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Econometrics II

WRITTEN EXAMINATION

Monday August 14, 2017

Tools allowed: Pocket calculator

Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description.

For the maximum number of points on each problem detailed and clear solutions are required.

Observe:: If not indicated otherwise, the error terms ϵ_t in the models are assumed independent and $N(0, \sigma^2)$.

1. (20p) The monthly average values y_t for a particular stock during seven months were:

Month	y_t
1	7.2
2	7.0
3	7.4
4	7.3
5	7.2
6	7.1
7	7.2

   (a) Would you consider y_t to be stationary? Why/why not?

   (b) Use an appropriate smoothing method to compute a forecast (prognosis) for month number 8. Use the discount factor 0.3 and the whole given series of values for computation of the starting value.

   (c) Compute $\hat{\rho}_1$ and $\hat{\rho}_2$.

   (d) Compute the value of a suitable model-fit measure with respect to your chosen smoothing method.

2. (25p) True or false? Short motivation/comment also needed.

   (a) The Yule-Walker equations are used to obtain autocorrelations for MA processes.
   (b) Rejection in the unit root test means that we have detected a random walk process.
   (c) The Hausmann test is used to determine the number of dependent variables in a regression model.
   (d) A dynamic model for y_t contains at least one lagged y-component.
   (e) An MA process is always stationary.
   (f) Doing the second order differencing for a process y_t means that we obtain $y_t - y_{t-1} + y_{t-2}$.
   (g) The Koyck model and the REM model are both examples of dynamic models.
   (h) The expectation of y_t in an AR model is equal to the constant term δ.

3. (20p) Consider the following situation: We have three similar companies and for each company we have observed values of three variables Y, X_1 and X_2 . Each variable is observed during the time points $t = 1, \ldots 10$. We want to formulate a model with the X-variables as regressors and Y as the dependent variable.

   (a) Is this a balanced model? Why/why not?
   (b) Formulate first the pooled OLS regression model with suitable notation and indices.

       $Y_{it} = \ldots$
   (c) As an alternative we would also like to try a random effects regression model. Formulate also this model.

       $Y_{it} = \ldots$

       In doing this, also explain the difference between the pooled OLS model and the random effects model and how they relate to each other. Especially: How is the error term of the pooled OLS model related to the the error term of the random effects model?
   (d) The Hausmann test is often used to choose between the fixed effects model and the random effects model and technically it is a test about a specific property of the error terms in the random effects model. Which property?

4. (20p) Below we have a model, which is essentially a regression model, but here we will look at it from a times series model perspective.

$$y_t = 3 - 2t + \epsilon_t,$$

where $t = 0, 1, 2, \ldots$, $\epsilon_t \sim N(0, 1)$ and $Cov(\epsilon_t, \epsilon_{t-k}) = \tau(k)$.

(a) Compute (derive expressions of) $E(y_t)$, $V(y_t)$ and $Cov(y_t, y_{t-k})$

(b) Why is y_t nonstationary? Is ϵ_t stationary? Why/why not?

(c) If we apply the method of first difference to this times series, do we obtain stationarity? Why/why not?

5. (15p) From a realization of a stationary time series the following estimators were computed: $\hat{\rho}_1 = 0.8$, $\hat{\rho}_2 = 0.5$ and $\hat{\rho}_3 = 0.4$.

(a) Which AR-model would you fit to these data? AR(1) or AR(2)?

(b) Derive estimates for the parameters in your chosen model.

(c) Compute an estimate for ρ_4.

Formula sheet, Econometrics II, Spring 2017

Under the simple linear model $y_t = \beta_1 + \beta_2 x_t + u_t$, where $u_t \sim N(0, \sigma^2)$ and given independent pairs of observations $(y_1, x_1), \ldots, (y_n, x_n)$, the OLS (and ML) estimators are:

$$\begin{aligned}
\widehat{\beta}_1 &= \overline{y} - \widehat{\beta}_2 \overline{x} \\
\widehat{\beta}_2 &= \frac{\sum (x_t - \overline{x})(y_t - \overline{y})}{\sum (x_t - \overline{x})^2} \\
\widehat{\sigma}^2 &= \frac{RSS}{n-2} = \frac{\sum (y_t - \widehat{y}_t)^2}{n-2}
\end{aligned}$$

where $\widehat{y}_t = \widehat{\beta}_1 + \widehat{\beta}_2 x_t$ and where $E(\widehat{\beta_1}) = \beta_1$, $E(\widehat{\beta_2}) = \beta_2$ and $E(\widehat{\sigma}^2) = \sigma^2$

Comparing an "old" model with a "new" (larger):

$$\begin{aligned}
F &= \frac{(ESS_{new} - ESS_{old})/\text{number of new regressors}}{RSS_{new}/(n - \text{number of parameters in the new model})} \\
&= \frac{(R^2_{new} - R^2_{old})/\text{number of new regressors}}{(1 - R^2_{new})/(n - \text{number of parameters in the new model})}
\end{aligned}$$

Comparing an "unrestricted" model with a "restricted":

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} = \frac{(R^2_{UR} - R^2_R)/m}{(1 - R^2_{UR})/(n-k)}$$

where m is the number of linear constraints and k is the number of parameters in the unrestricted model.

Dynamic models: $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_{t-1} + v_t$

Koyck: $y_t = \alpha(1 - \lambda) + \beta_0 x_t + \lambda y_{t-1} + v_t$
Adaptive expectations: $y_t = \gamma\beta_0 + \gamma\beta_1 x_t + (1 - \gamma) y_{t-1} + (u_t - (1 - \gamma) u_{t-1})$
Partial adjustment: $y_t = \delta\beta_0 + \delta\beta_1 x_t + (1 - \delta) y_{t-1} + \delta u_t$

The Durbin Watson d statistic:

$$d = \frac{\sum_{t=2}^{n} (\widehat{u}_t - \widehat{u}_{t-1})^2}{\sum_{t=1}^{n} \widehat{u}_t^2}$$

The Durbin h statistic:

$$h = \widehat{\rho} \sqrt{\frac{n}{1 - n\left[\widehat{V}(\widehat{\alpha}_2)\right]}} \approx N(0, 1), \text{ if } \rho = 0$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = \frac{1}{n}\sum_{t=1}^{n}[y_t - \widehat{y}_t(t-1)]^2$$

Autocorrelation function:

$$\rho_k = \frac{Cov(y_t, y_{t+k})}{V(y_t)}, \quad k = 0, 1, 2, \ldots$$

Sample correlation function:

$$\widehat{\rho}_k = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n-k}(y_t - \bar{y})^2}, \quad k = 0, 1, 2, \ldots$$

Simple moving average:

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} y_t$$

First-order exponential smoothing:

$$\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$$

Second-order exponential smoothing:

$$\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)},$$

where $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$.

Holt's method:

$$\begin{aligned} L_t &= \alpha y_t + (1-\alpha)(L_{t-1} + T_{t-1}) \\ T_t &= \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1} \end{aligned}$$

$$\widehat{y}_{T+\tau}(T) = L_T + \tau T_T, \quad \tau = 1, 2, \ldots$$

Forecast under a constant process:

$$\widehat{y}_{T+\tau}(T) = \widetilde{y}_T \quad \tau = 1, 2, \ldots$$

Forecast under a linear trend:

$$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T}\tau,$$

where $\hat{y}_T = \hat{\beta}_{0,T} + \hat{\beta}_{1,T}T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$

For white noise:

$$\hat{\rho}_k \approx N(0, 1/n)\ , k = 1, 2, \ldots$$

The Q statistic:

$$Q = n \sum_{k=1}^{K} \hat{\rho}_k^2 \approx \chi^2(K)$$

The Ljung-Box statistic:

$$Q_{LB} = n\,(n+2) \sum_{k=1}^{K} \left(\frac{\hat{\rho}_k^2}{n-k} \right) \approx \chi^2(K)$$

ARMA(p,q):

$$y_t = \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + \epsilon_t - \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$

Stationarity and invertibility conditions for some time series models:

Model	Stationarity conditions	Invertibility conditions
AR(1)	$\lvert\phi_1\rvert < 1$	None
AR(2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $\lvert\phi_2\rvert < 1$	None
MA(1)	None	$\lvert\theta_1\rvert < 1$
MA(2)	None	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\lvert\theta_2\rvert < 1$
ARMA(1,1)	$\lvert\phi_1\rvert < 1$	$\lvert\theta_1\rvert < 1$
ARMA(2,2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $\lvert\phi_2\rvert < 1$	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\lvert\theta_2\rvert < 1$

The Yule-Walker equations for AR(p):

$$\rho_k = \sum_{i=1}^{p} \phi_i \rho_{k-i}, \quad k = 1, 2, \ldots$$

TABLE D.3 Upper Percentage Points of the F Distribution

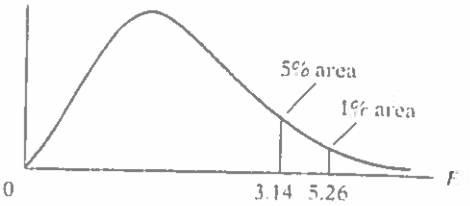
Example

$Pr(F > 1.59) = 0.25$

$Pr(F > 2.42) = 0.10$ for df $N_1 = 10$

$Pr(F > 3.14) = 0.05$ and $N_2 = 9$

$Pr(F > 5.26) = 0.01$



df for denominator N_2	Pr	df for numerator N_1											
		1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.05	161	200	216	225	230	234	237	239	241	242	243	244
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11

Source. From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 18, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

F-table continued

df for numerator N_1													df for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	
9.49	9.58	9.63	9.67	9.71	9.74	9.76	9.78	9.80	9.82	9.84	9.85	.25	1
61.2	61.7	62.0	62.3	62.5	62.7	62.8	63.0	63.1	63.2	63.3	63.3	.10	
246	248	249	250	251	252	252	253	253	254	254	254	.05	
3.41	3.43	3.43	3.44	3.45	3.45	3.46	3.47	3.47	3.48	3.48	3.48	.25	2
9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.48	9.49	9.49	9.49	.10	
19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	.05	
99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	.01	
2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	.25	3
5.20	5.18	5.18	5.17	5.16	5.15	5.15	5.14	5.14	5.14	5.14	5.13	.10	
8.70	8.66	8.64	8.62	8.59	8.58	8.57	8.55	8.55	8.54	8.53	8.53	.05	
26.9	26.7	26.6	26.5	26.4	26.4	26.3	26.2	26.2	26.2	26.1	26.1	.01	
2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	.25	4
3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.78	3.77	3.76	3.76	.10	
5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.66	5.65	5.64	5.63	.05	
14.2	14.0	13.9	13.8	13.7	13.7	13.7	13.6	13.6	13.5	13.5	13.5	.01	
1.89	1.88	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.87	1.87	.25	5
3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.10	.10	
4.62	4.56	4.53	4.50	4.46	4.44	4.43	4.41	4.40	4.39	4.37	4.36	.05	
9.72	9.55	9.47	9.38	9.29	9.24	9.20	9.13	9.11	9.08	9.04	9.02	.01	
1.76	1.76	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	.25	6
2.87	2.84	2.82	2.80	2.78	2.77	2.76	2.75	2.74	2.73	2.73	2.72	.10	
3.94	3.87	3.84	3.81	3.77	3.75	3.74	3.71	3.70	3.69	3.68	3.67	.05	
7.56	7.40	7.31	7.23	7.14	7.09	7.06	6.99	6.97	6.93	6.90	6.88	.01	
1.68	1.67	1.67	1.66	1.66	1.66	1.65	1.65	1.65	1.65	1.65	1.65	.25	7
2.63	2.59	2.58	2.56	2.54	2.52	2.51	2.50	2.49	2.48	2.48	2.47	.10	
3.51	3.44	3.41	3.38	3.34	3.32	3.30	3.27	3.27	3.25	3.24	3.23	.05	
6.31	6.16	6.07	5.99	5.91	5.86	5.82	5.75	5.74	5.70	5.67	5.65	.01	
1.62	1.61	1.60	1.60	1.59	1.59	1.59	1.58	1.58	1.58	1.58	1.58	.25	8
2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.32	2.31	2.30	2.29	.10	
3.22	3.15	3.12	3.08	3.04	2.02	3.01	2.97	2.97	2.95	2.94	2.93	.05	
5.52	5.36	5.28	5.20	5.12	5.07	5.03	4.96	4.95	4.91	4.88	4.86	.01	
1.57	1.56	1.56	1.55	1.55	1.54	1.54	1.53	1.53	1.53	1.53	1.53	.25	9
2.34	2.30	2.28	2.25	2.23	2.22	2.21	2.19	2.18	2.17	2.17	2.16	.10	
3.01	2.94	2.90	2.86	2.83	2.80	2.79	2.76	2.75	2.73	2.72	2.71	.05	
4.96	4.81	4.73	4.65	4.57	4.52	4.48	4.42	4.40	4.36	4.33	4.31	.01	

(Continued)

TABLE D.3 Upper Percentage Points of the F Distribution *(Continued)*

df for denominator N_2		df for numerator N_1											
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
11	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
13	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

F-table continued

df for numerator N_1													df for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	
1.53	1.52	1.52	1.51	1.51	1.50	1.50	1.49	1.49	1.49	1.48	1.48	.25	10
2.24	2.20	2.18	2.16	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.06	.10	
2.85	2.77	2.74	2.70	2.66	2.64	2.62	2.59	2.58	2.56	2.55	2.54	.05	
4.56	4.41	4.33	4.25	4.17	4.12	4.08	4.01	4.00	3.96	3.93	3.91	.01	
1.50	1.49	1.49	1.48	1.47	1.47	1.47	1.46	1.46	1.46	1.45	1.45	.25	11
2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	2.00	1.99	1.98	1.97	.10	
2.72	2.65	2.61	2.57	2.53	2.51	2.49	2.46	2.45	2.43	2.42	2.40	.05	
4.25	4.10	4.02	3.94	3.86	3.81	3.78	3.71	3.69	3.66	3.62	3.60	.01	
1.48	1.47	1.46	1.45	1.45	1.44	1.44	1.43	1.43	1.43	1.42	1.42	.25	12
2.10	2.06	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90	.10	
2.62	2.54	2.51	2.47	2.43	2.40	2.38	2.35	2.34	2.32	2.31	2.30	.05	
4.01	3.86	3.78	3.70	3.62	3.57	3.54	3.47	3.45	3.41	3.38	3.36	.01	
1.46	1.45	1.44	1.43	1.42	1.42	1.42	1.41	1.41	1.40	1.40	1.40	.25	13
2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.88	1.86	1.85	1.85	.10	
2.53	2.46	2.42	2.38	2.34	2.31	2.30	2.26	2.25	2.23	2.22	2.21	.05	
3.82	3.66	3.59	3.51	3.43	3.38	3.34	3.27	3.25	3.22	3.19	3.17	.01	
1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.39	1.39	1.39	1.38	1.38	.25	14
2.01	1.96	1.94	1.91	1.89	1.87	1.86	1.83	1.83	1.82	1.80	1.80	.10	
2.46	2.39	2.35	2.31	2.27	2.24	2.22	2.19	2.18	2.16	2.14	2.13	.05	
3.66	3.51	3.43	3.35	3.27	3.22	3.18	3.11	3.09	3.06	3.03	3.00	.01	
1.43	1.41	1.41	1.40	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.36	.25	15
1.97	1.92	1.90	1.87	1.85	1.83	1.82	1.79	1.79	1.77	1.76	1.76	.10	
2.40	2.33	2.29	2.25	2.20	2.18	2.16	2.12	2.11	2.10	2.08	2.07	.05	
3.52	3.37	3.29	3.21	3.13	3.08	3.05	2.98	2.96	2.92	2.89	2.87	.01	
1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.36	1.35	1.35	1.34	1.34	.25	16
1.94	1.89	1.87	1.84	1.81	1.79	1.78	1.76	1.75	1.74	1.73	1.72	.10	
2.35	2.28	2.24	2.19	2.15	2.12	2.11	2.07	2.06	2.04	2.02	2.01	.05	
3.41	3.26	3.18	3.10	3.02	2.97	2.93	2.86	2.84	2.81	2.78	2.75	.01	
1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34	1.34	1.34	1.33	1.33	.25	17
1.91	1.86	1.84	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.69	1.69	.10	
2.31	2.23	2.19	2.15	2.10	2.08	2.06	2.02	2.01	1.99	1.97	1.96	.05	
3.31	3.16	3.08	3.00	2.92	2.87	2.83	2.76	2.75	2.71	2.68	2.65	.01	
1.39	1.38	1.37	1.36	1.35	1.34	1.34	1.33	1.33	1.32	1.32	1.32	.25	18
1.89	1.84	1.81	1.78	1.75	1.74	1.72	1.70	1.69	1.68	1.67	1.66	.10	
2.27	2.19	2.15	2.11	2.06	2.04	2.02	1.98	1.97	1.95	1.93	1.92	.05	
3.23	3.08	3.00	2.92	2.84	2.78	2.75	2.68	2.66	2.62	2.59	2.57	.01	
1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31	1.30	.25	19
1.86	1.81	1.79	1.76	1.73	1.71	1.70	1.67	1.67	1.65	1.64	1.63	.10	
2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.94	1.93	1.91	1.89	1.88	.05	
3.15	3.00	2.92	2.84	2.76	2.71	2.67	2.60	2.58	2.55	2.51	2.49	.01	
1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31	1.30	1.30	1.29	.25	20
1.84	1.79	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	.10	
2.20	2.12	2.08	2.04	1.99	1.97	1.95	1.91	1.90	1.88	1.86	1.84	.05	
3.09	2.94	2.86	2.78	2.69	2.64	2.61	2.54	2.52	2.48	2.44	2.42	.01	

(Continue

TABLE D.3 **Upper Percentage Points of the F Distribution** (*Continued*)

df for denominator N_2		df for numerator N_1											
	Pr	1	2	3	4	5	6	7	8	9	10	11	12
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38	1.37
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37	1.36
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21	2.18
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84	1.81
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73	1.71
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62	1.60
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26	1.25
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60	1.57
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18

F-table continued

df for numerator N_1													
15	20	24	30	40	50	60	100	120	200	500	∞	Pr	df for denominator N_2
1.36	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.30	1.29	1.29	1.28	.25	22
1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.61	1.60	1.59	1.58	1.57	.10	
2.15	2.07	2.03	1.98	1.94	1.91	1.89	1.85	1.84	1.82	1.80	1.78	.05	
2.98	2.83	2.75	2.67	2.58	2.53	2.50	2.42	2.40	2.36	2.33	2.31	.01	
1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.28	1.28	1.27	1.27	1.26	.25	24
1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.58	1.57	1.56	1.54	1.53	.10	
2.11	2.03	1.98	1.94	1.89	1.86	1.84	1.80	1.79	1.77	1.75	1.73	.05	
2.89	2.74	2.66	2.58	2.49	2.44	2.40	2.33	2.31	2.27	2.24	2.21	.01	
1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.26	1.26	1.26	1.25	1.25	.25	26
1.76	1.71	1.68	1.65	1.61	1.59	1.58	1.55	1.54	1.53	1.51	1.50	.10	
2.07	1.99	1.95	1.90	1.85	1.82	1.80	1.76	1.75	1.73	1.71	1.69	.05	
2.81	2.66	2.58	2.50	2.42	2.36	2.33	2.25	2.23	2.19	2.16	2.13	.01	
1.33	1.31	1.30	1.29	1.28	1.27	1.27	1.26	1.25	1.25	1.24	1.24	.25	28
1.74	1.69	1.66	1.63	1.59	1.57	1.56	1.53	1.52	1.50	1.49	1.48	.10	
2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.73	1.71	1.69	1.67	1.65	.05	
2.75	2.60	2.52	2.44	2.35	2.30	2.26	2.19	2.17	2.13	2.09	2.06	.01	
1.32	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	.25	30
1.72	1.67	1.64	1.61	1.57	1.55	1.54	1.51	1.50	1.48	1.47	1.46	.10	
2.01	1.93	1.89	1.84	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62	.05	
2.70	2.55	2.47	2.39	2.30	2.25	2.21	2.13	2.11	2.07	2.03	2.01	.01	
1.30	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.21	1.20	1.19	1.19	.25	40
1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.43	1.42	1.41	1.39	1.38	.10	
1.92	1.84	1.79	1.74	1.69	1.66	1.64	1.59	1.58	1.55	1.53	1.51	.05	
2.52	2.37	2.29	2.20	2.11	2.06	2.02	1.94	1.92	1.87	1.83	1.80	.01	
1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.16	1.15	1.15	.25	60
1.60	1.54	1.51	1.48	1.44	1.41	1.40	1.36	1.35	1.33	1.31	1.29	.10	
1.84	1.75	1.70	1.65	1.59	1.56	1.53	1.48	1.47	1.44	1.41	1.39	.05	
2.35	2.20	2.12	2.03	1.94	1.88	1.84	1.75	1.73	1.68	1.63	1.60	.01	
1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.10	.25	120
1.55	1.48	1.45	1.41	1.37	1.34	1.32	1.27	1.26	1.24	1.21	1.19	.10	
1.75	1.66	1.61	1.55	1.50	1.46	1.43	1.37	1.35	1.32	1.28	1.25	.05	
2.19	2.03	1.95	1.86	1.76	1.70	1.66	1.56	1.53	1.48	1.42	1.38	.01	
1.23	1.21	1.20	1.18	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.06	.25	200
1.52	1.46	1.42	1.38	1.34	1.31	1.28	1.24	1.22	1.20	1.17	1.14	.10	
1.72	1.62	1.57	1.52	1.46	1.41	1.39	1.32	1.29	1.26	1.22	1.19	.05	
2.13	1.97	1.89	1.79	1.69	1.63	1.58	1.48	1.44	1.39	1.33	1.28	.01	
1.22	1.19	1.18	1.16	1.14	1.13	1.12	1.09	1.08	1.07	1.04	1.00	.25	∞
1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.18	1.17	1.13	1.08	1.00	.10	
1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.22	1.17	1.11	1.00	.05	
2.04	1.88	1.79	1.70	1.59	1.52	1.47	1.36	1.32	1.25	1.15	1.00	.01	

Stockholms
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Statistiska institutionen

Rättningsblad

Datum: 14/8-2017

Sal: Brunnsvikssalen

Tenta: Tidsserieanalys / Ekonometri II

Kurs: Ekonometri

ANONYMKOD: ETO-0014

☒ Jag godkänner att min tenta får läggas ut anonymt på hemsidan som studentsvar.

OBS! SKRIV ÄVEN PÅ BAKSIDAN AV SKRIVBLADEN

Markera besvarade uppgifter med kryss

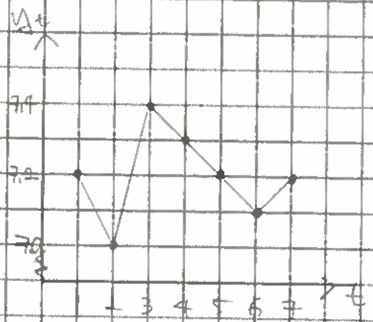
1	2	3	4	5	6	7	8	9	Antal inl. blad
X	X	X	X	X					6 34
Lär.ant. 16	25	15	6	14					

POÄNG	BETYG	Lärarens sign.
76	C	

SU, DEPARTMENT OF STATISTICS

Room: Brunnviksssalen **Anonymous code:** ETO-0014 **Sheet number:** 1

①



a) yes, Because it seems like that the process goes around its mean value.

OK

b) Since there is both upward and downward linear trend? I will use the second order exponential smoothing

first-order is enough here

Month	y_t
1	7.2
2	7.0
3	7.4
4	7.3
5	7.2
6	7.1
7	7.2

$\lambda = 0.3$

$\tilde{y}_0 = \bar{y} = 7.2$

$\tilde{y}_1^{(1)} = 0.3 y_1 + (1-0.3)\tilde{y}_0 = 0.3 \cdot 7.2 + 0.7 \cdot 7.2 = 7.2$

$\tilde{y}_2^{(1)} = 0.3 y_2 + (1-0.3)\tilde{y}_1 = 0.3 \cdot 7.0 + 0.7 \cdot 7.2 = 7.14$

$\tilde{y}_3^{(1)} = 0.3 y_3 + (1-0.3)\tilde{y}_2 = 0.3 \cdot 7.4 + 0.7 \cdot 7.14 = 7.22$

$\tilde{y}_4^{(1)} = 0.3 y_4 + (1-0.3)\tilde{y}_3 = 0.3 \cdot 7.3 + 0.7 \cdot 7.22 = 7.24$

$\tilde{y}_5^{(1)} = 0.3 y_5 + (1-0.3)\tilde{y}_4 = 0.3 \cdot 7.2 + 0.7 \cdot 7.24 = 7.23$

$\tilde{y}_6^{(1)} = 0.3 y_6 + (1-0.3)\tilde{y}_5 = 0.3 \cdot 7.1 + 0.7 \cdot 7.23 = 7.19$

$\tilde{y}_7^{(1)} = 0.3 y_7 + (1-0.3)\tilde{y}_6 = 0.3 \cdot 7.2 + 0.7 \cdot 7.19 = 7.19$

Second-order exponential smoothing $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$

$\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)}$

$\tilde{y}_1^{(2)} = 0,3\,\tilde{y}_1^{(1)} + (1-0,3)\,\tilde{y}_0^{(2)} = 0.3 \cdot 7,2 + 0,7 \cdot 7.2 = 7.2$

$\tilde{y}_2^{(2)} = 0,3\,\tilde{y}_2^{(1)} + 0,7\,\tilde{y}_1^{(2)} = 0,3 \cdot 7,14 + 0,7 \cdot 7,2 = 7.18$

$\tilde{y}_3^{(2)} = 0,3\,\tilde{y}_3^{(1)} + 0,7\,\tilde{y}_2^{(2)} = 0,3 \cdot 7,22 + 0,7 \cdot 7,18 = 7.19$

$\tilde{y}_4^{(2)} = 0,3\,\tilde{y}_4^{(1)} + 0,7\,\tilde{y}_3^{(2)} = 0,3 \cdot 7,24 + 0,7 \cdot 7,19 = 7.21$

$\tilde{y}_5^{(2)} = 0,3\,\tilde{y}_5^{(1)} + 0,7\,\tilde{y}_4^{(2)} = 0,3 \cdot 7,22 + 0,7 \cdot 7,21 = 7.22$

$\tilde{y}_6^{(2)} = 0,3\,\tilde{y}_6^{(1)} + 0,7\,\tilde{y}_5^{(2)} = 0,3 \cdot 7,19 + 0,7 \cdot 7,22 = 7.21$

$\tilde{y}_7^{(2)} = 0,3\,\tilde{y}_7^{(1)} + 0,7\,\tilde{y}_6^{(2)} = 0,3 \cdot 7,19 + 0,7 \cdot 7,21 = 7.20$

Forecast under a linear trend : $\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T} \cdot \tau$

$\hat{\beta}_1 = \dfrac{\Sigma(x_t-\bar{x})(y_t-\bar{y})}{\Sigma(x_t-\bar{x})^2} = \dfrac{0}{28} = 0 \Rightarrow \hat{\beta}_1 = 0$

x_t = month	y_t	$(x_t-\bar{x})$	$(x_t-\bar{x})^2$	$(x_t-\bar{x})(y_t-\bar{y})$
1	7,2	-3	9	0
2	7,0	-2	4	0.4
3	7,4	-1	1	-0,2
4	7,3	0	0	0
5	7,2	1	1	0
6	7,1	2	4	-0,2
7	7,2	3	9	0
$\bar{x}=4$	$\bar{y}=7.2$		Σ 28	Σ 0

Computation of $\hat{y}_T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$

$\hat{y}_1 = 2 \cdot \tilde{y}_1^{(1)} - \tilde{y}_1^{(2)} = 2 \cdot 7,2 - 7,2 = 7.2$

$\hat{y}_2 = 2 \cdot \tilde{y}_2^{(1)} - \tilde{y}_2^{(2)} = 2 \cdot 7,14 - 7,18 = 7.1$

$\hat{y}_3 = 2 \cdot \tilde{y}_3^{(1)} - \tilde{y}_3^{(2)} = 2 \cdot 7,22 - 7,19 = 7.25$

$\hat{y}_4 = 2 \cdot \tilde{y}_4^{(1)} + \tilde{y}_4^{(2)} = 2 \cdot 7,24 - 7,21 = 7.27$

$\hat{y}_5 = 2 \cdot \tilde{y}_5^{(1)} + \tilde{y}_5^{(2)} = 2 \cdot 7,23 - 7,22 = 7,24$

$\hat{y}_6 = 2 \cdot \tilde{y}_6^{(1)} + \tilde{y}_6^{(2)} = 2 \cdot 7,19 - 7,21 = 7,17$

$\hat{y}_7 = 2 \cdot \tilde{y}_7^{(1)} + \tilde{y}_7^{(2)} = 2 \cdot 7,19 - 7,20 = 7.18$

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Forecast for month 8.

$$\hat{y}_{7+1}(7) = \hat{y}_7 + 0 \cdot 1 = \hat{y}_7 = \underline{7.18}$$

d) For the second order exponential smoothing, a suitable model-fit measure is MSE (mean squared error)

$$MSE = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = \frac{1}{n}\left[(7.2-7.2)^2 + (7.0-7.1)^2 + \dots\right]$$

$$= \frac{1}{7}[0.0403] = \underline{0.0058}$$

Ok

e) $\hat{\rho}_1$ and $\hat{\rho}_2$

Sample autocorrelation function

$$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k}(y_t-\bar{y})(y_{t+k}-\bar{y})}{\sum_{t=1}^{n-k}(y_t-\bar{y})^2} \qquad n=7$$

$$\hat{\rho}_1 = \frac{\sum_{t=1}^{6}(y_t-\bar{y})(y_{t+1}-\bar{y})}{\sum_{t=1}^{6}(y_t-\bar{y})^2} = \frac{(7.2-7.2)(7.0-7.2)+(7.0-7.2)(7.4-7.2)+\dots}{(7.2-7.2)^2+(7.0-7.2)^2+(7.4-7.2)^2+\dots}$$

$$= \frac{-0.02}{0.1} = \underline{-0.2}$$

$$\hat{\rho}_2 = \frac{\sum_{t=1}^{5}(y_t-\bar{y})(y_{t+2}-\bar{y})}{\sum_{t=1}^{5}(y_t-\bar{y})^2} = \frac{-0.03}{0.09} = \underline{-0.33}$$

Ok

1/6

②

a) False.

The Yulewalker equations are used to obtain autocorrelations for AR-processes.

For example,

$$\rho_k = \sum_{i=1}^{p} \phi_i \rho_{k-i} \quad , \quad k=1,2,\ldots$$

AR(1) ⇒ $\rho_1 = \phi_1 \underbrace{\rho_0}_{1} = \phi_1$

$\rho_2 = \phi_1 \rho_1 = \phi_1 \phi_1 = \phi^2$

$\rho_3 = \phi_1 \rho_2 = \phi_1 \phi_1^2 = \phi_1^3$

$\rho_k = \phi_1^k$

AR(2) $\rho_1 = \phi_1 \underbrace{\rho_0}_{1} + \phi_2 \underbrace{\rho_{-1}}_{\rho_1} = \phi_1 + \phi_2 \rho_1$

$\Rightarrow \rho_1 = \dfrac{\phi_1}{1-\phi_2}$

$\rho_2 = \phi_1 \rho_1 + \phi_2 \underbrace{\rho_0}_{1} = \phi_1 \rho_1 + \phi_2$

Ok

b) False

For Unit root test, we start from the random walk process.

$y_t = \rho y_{t-1} + u_t$ if $\rho = 1$, the process is nonstationary.

we have a unit problem. Here, OLS method is not appropriate for estimation. So we transform the equation.

$y_t - y_{t-1} = (\rho - 1) y_{t-1} + u_t$

$\Delta y_t = \underbrace{(\rho-1)}_{\delta} y_{t-1} + u_t$ ← we use this equation to test for stationarity.

$H_0 : \delta = 0 \quad (\rho = 1)$ → the process is not stationary.

Therefore, rejection of the unit root test means that we do not have detected a random walk process.

Ok

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c) False

The Hausmann test is used to test if there is a serial correlation in REM.

H_0: No significant difference between FEM (the Fixed-Effect model) and REM (the Random effect model).

H_1: FEM is prefered due to a serial correlation in REM.

OK

d) True

According to Koyck transformation, the Koyck model is defined as $y_t = \alpha(1-\lambda) + \beta_0 x_t + \lambda y_{t-1} + \underbrace{v_t}_{\Rightarrow u_t - \lambda u_{t-1}}$

As a realization of Koyck model, there are two other models, such as the adaptive expectations model and the partial (stock) adjustment model.

Adaptive expections model: $y_t = \gamma\beta_0 + \gamma\beta_1 x_t + (1-\gamma)y_{t-1} + u_t - (1-\gamma)u_{t-1}$

Partial adjustment model: $y_t = \delta\beta_0 + \delta\beta_1 x_t + (1-\delta)y_{t-1} + \delta u_t$

For Koyck and Adaptive expectations,

$cov(v_t, v_{t-1}) = -\lambda\sigma^2$ for Koyck, $-\gamma\sigma^2$ for Adaptive expectations

$cov(y_{t-1}, v_t) = -\lambda\sigma^2$ " , $-\gamma\sigma^2$ "

On the other hand, for the partial adjustment model

$cov(v_t, v_{t-1}) = cov(y_{t-1}, v_t) = 0$.

As we see the formullas for these models, the dynamic models contain at least one lagged y-component

OK

e) True.

For example, a MA(1) process is $y_t = \delta - \theta_1 \varepsilon_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$

$$E(y_t) = E(\delta - \theta_1 \varepsilon_{t-1} + \varepsilon_t)$$
$$= \delta - \theta_1 \underbrace{E(\varepsilon_{t-1})}_{=0} + \underbrace{E(\varepsilon_t)}_{=0}$$

$E(y_t) = \delta$ ⇐ constant mean. It does not depend on time.

All pure MA(q) processes are stationary.

Ok

f) False

the second order differencing can be written as $(1-B)^2 y_t$.

$$(1-B)^2 y_t = (1-2B+B^2) y_t = \underline{y_t - 2y_{t-1} + y_{t-2}}$$

Ok

g) False

The Koyck model is a example of dynamic models.

But, the REM is an example of panel data models that take account in both cross-section data and time series data

Ok

h) False

For example, an AR(1) model ⇒ $y_t = \delta + \phi_1 y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma^2)$

$$E(y_t) = E(\delta + \phi_1 y_{t-1} + \varepsilon_t)$$
$$= \delta + \phi_1 \underbrace{E(y_{t-1})}_{=E(y_t)} + \underbrace{E(\varepsilon_t)}_{=0}$$

$$(1-\phi_1) E(y_t) = \delta$$

$$E(y_t) = \frac{\delta}{1-\phi_1}$$

Ok

/25

Room: Brunnsvikssalen **Anonymous code:** ETO-0014 **Sheet number:** 4

③ 3 companies, time points $t=1 \cdots 10$ $3 \times 3 \times 10 = \underline{90}$ obs.
3 variables Y, X_1, X_2

a) Yes, each company has same quantity of observations for all 3 variables and for all time points ($t=10$). OK

b) the pooled OLS regression model

$$Y_{it} = \beta_0 + \beta_1 X_{1it} + \beta_2 X_{2it} + u_{it}, \quad u_{it} \sim iid(0, \sigma^2)$$

All company has same intercept. OK

c) REM (random effects model)

$$Y_{it} = \beta_0 + \beta_1 X_{1it} + \beta_2 X_{2it} + \underbrace{W_{it}} \longrightarrow W_{it} = \varepsilon_i + u_{it}$$

REM treats β_{0i} as a random variable, $\beta_{0i} = \beta_0 + \varepsilon_i$
In this model, each company has its own intercept.
ε_i is a deviation from the mean value of Y_{it}.
But ε_i is not directly observable.

On the othe hand, the Pooled OLS model doesn't allow heterogeneity that may exist among the companies by lumping together all observations and forcing all company to have same intercept.

Regarding the error terms, the error term of the pooled OLS, (u_{it}) can be correlated with the regressors.
This results biased and inconsistent OLS estimators.
The error term of the REM is defined as $W_{it} = \varepsilon_i + u_{it}$, differently from the error term of the pooled OLS.

$\mathrm{COV}(w_{it_1}, w_{it_2}) = \mathrm{COV}(\varepsilon_i + u_{it_1}, \varepsilon_i + u_{it_2})$

$= E(\varepsilon_i^2) = V(\varepsilon_i) = \sigma^2$ ⇐ positive correlation

One big problem of REM is that there is possibility of a correlation between the error term and the regressors.

Ok

d) $E(\varepsilon_i \varepsilon_j) = 0$, $E(\varepsilon_i u_{it}) = 0$, $i \neq j$

$E(u_{it} u_{is}) = E(u_{ij} u_{ij}) = E(u_{it} u_{js}) = 0$, $i \neq j$, $t \neq s$

2

/15

④ $y_t = 3 - 2t + \varepsilon_t$ $t = 0, 1, 2 \cdots$ $\varepsilon_t \sim N(0, 1)$

$\mathrm{cov}(\varepsilon_t, \varepsilon_{t-1}) = \tau(k)$

a) $E(y_t) = E(3 - 2t + \varepsilon_t)$

$= 3 - 2E(t) + \underbrace{E(\varepsilon_t)}_{=0}$

$= 3 - 2t$ OK

?

$V(y_t) = V(3 - 2t + \boxed{\sum \varepsilon_t})$

$= \sum V(\varepsilon_t) = t \cdot \sigma^2$ ✓

$t=1$ $y_1 = 3 - 2 \cdot 1 + \varepsilon_1$

$y_2 = 3 - 2 \cdot 2 + \varepsilon_2$

$y_3 = 3 - 2 \cdot 3 + \varepsilon_3$

$\vdots$

$y_t = 3 - 2 \cdot t + \sum \varepsilon_t$

2,

$\mathrm{cov}(y_t, y_{t-k}) = \mathrm{cov}[(3 - 2t + \varepsilon_t), (3 - 2(t-k) + \varepsilon_{t+k})]$

$= \mathrm{cov}[(3 - 2t + \varepsilon_t), (3 - 2t + 2k + \varepsilon_{t+k})]$

$= E(9 - 6t + 6k + \cancel{3\varepsilon_{t+k}} - 6t + 4t^2 - 4tk \cancel{- 2t\varepsilon_{t+k}} + \cancel{3\varepsilon_t} \cancel{- 2t\varepsilon_t} \cancel{+ 2k\varepsilon_t} + \varepsilon_t\varepsilon_t$

$= 9 - 6t + 6k - 6t + 4t^2 - 4tk$ ✓

$= 4t^2 - 12t + 6k - 4tk + 9$ ✓

b) y_t is nonstationary because it does not have constant mean and constant variance over time. They depend on time.

OK

→ next page

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Room: Brunnvikssalen **Anonymous code:** ETO-0014 **Sheet number:** 5

④ – b) ε_t is stationary since its mean is zero and the variance of ε_t is 1. It does not depend on time. Covariance ?

c) NO. ✓

$$(1-B)y_t = 3-2t+\varepsilon_t$$

$$y_t - y_{t-1} = 3-2t+\varepsilon_t$$?

$$y_t = 3-2t+y_{t-1}+\varepsilon_t$$

$$E(y_t) = E(3-2t+y_{t-1}+\Sigma\varepsilon_t)$$

$$E(y_t) = 3-2t+E(y_{t-1})+0$$

$$E(y_t) = 0$$

$$V(y_t) = V(3-2t+y_{t-1}+\Sigma\varepsilon_t)$$

$$= V(y_{t-1}) + \Sigma\cdot V(\varepsilon_t)$$

$$= \underline{V(y_{t-1}) + t\cdot\sigma^2}$$

The variance still depends on time.

1/6

⑤ $\hat{\rho}_1 = 0{,}8$ $\hat{\rho}_2 = 0{,}5$ $\hat{\rho}_3 = 0{,}4$

a) I would say AR(2). According to the Yule-walker equation the autocorrelations of AR(1) can be computed as below.

$\rho_k = \phi_1^{k}$.

If $\hat{\rho}_1 = 0{,}8$, it means the parameter in AR(1), ϕ_1 should be equal to 0.8.

But $\hat{\rho}_2 = \phi_1^2 = 0{,}8^2 = 0{,}64$. It doesn't match with the estimation of $\hat{\rho}_2 = 0{,}5$.

Ok

b) AR(2) model is written as

$$y_t = \delta + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t$$

Yule-walker equation for the autocorrelation

$$\rho_k = \sum_{i=1}^{p} \phi_i \rho_{k-i} \qquad k=1,2\ldots$$

$$\rho = \phi_1\rho_0 + \phi_2\underbrace{\rho_{-1}}_{\rho_1} = \phi_1 + \phi_2\rho_1 = \frac{\phi_1}{1-\phi_2}$$

$$\rho_2 = \phi\rho_1 + \phi_2\rho_0 = \phi_1\rho_1 + \phi_2$$

$\hat{\rho}_1 = 0{,}8 \Rightarrow 0{,}8 = \dfrac{\phi_1}{1-\phi_2}$ $\qquad \phi_1 = 0{,}8(1-\phi_2)$

$\hat{\rho}_2 = 0{,}5 \Rightarrow 0{,}5 = \phi_1 0.8 + \phi_2$ $\qquad \phi_1 = 0{,}8(1+0{,}39)$

$0{,}5 = (0{,}8 - 0{,}8\phi_2)\,0{,}8 + \phi_2$

$\underline{\hat{\phi}_2 = -0{,}39}$ $\qquad \underline{\hat{\phi}_1 = 1.112}$ Ok

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c) $\rho_4 = \phi_1 \rho_3 + \phi_2 \rho_2$

k=4

$\hat{\rho} = -\cancel{0.39} \cdot 0.4 + \cancel{1.112} \cdot 0.5 = \cancel{0.4}$ 0,25 (Slarv!)

1,11 -0,39

1/2

Statistiska institutionen

Stockholms universitet

Rättningsblad

Datum: 14/8-2017

Sal: Brunnsvikssalen

Tenta: Tidsserieanalys /Ekonometri II

Kurs: Ekonometri

ANONYMKOD: ET0-0020

☒ Jag godkänner att min tenta får läggas ut anonymt på hemsidan som studentsvar.

OBS! SKRIV ÄVEN PÅ BAKSIDAN AV SKRIVBLADEN

Markera besvarade uppgifter med kryss

1	2	3	4	5	6	7	8	9	Antal inl. blad
X	X	X	X	X					5 3x
Lär.ant. 18	20	14	18	5					

POÄNG	BETYG	Lärarens sign.
75	C	

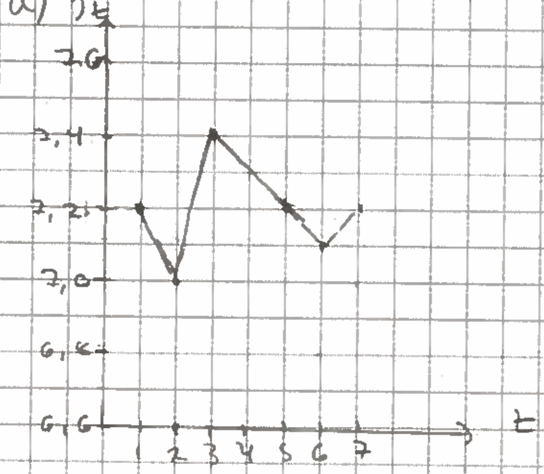
SU, STATISTIK

Skrivsal: Brunnsvikssalen **Anonymkod:** ETC-0020 **Blad nr:** 1

Uppgift 1

Month	y_t	$\tilde{y}_T$	$y_t - \tilde{y}_{T-1}$
1	7,2	7,2	0
2	7,0	7,14	-0,20
3	7,4	7,218	0,26
4	7,3	7,2426	0,08
5	7,2	7,22982	-0,04
6	7,1	7,190874	-0,01
7	7,2	7,1936118	0,01

a)



Ovan är observationerna plottade och utifrån denna ser y_t ut att vara stationär då det bara är små fluktuationer kring väntevärdet. OK

b) För en konstant process kan vi använda 1 order exponential smoothing: $\lambda = 0,3$ $\tilde{y}_0 = \bar{y} = \frac{1}{7}\sum_{t=1}^{7} y_t = 7,2$

$\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$

$\tilde{y}_1 = 0,3 \cdot 7,2 + 0,7 \cdot 7,2 = 7,2$

Resterande utjämnade värden beräknas enligt ovan och redovisas i tabellen.

Forecast under en konstant process ges av:

$\hat{y}_{T+\tau}(T) = \tilde{y}_T$ dvs

$\hat{y}_8(7) = \tilde{y}_7 \approx 7,194$

OK

c) $$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k} (y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n-k} (y_t - \bar{y})^2}$$

$$\hat{\rho}_1 = \frac{\sum_{t=1}^{6} (y_t - \bar{y})(y_{t+1} - \bar{y})}{\sum_{t=1}^{6} (y_t - \bar{y})^2}$$

$$= \frac{0 + (-0,2) \cdot 0,2 + 0,2 \cdot 0,1 + 0 + 0 + 0}{0^2 + (-0,2)^2 + 0,2^2 + 0,1^2 + 0^2 + (0,1)^2} = \frac{-0,02}{0,1} = 0,2 \;?$$

OK

$$\hat{\rho}_2 = \frac{\sum_{t=1}^{5} (y_t - \bar{y})(y_{t+2} - \bar{y})}{\sum_{t=1}^{5} (y_t - \bar{y})^2}$$

$$= \frac{0 + (-0,2) \cdot 0,1 + 0 + 0,1 \cdot (-0,1) + 0}{0^2 + (-0,2)^2 + 0,2^2 + 0,1^2 + 0^2} = \frac{-0,03}{0,09} = \underline{-0,33}$$

OK

d) Vi kan använda MSE (Mean Square Error) som ett mått på model fit:

$$MSE = \frac{1}{7} \cdot \sum_{t=1}^{7} (y_t - \tilde{y}_{t-1})^2 = \frac{1}{7} \cdot 0,1158 = \underline{0,016}$$

OK

/18

SU, STATISTIK

Skrivsal: Brunnsvikssalen **Anonymkod:** ETC-0020 **Blad nr:** 2

Uppgift 2

a) Falskt. Yule-Walker ekvationerna används för att få autokorrelationerna för en AR-process. Ok

b) Falskt. Nollhypotesen är att $\delta = 0$ där $\delta = 1 - \rho$, dvs om $\delta = 0 \Leftrightarrow \rho = 1$ och vi får en random walk process. Ok

c) Falskt. Hausmann-testet används för att avgöra om en Fixed-effects-modell är lämpligare än en Random-effects-modell. Ok

d) Falskt ✓ En dynamisk modell ges av $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_{t-1} + v_t$, dvs bara en laggad y-term

e) Sant. Eftersom en MA-process är en variant på en infinite moving average-modell, som alltid är stationär, gäller detta. Ok

f) Falskt. Ok Vi får $y_t - 2y_{t-1} + y_{t-2}$.

g) Falskt. REM är inte en dynamisk modell, däremot är Koyck det. Ok

h) Falskt. För en AR(p) gäller att:

$$E(y_t) = \mu = \frac{\delta}{1 - \phi_1 - \ldots - \phi_p}$$

ok

/20

SU, STATISTIK

Skrivsal: Brunnsviksalen **Anonymkod:** ETC-0020 **Blad nr:** 3

Uppgift 3

a) Ja, det är en balanserad modell eftersom vi har observerade värden på varje variabel i alla tidpunkter

Ok

b) $Y_{it} = \beta_0 + \beta_1 X_{1it} + \beta_2 X_{2it} + u_{it}$ $i = 1, 2, 3$ $t = 1, 2, \ldots, 10$

Ok

c)

d) Egenskapen som testas är om feltermerna är okorrelerade med regressorerna. Om de är det genererar REM inkonsistenta skattningar och FEM är då ett bättre val

Ok

/14

SU, STATISTIK

Skrivsal: Brunnsvikssalen **Anonymkod:** FTO-0020 **Blad nr:** 4

4. $y_t = 3 - 2_t + \varepsilon_t$ $t = 0, 1, 2 \ldots$ $\varepsilon_t \sim N(0,1)$

$Cov(\varepsilon_t, \varepsilon_{t-k}) = \tau(k)$

a) $E(y_t) = E(3-2t+\varepsilon_t)$

$E(y_t = \underline{3-2t}$

$V(y_t) = V(3-2_t+\varepsilon_t)$

$= \underbrace{V(3)}_{0} - \underbrace{V(2 \cdot t)}_{0} + \underbrace{V(\varepsilon_t)}_{1}$

$V(y_t = \underline{\underline{1}}$

$Cov(y_t, y_{t-k}) = Cov(3-2t+\varepsilon_t, 3-2(t-k)+\varepsilon_{t-k})$

$= \underbrace{-2(Cov(t, t-k)}_{0} + \underbrace{Cov(\varepsilon_t, \varepsilon_{t-k})}_{\tau(k)}$

$Cov(y_t, y_{t-k}) = Cov(\varepsilon_t, \varepsilon_{t-k}) = \underline{\underline{\tau(k)}}$ Ok

b) För att en tidsserie ska vara stationär ska väntevärdet och variansen vara konstanta och kovariansen mellan y_t och y_{t-k} ska bara bero på laggen och inte tiden. I modellen ovan minskar väntevärdet med tiden men variansen är konstant Därav är den ej stationär

ε_t är dock stationär eftersom den har väntevärdet 0, variansen 1 och en kovarians som bara är en funktion av laggen k, vilket anges i uppgiften.

Ok

c) Ja, den blir stationär. Om vi tar i a differensen får vi:

$y_t - y_{t-1} = 3 - 2t + \varepsilon_t - (3 - 2(t-1) + \varepsilon_{t-1})$

$\Delta y_t = \cancel{3} - 2t + \varepsilon_t - 3 + 2(t-1) - \varepsilon_{t-1}$

$\Delta y_t = -2 + \varepsilon_t - \varepsilon_{t-1}$

$E(\Delta y_t) = E(-2 + \underset{0}{\varepsilon_t} - \underset{0}{\varepsilon_{t-1}})$

$= \underline{\underline{-2}}$

Variansen blir även nu konstant

Kovarianser ?

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SU, STATISTIK

Skrivsal: Brunnsviksalen **Anonymkod:** ETO-CC2C **Blad nr:** 5

Uppgift 5

$\hat{\rho}_1 = 0,8 \quad \hat{\rho}_2 = 0,5 \quad \hat{\rho}_3 = 0,4$

a) Vi använde Yule-Walker-ekvationerna för att beräkna autokorrelationerna för en AR(p). Då tänker jag att man skulle kunna testa om uträkningarna stämmer för en AR(1) och AR(2)

För en AR(1) ger Yule-Walker:

$\rho_k = \sum_{i=1}^{1} \phi_i \rho_{k-1} \Rightarrow$

$\hat{\rho}_1 = 0,8 = \hat{\phi}_1 \cdot 1 \rightarrow \hat{\phi}_1 = 0,8$

$\hat{\rho}_2 = 0,5 = \hat{\phi}_1 \cdot 0,8 \Rightarrow \hat{\phi}_1 = 0,625$

$\hat{\rho}_3 = 0,4\ \hat{\phi}_1 \cdot 0,5 \Rightarrow \hat{\phi}_1 = 0,8$

Eftersom detta är en realization av en stationär process och $\hat{\phi}_1 = 0,8$ i 2 av 3 fall ? skulle detta kunna vara en AR(1). Om jag ställde upp det på samma sätt under antagandet att det var en AR(2) och därefter gjorde ett ekvationssystem blev resultatet att $\hat{\phi}_1$ eller $\hat{\phi}_2$ alltid blev 0, så jag antar att detta måste vara en AR(1) (förutsatt att det är rätt tillvägagångssätt över huvud taget ☺) Njå!

b) En AR(1) ges av:

$y_t = \delta + \phi_1 y_{t-1} + \varepsilon_t$ $\qquad \varepsilon \sim (0, \sigma^2_\varepsilon)$

I a) antog jag att $\hat{\phi}_1 = 0,8$.

Eftersom $E(y_t) = \frac{\delta}{1-\phi_1}$ för en stationär AR(p).

borde vi kunna skatta $\hat{\delta} = \frac{E(y_t)}{1-0,8}$

Längre än så kommer jag dessvärre inte....

c) $\hat{\rho}_4 = \sum_{i=1}^{1} \hat{\phi}_1 \cdot \hat{\rho}_3 = 0,8 \cdot 0,4 = \underline{\underline{0,32}}$ OK, givet tidigare fel.

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