.60	.01	
1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.10	.25	120
1.55	1.48	1.45	1.41	1.37	1.34	1.32	1.27	1.26	1.24	1.21	1.19	.10	
1.75	1.66	1.61	1.55	1.50	1.46	1.43	1.37	1.35	1.32	1.28	1.25	.05	
2.19	2.03	1.95	1.86	1.76	1.70	1.66	1.56	1.53	1.48	1.42	1.38	.01	
1.23	1.21	1.20	1.18	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.06	.25	200
1.52	1.46	1.42	1.38	1.34	1.31	1.28	1.24	1.22	1.20	1.17	1.14	.10	
1.72	1.62	1.57	1.52	1.46	1.41	1.39	1.32	1.29	1.26	1.22	1.19	.05	
2.13	1.97	1.89	1.79	1.69	1.63	1.58	1.48	1.44	1.39	1.33	1.28	.01	
1.22	1.19	1.18	1.16	1.14	1.13	1.12	1.09	1.08	1.07	1.04	1.00	.25	∞
1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.18	1.17	1.13	1.08	1.00	.10	
1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.22	1.17	1.11	1.00	.05	
2.04	1.88	1.79	1.70	1.59	1.52	1.47	1.36	1.32	1.25	1.15	1.00	.01	

Statistiska institutionen

Stockholms universitet

Rättningsblad

Datum: 31/5-2017

Sal: Värtasalen

Tenta: Tidsserieanalys/Ekonometri II

Kurs: Ekonometri

ANONYMKOD: ETI-0021

☑ Jag godkänner att min tenta får läggas ut anonymt på hemsidan som studentsvar.

OBS! SKRIV ÄVEN PÅ BAKSIDAN AV SKRIVBLADEN

Markera besvarade uppgifter med kryss

1	2	3	4	5	6	7	8	9	Antal inl. blad
X	X	X	X	X	X				7
Lär.ant. 18	15	18	18	12	13				

POÄNG	BETYG	Lärarens sign.
94	A	PGA

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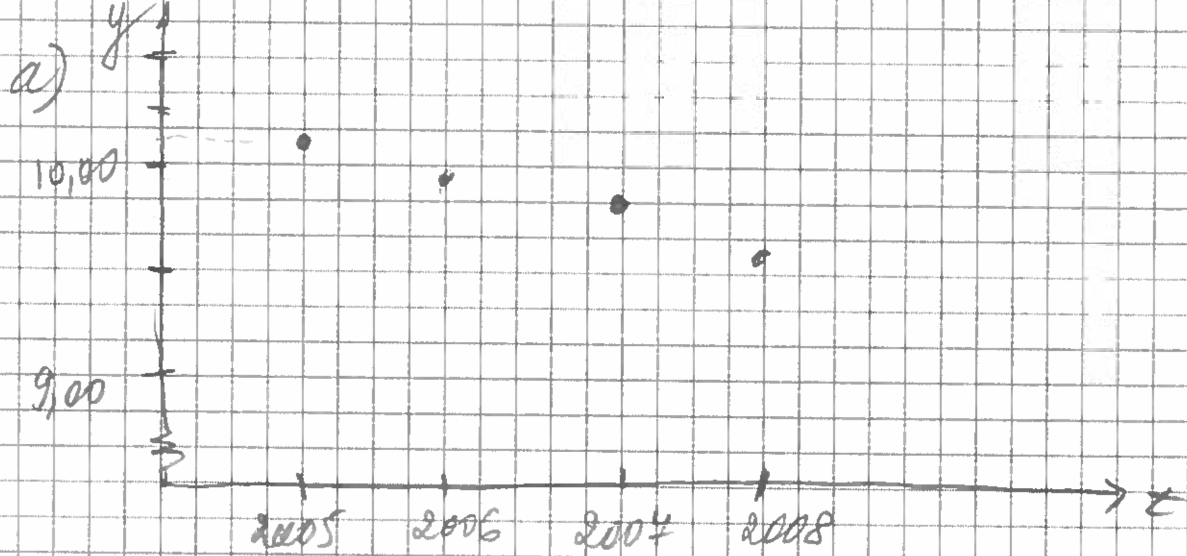
Exercise 1

a)



Solution:

- first, we make a plot of the units over time to understand what smoothing method will be most appropriate;
- although we have quite a few observations, we can however spot a somewhat negative (downward) trend characterizing the series. Therefore, I chose the second-order expon. smoothing method in this case;
- $\tilde{y}_T^{(2)} = \lambda \cdot \tilde{y}_T^{(1)} + (1-\lambda) \cdot \tilde{y}_{T-1}^{(2)}$ (2nd order)

  $\tilde{y}_T^{(1)} = \lambda \cdot y_T + (1-\lambda) \cdot \tilde{y}_{T-1}$ (based on the 1st order)
- $\lambda = 0,3$
- $\tilde{y}_0 = \bar{y} = \frac{\sum_{t=1}^{4} y_t}{4} = \frac{10,18 + 9,95 + 9,85 + 9,69}{4} = \frac{39,68}{4} =$

  $= 9,92.$

Ad. Ex. 1

- Results from the first-order exp smoothing

$\tilde{y}_1 = 0,3 \cdot y_1 + 0,7 \cdot \tilde{y}_0 = 0,3 \cdot 10,18 + 0,7 \cdot 9,92 =$
$\approx 9,998$

$\tilde{y}_2 = 0,3\ y_2 + 0,7 \cdot \tilde{y}_1 = 0,3 \cdot 9,96 + 0,7 \cdot 9,998 =$
$\approx 9,986$

$\tilde{y}_3 = 0,3 \cdot 9,85 + 0,7 \cdot 9,986 \approx 9,946$

$\tilde{y}_4 = 0,3 \cdot 9,69 + 0,7 \cdot 9,946 \approx 9,869$

- Results from the second ord. exp. smoothing:

$\rightarrow \tilde{y}_1^{(2)} = 0,3 \cdot \tilde{y}_1^{(1)} + 0,7 \cdot \tilde{y}_0^{(2)} = \quad (\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)} = 9,998)$
$= 0,3 \cdot 9,998 + 0,7 \cdot 9,998 = \underline{9,998}$

$\tilde{y}_2^{(2)} = 0,3 \cdot \tilde{y}_2^{(1)} + 0,7 \cdot \tilde{y}_1^{(2)} = 0,3 \cdot 9,986 + 0,7 \cdot 9,998 =$
$= 9,994$

$\tilde{y}_3^{(2)} = 0,3 \cdot \tilde{y}_3^{(1)} + 0,7\ \tilde{y}_2^{(2)} = 0,3 \cdot 9,946 + 0,7 \cdot 9,994 =$
$\approx 9,979$

$\tilde{y}_4^{(2)} = 0,3 \cdot \tilde{y}_4^{(1)} + 0,7 \cdot \tilde{y}_3^{(2)} = 0,3 \cdot 9,869 + 0,7 \cdot 9,979 =$
$= 9,946$

$\Rightarrow \underline{\hat{y}_T = 2\ \tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}}$

$\hat{y}_1 = 2 \cdot \tilde{y}_1^{(1)} - \tilde{y}_1^{(2)} = 2 \cdot 9,998 - 9,998 = 9,998$

$\hat{y}_2 = 2 \cdot \tilde{y}_2^{(1)} - \tilde{y}_2^{(2)} = 2 \cdot 9,986 - 9,994 = 9,978$

$\hat{y}_3 = 2\ \tilde{y}_3^{(1)} - \tilde{y}_3^{(2)} = 2 \cdot 9,946 - 9,979 = 9,913$

$\hat{y}_4 = 2\ \tilde{y}_4^{(1)} - \tilde{y}_4^{(2)} = 2 \cdot 9,869 - 9,946 = 9,792$

OK

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(ctd., Ex. 1)

- Forecasting under a linear trend:

$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T} \cdot \tau$

$\hat{y}_{4+1}(4) = \hat{y}_4 + \hat{\beta}_{1,T} \cdot 1$ $\left(\begin{array}{l} 4 = 2008 \\ 4+1 = 5 = 2009 \end{array}\right)$

$\rightarrow \hat{\beta}_{1,T} = \dfrac{\sum(x_t - \bar{x})(y_t - \bar{y})}{\sum(x_t - \bar{x})^2} =$ (we use here the OLS estimation for calculations simplicity. In practice, we should have used the WLS method)

$= \dfrac{\sum x_t \cdot y_t - n \cdot \bar{x} \cdot \bar{y}}{\sum x_t^2 - n \cdot \bar{x}^2}$

$\rightarrow x_t = \{1, 2, 3, 4\}$, $\bar{x} = \dfrac{1+2+3+4}{4} = 2{,}5$

$y_t = \{10{,}18; 9{,}96; 9{,}85; 9{,}69\}$, $\bar{y} = 9{,}92$

	x_t	y_t	$x_t y_t$	x_t^2
2005	1	10,18	10,18	1
2006	2	9,96	19,92	4
2007	3	9,85	29,55	9
2008	4	9,69	38,76	16
	$\sum$		98,41	30

$\bar{x} \cdot \bar{y} = 2{,}5 \cdot 9{,}92 = 24{,}8$

$\bar{x}^2 = 2{,}5^2 = 6{,}25$

$\rightarrow \hat{\beta}_{1,4} = \dfrac{98{,}41 - 4 \cdot 24{,}8}{30 - 4 \cdot 6{,}25} = -\dfrac{0{,}79}{5} = -0{,}158$

$\rightarrow \boxed{\hat{y}_5(4)} = 9{,}792 - 0{,}158 = \boxed{9{,}634}$ ← prediction for 2009

OK

The predicted result for Usain Bolt in 2009 is 9,634 (which seems quite a good prediction when compared to the actual value of 9,5)

Exercise 2

a) 2 AR(1) processes; value of φ in both cases?

- We have two AR(1) processes.
- An AR(1) process in general can be modelled as following:

$$y_t = \delta + \varphi_1 \cdot y_{t-1} + \varepsilon_t \quad (\varepsilon_t \sim \text{white noise}).$$

- For AR(1) process the relationship between ρ_k (autocorrelation at lag k) and φ_1 is given by: $\boxed{\rho_k = \varphi_1^k}$
- Therefore, for the process (1):

$\rho_1 = -0,5 \rightarrow -0,5 = \varphi_1^1$ $\quad (y_t = \delta - 0,5 \cdot y_{t-1} + \varepsilon_t$

$\Rightarrow \boxed{\varphi_1 = -0,5}$ (process (1))

- For the process (2):

$\rho_1 = 0,5 \rightarrow 0,5 = \varphi_1^1$

$\Rightarrow \boxed{\varphi_1 = 0,5}$ (process (2) : $y_t = \delta + 0,5 \cdot y_{t-1} + \varepsilon_t$).

OK

b) We have ACF and PACF for two stat. process

Type of models?

(1): ACF: we have one sign. spike at lag 1, all other corr. coefficients are equal to zero.

PACF: the partial autocorr. coeff. are exponen. decaying to zero.

Such a pattern is typical for a MA process. In our case it's MA(1).

(2): ACF: one sign. spike at lag 1 (as in (1));

PACF: expon. decay to zero (with interchanging coefficients' signs (+, -, +, -)).

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cont. Ex. 2

The same model type as for the process (1): MA(1)

What can we say about parameters? Diff!

Generally, a MA(1) process looks as following:

$$y_t = \delta + \varepsilon_t - \theta_1 \cdot \varepsilon_{t-1}$$

→ For the process (1):

$y_t = \delta + \varepsilon_t - \theta_1 \cdot \varepsilon_{t-1}$; θ_1 will be positive $(\theta_1 > 0)$ as we have negative autocorr for lag 1.

→ For the process (2):

$y_t = \delta + \varepsilon_t + \theta_1 \cdot \varepsilon_{t-1}$; θ_1 will be negative $(\theta_1 < 0)$ to account for the positive autocorr. at lag 1.

Note: $\rho_1 = -\frac{\theta_1}{1+\theta_1^2}$ (If $\theta_1 > 0 \Rightarrow \rho_1 < 0$, if $\theta_1 < 0 \Rightarrow \rho_1 > 0$)

Ok /15

Exercise 3.

- n = 369, IBM closing stock prices.

a) We can see that the series is not stationary both from the plot itself and from the correlogram.

Ex 3

→ Plot: it's clear, e.g., that the series possesses neither a constant mean or variance.

→ ACF: the autocorrelation coefficients are persistently close to one up till lag 25 (no lag 1 is 1) (extremely slow decay).

→ PACF: one significant lag 1 which is equal to 1, all others insignificant. OK

b) y_t behaves like a random walk model; I would suggest RWM with drift

$$y_t = \delta + y_{t-1} + \varepsilon_t \quad (\varepsilon_t \text{ a white noise}),$$ OK

c) I suggest differencing the model (first difference procedure) in order to obtain stationarity

$$y_t - y_{t-1} = \delta + \varepsilon_t$$

$$w_t = (1 - B)\, y_t = \delta + \varepsilon_t$$

→ the w_t (transformed) model is $I(0)$, a constant stationary process

→ y_t (the original time series) is ARIMA $(0, 1, 0)$ or $I(1)$. OK

d) the forecasted value at $t = 400$,

→ $\hat{y}_{369+31}(369) = 31 \cdot \delta + y_{369} = 31\delta + 350$

(from the plot $y_{369} \approx 350$). (for the RWM with drift)

→ If we apply the first order smoothing for the transformed data, w_t, then

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cont, Ex. 3

$\hat{y}_{369+31}(369) = \hat{y}_{369}$

If we had a RWM without drift, $\hat{y}_{400}(369) = \hat{y}(369) \approx 350$

Ok

c) We can use, e.g., the DF unit root test. It test whether $\rho = 1$ (the autocorr coeff at lag 1 is equal to 1). To test this the data is transfor. so that $\delta = \rho - 1$. $\Rightarrow$

→ $H_0: \delta = 0 \ (\rho = 1) \rightarrow$ non-stationarity
$H_1: \delta < 0 \rightarrow$ stationarity.

→ as usual t-tests will be misleading here if the data is non-stationary the assumption is that under H_0 the test-statistic follows the Tau distribution

→ if H_0 is rejected, then the data is stationary and we can then use the t-values for our estimator

→ the DF tests three hypotheses:

different τ-values in the tables

(1) the data is RWM without drift
(2) - " - RWM with drift
(3) - " - has a deterministic and a stochastic trend and therefore can be stationary around the trend.

+ $\boxed{8.3}$

→ If the series exhibit serial correlation in the error terms then the ADF (augmented DF) test is used. Ok

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Exercise 4

- $n = 7$ (seven countries), $t = 1 \dots 10$
- y (dependent)
  X_1 - ...
- Pooled OLS and FEM

a) OLS model:

$$y_{it} = \beta_0 + \beta_1 \cdot X_{1it} + u_{it} \quad (u_{it} - \text{white noise})$$

Ok

b) • $F_{test}\binom{1}{68} = \frac{ESS/1}{RSS/68} = \frac{3,4039 \cdot 10^{18}/1}{6,2359 \cdot 10^{20}/68} =$

$$= \frac{3,4039/1}{623,59/68} = \frac{3,4039}{9,1704} \approx 0,40$$

• $H_0: \beta_1 = 0$
  $H_a: \beta_1 \neq 0$

• p-value of the F-test is equal to 0,5272 therefore we can not reject H_0 at any reasonable sign. level. The conclusion is that the model is not significant (on the whole)

Ok

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Ch4, Ex.3

c) FEM, A(1) → Benchmark
(we have 4 countries)

$$Y_{it} = \alpha_1 + \alpha_2 \cdot D_{2i} + \alpha_3 \cdot D_{3i} + \alpha_4 \cdot D_{4i} + \alpha_5 \cdot D_{5i} + \alpha_6 \cdot D_{6i} + \alpha_7 \cdot D_{7i} + \beta_1 \cdot X_{1t} + u_{it}$$

where $D_i = \begin{cases} 1 & \text{if country } i \\ 0, & \text{otherwise} \end{cases}$

OK

d) If we have to find one main improvement it 'would' say that the variable X_1 is significant (p-value = 0,029, at 2,9% sign. level if we use the FEM model. The OLS results produced a highly insign. result for X_1 (p-value = 0,527).

This improvement is quite typically reflected with the F-test. The result for the FEM model is significant now (p-value = 0,0199)

Ok

e) To compare the pooled OLS model with the FEM, we can use Restricted F-test.

$$F = \frac{(R^2_{ur} - R^2_r)/m}{(1 - R^2_{ur})/(n-k)} \overset{H_0}{\sim} F\left(\frac{m}{n-k}\right)$$

cont. Qu 4

- H_0: $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = 0$.
  H_a: at least one of the differential intercept coefficients is not equal to zero

- m = {number of linear constraints} = 6
  n = 70
  k = {number of parameters in the unrestricted model} = 8

- $F_{obs} = \dfrac{(0{,}2246 - 0{,}0059)/6}{(1 - 0{,}2276)/(70-8)} = \dfrac{0{,}2217/6}{0{,}7724/62} =$
  $\approx \dfrac{0{,}03695}{0{,}01246} = 2{,}9654.$

- $F^{(0{,}05)}_{(6;\,62)}(crit) = 2{,}25$ (we use $F^{0{,}05}_{(6,60)}$ no value specified for 62 in the Table)
  $F^{(0{,}01)}_{(6,62)}(crit) = 3{,}12.$

- $F_{obs} = 2{,}97 > F^{(0{,}05)}_{crit} = 2{,}25$, but
  $F_{obs} \approx 2{,}97 < F^{(0{,}01)}_{crit} = 3{,}12.$

Therefore, at 5% sign. level we can reject the null hypothesis in favor of the FEM model, however at 1% sign. level we can not reject the H_0 and therefore can not draw conclusion that the FEM model will perform better than the OLS pooled approach

OK

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Exercise 5

a) FALSE. The Ljung-Box test is one of the tests that tests for the stationarity of the times series, more specifically if the autocorrel. coefficients up to a certain lag are jointly equal to zero. The Durbin h statistic (or Durbin stat) can be used to test for the autocorrelation in the se

b) TRUE:

$$y_t = \delta + \varphi_1 \cdot y_{t-1} + \varphi_2 \cdot y_{t-2} + \dots + \varphi_p \cdot y_{t-p} + \varepsilon_t$$

$$E(y_t) = E(\delta) + E(\varphi_1 \cdot y_{t-1}) + E(\varphi_2 \cdot y_{t-2}) + \dots E(\varphi_p \cdot y_{t-p})$$

If the series is stationary then

$$\mu = E(y_t) = E(y_{t-1}) = \dots$$

$$\mu = \delta + \varphi_1 \cdot \mu + \varphi_2 \mu + \dots \varphi_p \cdot \mu$$

$$\mu(1 - \varphi_1 - \varphi_2 - \dots - \varphi_p) = \delta$$

$$\mu = \frac{\delta}{1 - \varphi_1 - \varphi_2 - \dots - \varphi_p} \Rightarrow$$ therefore, if $\delta = 0 \Rightarrow \mu = 0 \Rightarrow$ $E(y_t) = 0$. OK

c) TRUE: Durbin-Watson statistic is not valid for ~~detect~~ detective autocorrelation in the models that contain autoregressive elements. Therefor another test, Durbin h statistic, has been developed that takes into account the autoregressive elements. Since AR-models contain cd. Ex. 5

autoregr. elements we can use the Durbin h-test

OK

d) TRUE. the LB-statistics test the joint hypothesis.

$H_0: \rho_1 = \rho_2 = \dots \rho_k = 0$ (up to lag k).

If reject the H_0, it means that we can not assume that the process is stationary, as we've got evidence that the squared sum of the autocorr. coeff. up to lag k is not equal to zero, therefore at least one of the coefficients must be distinct from zero indicating nonstationarity.

e) FALSE. In the Koyck model

$V_t = U_t - \lambda \cdot U_{t-1}$

$cov(V_t, V_{t-1}) = cov(U_t - \lambda \cdot U_{t-1}) \cdot (U_{t-1} - \lambda U_{t-2}) =$

$= \{ U_t \overset{iid}{\sim} (0, \sigma^2) \} = cov(-\lambda \cdot U_{t-1}^2) + cov($cross-products$) = \{$ cov(cross-products)$=$ $= 0 \} = -\lambda \sigma^2 \neq 0.$

OK

f) FALSE.

$\tilde{y}_T = \lambda \cdot y_T + (1-\lambda) \cdot \tilde{y}_{T-1}, \quad 0 < \lambda < 1$

The more λ is close to 1, the more weight is put on the last observation, and the less smoothed data will be produced. Therefore, an increased λ means less smoothing.

OK

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Exercise 6

- $y_t = \delta + \frac{\sum_{k=1}^{m+1} Z_{t-k}}{m} + Z_t$,

$Z_t \overset{iid}{\sim} N(0,1)$, m – some positive integer,

a) $y_t = \delta + Z_t + \frac{1}{m} \cdot Z_{t-1} + \ldots + \frac{1}{m} \cdot Z_{t-(m+1)}$

the model is: $MA(m+1)$. OK

b) We have the $MA(m+1)$ model, which is stationary by definition.

- $E(y_t) = E\left(\delta + Z_t + \frac{\sum_{k=1}^{m+1} Z_{t-k}}{m}\right) =$

$= \{Z_t \overset{iid}{\sim} (0,1)\} = E(\delta) + 0 = \delta$ OK

- $var(y_t) = var\left(\delta + Z_t + \frac{\sum_{k=1}^{m+1} Z_{t-k}}{m}\right) =$

$= \{var(\delta) = 0\} = 1 + var\left(\frac{1}{m} \cdot \sum_{k=1}^{m+1} Z_{t-k}\right) =$

$= 1 + \frac{m+1}{m^2} = \frac{m^2+m+1}{m^2} = \gamma(0)$ OK

- $\rho_1 = \frac{\gamma(1)}{\gamma(0)} = \frac{cov(y_t, y_{t-1})}{var(y_t)}$

$\boxed{k \le m+1}$

- $\gamma(1) = cov(y_t, y_{t-1}) = cov\left[\left(\frac{\sum_{k=1}^{m+1} Z_{t-k}}{m} + Z_t\right) \times\right.$

$\left.\times \left(\frac{\sum_{k=1}^{m+1} Z_{t-k-1}}{m} + Z_{t-1}\right)\right] = cov\left(\frac{\sum_{k=1}^{m+1} Z_{t-k}}{m}, \frac{\sum_{k=1}^{m+1} Z_{t-k-1}}{m}\right) +$

$\boxed{\text{ctd Ex. 6}}$

$$+ \operatorname{cov}\left(\frac{\sum_{k=1}^{m+1} Z_{t-k}}{m} \cdot Z_{t-1}\right) + \underbrace{\operatorname{cov}\left(Z_t \cdot \frac{\sum_{k=1}^{m+1} Z_{t-k-1}}{m}\right)}_{\left[\substack{0 \\ k=1,\dots}\ \left(Z_t \overset{iid}{\sim} (0,1)\right)\right]} +$$

$$+ \underbrace{\operatorname{cov}(Z_t, Z_{t-1})}_{0\ \left(Z_t \overset{iid}{\sim} (0,1)\right)} =$$

$$= \frac{1}{m^2} \operatorname{cov}\left(\sum_{k=1}^{m+1} Z_{t-k} \cdot \sum_{k=1}^{m+1} Z_{t-k-1}\right) +$$

$$+ \frac{1}{m} \cdot \operatorname{cov}\left(Z_{t-1} \cdot \sum_{k=1}^{m+1} Z_{t-k}\right) = \left\{ \begin{array}{l} u_t \overset{iid}{\sim} N(0,1) \to \\ \operatorname{cov} \text{ of cross-prod.} \\ = 0;\ \operatorname{var}(Z_t) = \operatorname{var} \end{array} \right\} =$$

$$= \frac{m-k+1}{m^2} \cdot \operatorname{Var}(Z_t) + \frac{1}{m} \operatorname{cov}\left(Z_{t-1} \cdot \sum_{k=1}^{m+1} Z_{t-k}\right) =$$

$$= \begin{cases} k = 1: = \dfrac{m-k+1}{m^2} + \dfrac{1}{m} = \dfrac{m-k+1+m}{m^2} = \dfrac{2m-k+1}{m^2} \\[2ex] k > 1: = \dfrac{m-k+1}{m^2} + 0 \end{cases}$$

- $k > 1$: $\rho_1 = \dfrac{\gamma(1)}{\gamma(0)} = \dfrac{m-k+1}{m^2} \cdot \dfrac{m^2}{m^2+m+1} =$

$$= \frac{m-k+1}{m^2+m+1}$$

- $k = 1$: $\rho_1 = \dfrac{2m-k+1}{m^2} \cdot \dfrac{m^2}{m^2+m+1} = \dfrac{2m-k+1}{m^2+m+1}$

OK

1/3

Statistiska institutionen

Stockholms universitet

Rättningsblad

Datum: 31/5-2017

Sal: Värtasalen

Tenta: Tidsserieanalys/Ekonometri II

Kurs: Ekonometri

ANONYMKOD: ETI-0037

☒ Jag godkänner att min tenta får läggas ut anonymt på hemsidan som studentsvar.

OBS! SKRIV ÄVEN PÅ BAKSIDAN AV SKRIVBLADEN

Markera besvarade uppgifter med kryss

1	2	3	4	5	6	7	8	9	Antal inl. blad
X	X	X	X	X	X				6
Lär.ant. 17	15	16	17	15	11				

1k

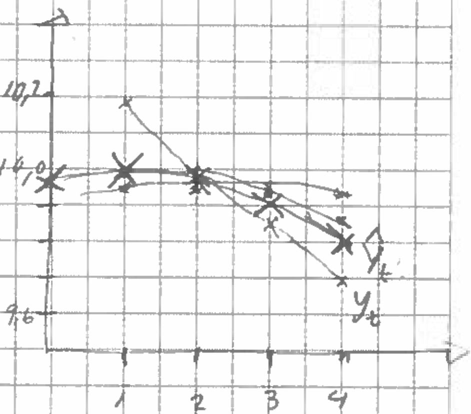
POÄNG	BETYG	Lärarens sign.
91	A	PGA

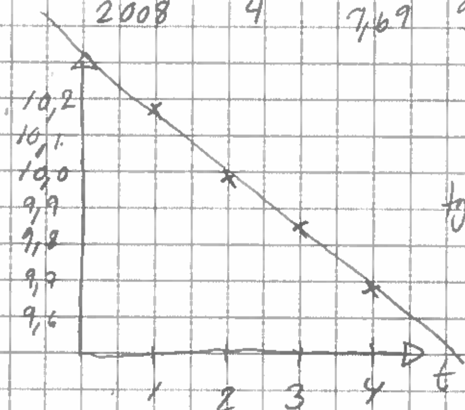
SU, STATISTIK

Skrivsal: Värta **Anonymkod:** ETI - 0037 **Blad nr:** 1

1. a)

År	t	y_t	$\tilde{y}_t^{(1)}$	$\tilde{y}_t^{(2)}$	$\hat{y}_t$
	0		9,92	9,92	9,920
2005	1	10,18	9,98	9,938	10,022
2006	2	9,96	9,974	9,949	9,999
2007	3	9,85	9,937	9,945	9,929
2008	4	9,69	9,863	9,920	9,806





tydlig linjär trend.

$\lambda = 0,3$

$$\bar{y}_t = \frac{10,18 + 9,96 + 9,85 + 9,69}{4} = 9,92$$

EXP. SMOOTHING

(1) $\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$

(2) $\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)}$

where $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$

(1) $\tilde{y}_1^{(1)} = 0,3 \cdot 10,18 + (0,7) \cdot 9,92 = 9,98$ (Skriv)

$\tilde{y}_2^{(1)} = 0,3 \cdot 9,96 + 0,7 \cdot 9,98 = 9,974$

$\tilde{y}_3^{(1)} = 0,3 \cdot 9,85 + 0,7 \cdot 9,974 = 9,937$

$\tilde{y}_4^{(1)} = 0,3 \cdot 9,69 + 0,7 \cdot 9,937 \quad 9,863$

(2) $\tilde{y}_1^{(2)} = 0,3 \cdot 9,98 + 0,7 \cdot 9,92 = 9,938$

$\tilde{y}_2^{(2)} = 0,3 \cdot 9,974 + 0,7 \cdot 9,938 = 9,949$

$\tilde{y}_3^{(2)} = 0,3 \cdot 9,937 + 0,7 \cdot 9,949 = 9,945$

$\tilde{y}_4^{(2)} = 0,3 \cdot 9,863 + 0,7 \cdot 9,945 = 9,920$

Forecast under linear trend:

$$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T}\,\tau$$

where $\hat{y}_T = \hat{\beta}_{0,T} + \hat{\beta}_{1,T} T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$

För att bli av med bias använder vi formeln: $\hat{y}_T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$ och får värdena: $\hat{y}_1 = 10,022$, $\hat{y}_2 = 9,999$, $\hat{y}_3 = 9,929$ & $\hat{y}_4 = 9,806$

Svar: Eftersom vi ser en tydlig linjär trend använder vi dubbel exp. utjämning och kompenserar därefter för bias genom att ta: $\hat{y}_T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$, den utjämnade serien blir som följer: $\hat{y}_1 = 10,22$ $\hat{y}_2 = 9,997$ $\hat{y}_3 = 9,929$ $\hat{y}_4 = 9,806$

$T = 2008$

1b) Forecast under linear trend:

$$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T} \cdot \tau \qquad \tau(\text{här}) = 1$$

$$\hat{y}_T = 9{,}806$$

Vi skattar $\hat{\beta}_{1,T}$ med OLS (ist. f. WLS som vore mer lämpligt)

$$\hat{\beta}_{1,T} = \frac{\sum(x_t - \bar{x})(y_t - \bar{y})}{\sum (x_t - \bar{x})^2}$$

Vi har $x_t = t$

$$\bar{x} = \bar{t} = \frac{1+2+3+4}{4} = 2{,}5$$

$$\bar{y} = \frac{10{,}18 + 9{,}96 + 9{,}85 + 9{,}69}{4} = 9{,}92$$

$$\beta_1 = \frac{(1-2{,}5)(10{,}18-9{,}92)+(2-2{,}5)(9{,}96-9{,}92)+(3-2{,}5)(9{,}85-9{,}92)+(4-2{,}5)(9{,}69-9{,}92)}{(1-2{,}5)^2+(2-2{,}5)^2+(3-2{,}5)^2+(4-2{,}5)^2}$$

$$= \frac{-0{,}79}{5} = \underline{-0{,}158}$$

$$\hat{y}_{T+1}(T) = 9{,}806 - 0{,}158 \cdot 1 = \underline{\underline{9{,}698}} \quad (9{,}63)$$

Svar: Prognosen för 2009 blir att Usain Bolt springer på 9,698 sekunder.

/17

SU, STATISTIK

Skrivsal: Värta Anonymkod: ETI-0037 Blad nr: 2.

2. De är stationära, alltså är $\mu = \frac{\delta}{1-\phi}$

a) då $Y_t = \delta + \phi Y_{t-1} + \varepsilon_t$

och $E(Y_t) = \delta + \phi E(Y_t)$ så

$$\mu = \delta + \phi\mu$$

$$\mu - \phi\mu = \delta$$

$$\mu = \frac{\delta}{1-\phi}$$

$\rho_k = \phi^k$ för AR(p) eftersom detta är en AR(1)

så är $\rho_1 = \phi^1 = \phi$.

I modell (1) skattar jag $\rho_1 = \phi^{(1)} = -0,5$

I modell (2) — " — $\rho_1 = \phi^{(2)} = +0,5$

Svar: ϕ för modell 1 är $-0,5$, ϕ för modell 2 är $+0,5$.

OK

b) Stationära processer

I både modell 1 och 2 har vi en spik på ACF, samt dämpat exp. förlopp i PACF. Detta tyder på MA(1)-processer. Eftersom θ i en MA(1) har motsatt tecken mot ρ_1 kan vi konstatera att θ är pos för modell (1) & neg för modell (2), vilket stämmer med utseendet på PACF.

OK

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SU, STATISTIK

Skrivsal: Värta. Anonymkod: ETI-0037 Blad nr: 3

3. $n = 369$ (daily closing IBM stock prices)

a) Det ser inte ut som om väntevärdet är konstant, (tidsserien rör sig inte kring ngt särskilt värde. OK

b) Vi ser en hög långsamt avtagande autokorrelation på ACF:en, men bara en spik på PACF. Jag föreslår en differentiering. OK

c) Kanske har vi att göra med en Arima (0,1,0), dvs en slumpvandring med drift $Y_t = \delta + Y_{t-1} + \varepsilon_t$. Efter en differentiering kommer vi då att få en ~~slumpvandring~~ = $\Delta Y_t = w_t = \delta + \varepsilon_t$. (Föreslagen åtgärd: differentiering)

d) Eftersom vi har att göra med en slumpvandring med drift då $E(Y_t) = t\delta + Y_0$ har vi inget bättre prognos än det sista värdet på tidsserien som är ca ≈ 350. OK

e) Vi kan göra ett unit root test, dvs testa om λ i formeln $Y_t = \delta + \lambda Y_{t-1} + \varepsilon_t$ verkligen är 1, vilket är detsamma som att testa om $Y_t - Y_{t-1} = \delta + (1-\lambda) Y_{t-1} + \varepsilon_t$

$$\underset{\delta = 0}{(1-\lambda)}$$

$H_0: \delta = 0$

$H_A: |\delta| < 0$

Om H_0 inte förkastas har vi att göra med en slumpvandring om H_0 förkastas är proc. stationär. OK

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SU, STATISTIK

Skrivsal: Värta **Anonymkod:** ETI-0037 **Blad nr:** 4

4

Y

X_1

7 länder

10 tidpunkter

observationer = 70

a) $Y_{it} = \beta_1 + \beta_2 X_{1it} + u_{it}$

b) F-testet testar om variablerna tillsammans kan förklara modellen (om någon av dem gör det) I det här fallet har vi ju bara en förklaringsvariabel X_1, som inte är signifikant, vilket förklarar det mycket låga F-värdet.

$H_0 : \beta_2 \neq 0$

$H_A : \beta_2 = 0$

$F_{0,05}(1, 62) \approx 4$

vi förkastar H_0 om $F_{obs} > F_{0,05}(1, 62)$

$$F_{obs} = \frac{ESS/1}{TSS/68} = \frac{3,7 \cdot 10^{18}}{6,27 \cdot 10^{20}} = 0,40$$

p-värde: 0,53
långt långt ifrån signifikant.

SVAR: DVS: H_0 kan inte förkastas, det är inte säkert att X_{7it} förklarar Y_{it}.

OK

c) $Y_{ti} = \alpha_1 + \alpha_2 D_{2i} + \alpha_3 D_{3i} + \alpha_4 D_{4i} + \alpha_5 D_{5i} + \alpha_6 D_{6i} + \alpha_7 D_{7i} +$

$+ \beta_1 X_{1ti} + u_{ti}$

OK

d) Svar: förklaringsvariabeln X_1 är nu signifikant på $\alpha = 5\%$
(I OLS-modellen var p-värdet 0,527, i FEM-modellen är det 0,029)

* F-test för hela modellen är även det signifikant (p-värde 0,02)

OK

e) Vi kan göra ett restricted F-test.

OLS modell: (restricted)	FEM-modell:
obs: 70	obs: 70
k: 2	k: 8
R^2: 0,0059	$R^2 = 0,2276$

$m = 6$

$H_0: \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = 0$

H_A: Någon av $\alpha_2 \ldots \alpha_7$ är inte 0.

$$F_{obs} = \frac{(R^2_{UR} - R^2_R)/m}{(1 - R^2_{UR})/(n-k)} = \frac{(0,2276 - 0,0059)/6}{(1 - 0,2276)/(70-8)} = 2,966$$

Vi förkastar H_0 om $F_{obs} > F_{0,05}(6, 62)$ $\quad F_{0,05}(6,62) \approx 2,25$

Svar: H_0 förkastas. Den utökade FEM-modellen är bättre.

OK

/17

SU, STATISTIK

Skrivsal: Väta. Anonymkod: ETT-0037 Blad nr: 5

5\. a) False. L-B testar för autokorrelation mellan Y_t och laggade värden av Y_t: Y_{t-1}, Y_{t-2}, Y_{t-3} etc.

b) I en stationär AR(p) är $\mu = \frac{\delta}{1-\phi_1-\phi_2 \text{ osv}}$ om $\delta = 0$ så är $\mu = 0$. TRUE OK

c) True. Durbin h-test is used for detecting autocorrelation in autoregressive models. The standard Durbin-Watson test is biased for first order autocorrelation. OK

d) False. H_0 in L-B is $H_0: \rho = 0$, no correlation. A rejection of H_0 means there is correlation, but correlation in itself doesn't make a process non-stationary. OK

e) False, $Cov(u_t, u_{t-1})$ in a Koyck model is not 0, it is negative. OK

f) False, an increased value of λ means less smoothing. OK

/15

$m=2$

6. $$y_t = \delta + \frac{\sum_{k=1}^{m+1} z_{t-k}}{m} + z_t \qquad z_t \sim N(0,1)$$

$m =$ pos. tal

Om vi ex sätter $t=1$, $m=5$ ger $\sum_1^6 z_{t-k}$

a) $$y_1 = \delta + \frac{(z_0 + z_{-1} + z_{-2} + z_{-3} + z_{-4} + z_{-5})}{5} + z_t$$

y_t är alltså summan av en konstant + en summa av felternerna $m+1$ tsp +6 i tiden delat med m. Väntevärdet blir således konstanten δ. Om vi sätter

$m=1$ så får vi $\quad y_t = \delta + z_{t-1} + z_{t-2} + z_t$.

$m=2$ så får vi $\quad y_t = \delta + \frac{z_{t-1} + z_{t-2} + z_{t-3}}{2} + z_t$

Skulle kunna skrivas $\quad y_t = \delta + 0{,}5\varepsilon_{t-1} + 0{,}5\varepsilon_{t-2} + 0{,}5\varepsilon_{t-3} + \varepsilon_t$

I så fall har vi att göra med en $MA(q)$ modell där $q = m+1$ & $\theta_1 = \theta_2 = \theta_3$ osv $= \frac{1}{m}$ $\quad m = \frac{1}{\theta}$

Svar: Någon typ av moving average modell för det att som. MA (m+

b) $$y_t = \delta + \frac{\sum_{k=1}^{m+1} z_{t-k}}{m} + z_t$$

$$E(y_t) = \delta + \frac{1}{m} \cdot \underbrace{E\left(\sum_{k=1}^{m+1} z_{t-k}\right)}_{=0} + \underbrace{E(z_t)}_{=0} = \delta \quad \text{OK}$$

$$V(y_t) = V(\delta) + \frac{1}{m^2} V\underbrace{\sum(z_{t-k})}_{\sum V(z_{t-k})} + \underbrace{V(z_t)}_{=1}$$

$$V(y_t) = 0 + \frac{(m+1)\cdot 1}{m^2} + 1 = 1 + \frac{(m+1)}{m^2} \quad \text{OK}$$

SU, STATISTIK

Skrivsal: Västra. Anonymkod: E17-0037 Blad nr: 6

6 b) FORTS.

$$\rho_1 = \frac{Cov(y_{t-1}, y_t)}{V(y_t)}$$

$\sum_{k=1}^{3}$

$\frac{1}{2}(z_{-1} + z_{-2} + z_{-3})$ m gemensamma termer

$\frac{1}{2}(z_{-2} + z_{-3} + z_{-4})$ m gemensamma termer

$$Cov(y_{t-1}, y_t) = Cov\left(\delta + \frac{1}{m}\sum_{k=1}^{m+1} z_{t-k} + z_t,\ \delta + \frac{1}{m}\sum_{k=1}^{m+1} z_{t-1-k} + z_{t-1}\right)$$

$$= \left(\frac{1}{m^2} Cov(z_{t-2}, z_{t-2}) + \frac{1}{m^2} Cov(z_{t-3}, z_{t-3})\right) + \frac{1}{m} Cov(z_{t-1}, z_{t-1})$$

$$= m\left(\frac{1}{m^2}\right) + \frac{1}{m} = \frac{2}{m} \quad (= 2\theta)$$

$$\rho_1 = \frac{\frac{2}{m}}{\frac{m^2+m+1}{m^2}} = \frac{2\, m}{m(m^2+m+1)} = \frac{2m}{m^2+m+1}$$

OK

Svar 6b: $E(y_t) = \delta$

$$V(y_t) = \frac{m^2+m+1}{m^2}$$

$$\rho_1 = \frac{2m}{m^2+m+1}$$

/||