Stockholm University
Department of Statistics
Per Gösta Andersson

**Econometrics II**

**WRITTEN EXAMINATION**
Wednesday January 10, 2018

Tools allowed: Pocket calculator

Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description.

For the maximum number of points on each problem detailed and clear solutions are required.

**Observe:**: If not indicated otherwise, the error terms $\epsilon_t$ in the models are assumed independent and $N(0, \sigma^2)$.

You may answer in Swedish.

1. (20p) Yearly values for some financial variable $y_t$ are recorded for 11 years ($t = 1, \ldots 11$):

| Year | $y_t$ |
|---|---|
| 1 | 7 |
| 2 | 9 |
| 3 | 5 |
| 4 | 9 |
| 5 | 13 |
| 6 | 8 |
| 7 | 12 |
| 8 | 13 |
| 9 | 9 |
| 10 | 11 |
| 11 | 7 |

   (a) Plot the data. By visual inspection would you consider $y_t$ to be stationary? Why/why not?

   (b) Use an appropriate exponential smoothing method to compute a forecast (prediction) for year 12. Use the discount factor 0.2 and the whole given series of values for computation of the starting value.

(c) Another smoothing possibility is the use of the simple moving average with span $N$. Describe in short terms the main principal theoretical differencies between the smoothing you used in (b) and the simple moving average smoothing.

2. (21p) True or false? Short motivation/comment also needed.

   (a) The partial adjustment (stock) model is constructed for special use on panel data.

   (b) The expectation of $y_t$ in an MA-model is zero if the parameter $\mu$ is zero.

   (c) The test statistic in Durbins $h$-test is approximately $F$-distributed under $H_0$.

   (d) In the Koyck model $Cov(v_t, y_{t-1}) = 0$, where $v_t$ is the error term.

   (e) The null hypothesis in unit root tests means that the investigated process is stationary.

   (f) A natural choice of model for panel data is REM, if the data is assumed obtained as a sample from a larger population.

   (g) A GARCH model is used to account for autocorelation in a time series model.

3. (16p) Consider the following estimated autocorrelation coefficients using 500 observations for some stationary process:

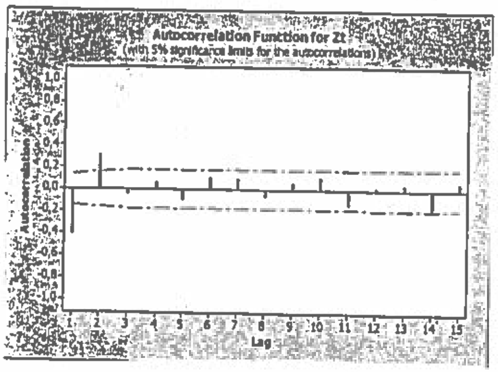
| Lag | ACF |
|---|---|
| 1 | 0.307 |
| 2 | -0.013 |
| 3 | 0.086 |
| 4 | 0.031 |
| 5 | -0.079 |

   (a) Test the null hypothesis that the autocorrelation functions for lag 1 to 5 are all zero at significance level 5%.

   (b) Which underlying time series model would you suggest (with motivation) given the information in this situation?

   (c) Derive an expression for $\rho_1$, given your chosen model in (b).

4. (16p) Based on observed values of a time series $y_t$, an ARIMA(0,1,2) was fitted with results as below. ($z_t = y_t - y_{t-1}$)




**ARIMA Model: Xt**

```
Final Estimates of Parameters

Type         Coef   SE Coef      T      P
MA   1     0,3565    0,0653   5,46  0,000
MA   2    -0,4202    0,0653  -6,44  0,000
Constant  0,20457   0,08959   2,28  0,023

Differencing: 1 regular difference
Number of observations:  Original series 200, after differencing 199
Residuals:      SS =  276,173 (backforecasts excluded)
                MS =  1,409   DF = 196
```

(a) Do the results of the estimated ACF:s for $z_t$ support the chosen fitted model? Why/why not?

(b) Write out the estimated model of $z_t$ as $\hat{z}_t = \ldots$, with the parameter estimates inserted.

Also, compute $E(\hat{z}_t)$ and $V(\hat{z}_t)$. (This means the same as computing $E(z_t)$ and $V(z_t)$ and then inserting the estimated parameters).

5. (14p) Let $y_t$ be a stationary AR(2) process. From data we have obtained estimates of the autocorrelation functions: $\hat{\rho}_1 = 0.50$ and $\hat{\rho}_2 = 0.17$.

Compute an estimate of $\rho_3$.

6. (13p) Consider the stochastic processes below. For each process determine if it is stationary or nonstationary. If the latter case applies, determine a transformation to make it stationary.

1. $y_t = 1 + t + \epsilon_t$

2. $y_t = \epsilon_t \epsilon_{t-1}$

# Formula sheet, Econometrics II, Spring 2017

Under the simple linear model $y_t = \beta_1 + \beta_2 x_t + u_t$, where $u_t \sim N(0, \sigma^2)$ and given independent pairs of observations $(y_1, x_1), \ldots, (y_n, x_n)$, the OLS (and ML) estimators are:

$$\begin{aligned}
\widehat{\beta}_1 &= \overline{y} - \widehat{\beta}_2 \overline{x} \\
\widehat{\beta}_2 &= \frac{\sum (x_t - \overline{x})(y_t - \overline{y})}{\sum (x_t - \overline{x})^2} \\
\widehat{\sigma}^2 &= \frac{RSS}{n-2} = \frac{\sum (y_t - \widehat{y}_t)^2}{n-2}
\end{aligned}$$

where $\widehat{y}_t = \widehat{\beta}_1 + \widehat{\beta}_2 x_t$ and where $E(\widehat{\beta_1}) = \beta_1$, $E(\widehat{\beta_2}) = \beta_2$ and $E(\widehat{\sigma}^2) = \sigma^2$

Comparing an "old" model with a "new" (larger):

$$\begin{aligned}
F &= \frac{(ESS_{new} - ESS_{old})/\text{number of new regressors}}{RSS_{new}/(n - \text{number of parameters in the new model})} \\
&= \frac{(R^2_{new} - R^2_{old})/\text{number of new regressors}}{(1 - R^2_{new})/(n - \text{number of parameters in the new model})}
\end{aligned}$$

Comparing an "unrestricted" model with a "restricted":

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} = \frac{(R^2_{UR} - R^2_R)/m}{(1 - R^2_{UR})/(n-k)}$$

where $m$ is the number of linear constraints and $k$ is the number of parameters in the unrestricted model.

Dynamic models: $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_{t-1} + v_t$

Koyck: $y_t = \alpha(1-\lambda) + \beta_0 x_t + \lambda y_{t-1} + v_t$
Adaptive expectations: $y_t = \gamma\beta_0 + \gamma\beta_1 x_t + (1-\gamma)y_{t-1} + (u_t - (1-\gamma)u_{t-1})$
Partial adjustment: $y_t = \delta\beta_0 + \delta\beta_1 x_t + (1-\delta)y_{t-1} + \delta u_t$

The Durbin Watson $d$ statistic:

$$d = \frac{\sum_{t=2}^{n} (\widehat{u}_t - \widehat{u}_{t-1})^2}{\sum_{t=1}^{n} \widehat{u}_t^2}$$

The Durbin $h$ statistic:

$$h = \widehat{\rho}\sqrt{\frac{n}{1 - n\left[\hat{V}(\widehat{\alpha}_2)\right]}} \approx N(0,1), \text{ if } \rho = 0$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = \frac{1}{n}\sum_{t=1}^{n}[y_t - \widehat{y}_t(t-1)]^2$$

Autocorrelation function:

$$\rho_k = \frac{Cov(y_t, y_{t+k})}{V(y_t)}, \quad k = 0, 1, 2, \ldots$$

Sample correlation function:

$$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n-k}(y_t - \bar{y})^2}, \quad k = 0, 1, 2, \ldots$$

Simple moving average:

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} y_t$$

First-order exponential smoothing:

$$\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$$

Second-order exponential smoothing:

$$\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)},$$

where $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$

Holt's method:

$$\begin{aligned} L_t &= \alpha y_t + (1-\alpha)(L_{t-1} + T_{t-1}) \\ T_t &= \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1} \end{aligned}$$

$$\widehat{y}_{T+\tau}(T) = L_T + \tau T_T, \quad \tau = 1, 2, \ldots$$

Forecast under a constant process:

$$\tilde{y}_{T+\tau}(T) = \tilde{y}_T \quad \tau = 1, 2, ...$$

Forecast under a linear trend:

$$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T}\tau,$$

where $\hat{y}_T = \hat{\beta}_{0,T} + \hat{\beta}_{1,T}T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$

For white noise:

$$\hat{\rho}_k \approx N(0, 1/n)\ , k = 1, 2, \ldots$$

The Q statistic:

$$Q = n \sum_{k=1}^{K} \hat{\rho}_k^2 \approx \chi^2(K)$$

The Ljung-Box statistic:

$$Q_{LB} = n\,(n+2) \sum_{k=1}^{K} \left( \frac{\hat{\rho}_k^2}{n-k} \right) \approx \chi^2(K)$$

ARMA(p,q):

$$y_t = \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + \epsilon_t - \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$

Stationarity and invertibility conditions for some time series models:

| Model | Stationarity conditions | Invertibility conditions |
|---|---|---|
| AR(1) | $\|\phi_1\| < 1$ | None |
| AR(2) | $\phi_1 + \phi_2 < 1$<br>$\phi_2 - \phi_1 < 1$<br>$\|\phi_2\| < 1$ | None |
| MA(1) | None | $\|\theta_1\| < 1$ |
| MA(2) | None | $\theta_1 + \theta_2 < 1$<br>$\theta_2 - \theta_1 < 1$<br>$\|\theta_2\| < 1$ |
| ARMA(1,1) | $\|\phi_1\| < 1$ | $\|\theta_1\| < 1$ |
| ARMA(2,2) | $\phi_1 + \phi_2 < 1$<br>$\phi_2 - \phi_1 < 1$<br>$\|\phi_2\| < 1$ | $\theta_1 + \theta_2 < 1$<br>$\theta_2 - \theta_1 < 1$<br>$\|\theta_2\| < 1$ |

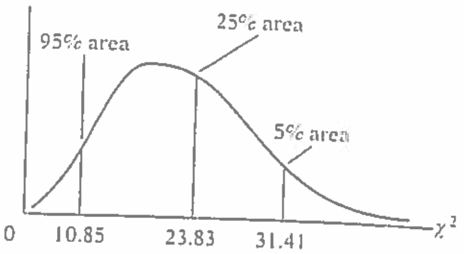
The Yule-Walker equations for AR(p):

$$\rho_k = \sum_{i=1}^{p} \phi_i \rho_{k-i}, \quad k = 1, 2, \ldots$$

**TABLE D.4**
**Upper Percentage Points of the $\chi^2$ Distribution**

**Example**

$\Pr(\chi^2 > 10.85) = 0.95$
$\Pr(\chi^2 > 23.83) = 0.25$ for df = 20
$\Pr(\chi^2 > 31.41) = 0.05$




| Degrees of freedom \ Pr | .995 | .990 | .975 | .950 | .900 |
|---|---|---|---|---|---|
| 1 | 392704 × 10⁻¹⁰ | 157088 × 10⁻⁹ | 982069 × 10⁻⁹ | 393214 × 10⁻⁸ | .0157908 |
| 2 | .0100251 | .0201007 | .0506356 | .102587 | .210720 |
| 3 | .0717212 | .114832 | .215795 | .351846 | .584375 |
| 4 | .206990 | .297110 | .484419 | .710721 | 1.063623 |
| 5 | .411740 | .554300 | .831211 | 1.145476 | 1.61031 |
| 6 | .675727 | .872085 | 1.237347 | 1.63539 | 2.20413 |
| 7 | .989265 | 1.239043 | 1.68987 | 2.16735 | 2.83311 |
| 8 | 1.344419 | 1.646482 | 2.17973 | 2.73264 | 3.48954 |
| 9 | 1.734926 | 2.087912 | 2.70039 | 3.32511 | 4.16816 |
| 10 | 2.15585 | 2.55821 | 3.24697 | 3.94030 | 4.86518 |
| 11 | 2.60321 | 3.05347 | 3.81575 | 4.57481 | 5.57779 |
| 12 | 3.07382 | 3.57056 | 4.40379 | 5.22603 | 6.30380 |
| 13 | 3.56503 | 4.10691 | 5.00874 | 5.89186 | 7.04150 |
| 14 | 4.07468 | 4.66043 | 5.62872 | 6.57063 | 7.78953 |
| 15 | 4.60094 | 5.22935 | 6.26214 | 7.26094 | 8.54675 |
| 16 | 5.14224 | 5.81221 | 6.90766 | 7.96164 | 9.31223 |
| 17 | 5.69724 | 6.40776 | 7.56418 | 8.67176 | 10.0852 |
| 18 | 6.26481 | 7.01491 | 8.23075 | 9.39046 | 10.8649 |
| 19 | 6.84398 | 7.63273 | 8.90655 | 10.1170 | 11.6509 |
| 20 | 7.43386 | 8.26040 | 9.59083 | 10.8508 | 12.4426 |
| 21 | 8.03366 | 8.89720 | 10.28293 | 11.5913 | 13.2396 |
| 22 | 8.64272 | 9.54249 | 10.9823 | 12.3380 | 14.0415 |
| 23 | 9.26042 | 10.19567 | 11.6885 | 13.0905 | 14.8479 |
| 24 | 9.88623 | 10.8564 | 12.4011 | 13.8484 | 15.6587 |
| 25 | 10.5197 | 11.5240 | 13.1197 | 14.6114 | 16.4734 |
| 26 | 11.1603 | 12.1981 | 13.8439 | 15.3791 | 17.2919 |
| 27 | 11.8076 | 12.8786 | 14.5733 | 16.1513 | 18.1138 |
| 28 | 12.4613 | 13.5648 | 15.3079 | 16.9279 | 18.9392 |
| 29 | 13.1211 | 14.2565 | 16.0471 | 17.7083 | 19.7677 |
| 30 | 13.7867 | 14.9535 | 16.7908 | 18.4926 | 20.5992 |
| 40 | 20.7065 | 22.1643 | 24.4331 | 26.5093 | 29.0505 |
| 50 | 27.9907 | 29.7067 | 32.3574 | 34.7642 | 37.6886 |
| 60 | 35.5346 | 37.4848 | 40.4817 | 43.1879 | 46.4589 |
| 70 | 43.2752 | 45.4418 | 48.7576 | 51.7393 | 55.3290 |
| 80 | 51.1720 | 53.5400 | 57.1532 | 60.3915 | 64.2778 |
| 90 | 59.1963 | 61.7541 | 65.6466 | 69.1260 | 73.2912 |
| 100* | 67.3276 | 70.0648 | 74.2219 | 77.9295 | 82.3581 |

*For df greater than 100 the expression $\sqrt{2\chi^2} - \sqrt{(2k-1)} = Z$ follows the standardized normal distribution, where $k$ represents the degrees of freedom.

$\chi^2$-table continued

| .750 | .500 | .250 | .100 | .050 | .025 | .010 | .005 |
|---|---|---|---|---|---|---|---|
| .1015308 | .454937 | 1.32330 | 2.70554 | 3.84146 | 5.02389 | 6.63490 | 7.87944 |
| .575364 | 1.38629 | 2.77259 | 4.60517 | 5.99147 | 7.37776 | 9.21034 | 10.5966 |
| 1.212534 | 2.36597 | 4.10835 | 6.25139 | 7.81473 | 9.34840 | 11.3449 | 12.8381 |
| 1.92255 | 3.35670 | 5.38527 | 7.77944 | 9.48773 | 11.1433 | 13.2767 | 14.8602 |
| .67460 | 4.35146 | 6.62568 | 9.23635 | 11.0705 | 12.8325 | 15.0863 | 16.7496 |
| 3.45460 | 5.34812 | 7.84080 | 10.6446 | 12.5916 | 14.4494 | 16.8119 | 18.5476 |
| 4.25485 | 6.34581 | 9.03715 | 12.0170 | 14.0671 | 16.0128 | 18.4753 | 20.2777 |
| 5.07064 | 7.34412 | 10.2188 | 13.3616 | 15.5073 | 17.5346 | 20.0902 | 21.9550 |
| 5.89883 | 8.34283 | 11.3887 | 14.6837 | 16.9190 | 19.0228 | 21.6660 | 23.5893 |
| .73720 | 9.34182 | 12.5489 | 15.9871 | 18.3070 | 20.4831 | 23.2093 | 25.1882 |
| .58412 | 10.3410 | 13.7007 | 17.2750 | 19.6751 | 21.9200 | 24.7250 | 26.7569 |
| 8.43842 | 11.3403 | 14.8454 | 18.5494 | 21.0261 | 23.3367 | 26.2170 | 28.2995 |
| 9.29906 | 12.3398 | 15.9839 | 19.8119 | 22.3621 | 24.7356 | 27.6883 | 29.8194 |
| 10.1653 | 13.3393 | 17.1170 | 21.0642 | 23.6848 | 26.1190 | 29.1413 | 31.3193 |
| 11.0365 | 14.3389 | 18.2451 | 22.3072 | 24.9958 | 27.4884 | 30.5779 | 32.8013 |
| 11.9122 | 15.3385 | 19.3688 | 23.5418 | 26.2962 | 28.8454 | 31.9999 | 34.2672 |
| 12.7919 | 16.3381 | 20.4887 | 24.7690 | 27.5871 | 30.1910 | 33.4087 | 35.7185 |
| 13.6753 | 17.3379 | 21.6049 | 25.9894 | 28.8693 | 31.5264 | 34.8053 | 37.1564 |
| 14.5620 | 18.3376 | 22.7178 | 27.2036 | 30.1435 | 32.8523 | 36.1908 | 38.5822 |
| 15.4518 | 19.3374 | 23.8277 | 28.4120 | 31.4104 | 34.1696 | 37.5662 | 39.9968 |
| 16.3444 | 20.3372 | 24.9348 | 29.6151 | 32.6705 | 35.4789 | 38.9321 | 41.4010 |
| 17.2396 | 21.3370 | 26.0393 | 30.8133 | 33.9244 | 36.7807 | 40.2894 | 42.7956 |
| 18.1373 | 22.3369 | 27.1413 | 32.0069 | 35.1725 | 38.0757 | 41.6384 | 44.1813 |
| 19.0372 | 23.3367 | 28.2412 | 33.1963 | 36.4151 | 39.3641 | 42.9798 | 45.5585 |
| 19.9393 | 24.3366 | 29.3389 | 34.3816 | 37.6525 | 40.6465 | 44.3141 | 46.9278 |
| 20.8434 | 25.3364 | 30.4345 | 35.5631 | 38.8852 | 41.9232 | 45.6417 | 48.2899 |
| 21.7494 | 26.3363 | 31.5284 | 36.7412 | 40.1133 | 43.1944 | 46.9630 | 49.6449 |
| 22.6572 | 27.3363 | 32.6205 | 37.9159 | 41.3372 | 44.4607 | 48.2782 | 50.9933 |
| .5666 | 28.3362 | 33.7109 | 39.0875 | 42.5569 | 45.7222 | 49.5879 | 52.3356 |
| 24.4776 | 29.3360 | 34.7998 | 40.2560 | 43.7729 | 46.9792 | 50.8922 | 53.6720 |
| 33.6603 | 39.3354 | 45.6160 | 51.8050 | 55.7585 | 59.3417 | 63.6907 | 66.7659 |
| 42.9421 | 49.3349 | 56.3336 | 63.1671 | 67.5048 | 71.4202 | 76.1539 | 79.4900 |
| 52.2938 | 59.3347 | 66.9814 | 74.3970 | 79.0819 | 83.2976 | 88.3794 | 91.9517 |
| 1.6983 | 69.3344 | 77.5766 | 85.5271 | 90.5312 | 95.0231 | 100.425 | 104.215 |
| 1.1445 | 79.3343 | 88.1303 | 96.5782 | 101.879 | 106.629 | 112.329 | 116.321 |
| 80.6247 | 89.3342 | 98.6499 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 90.1332 | 99.3341 | 109.141 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 8, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

Department of Statistics

Stockholms universitet

# Correction sheet

**Date:** 10/01/2018

**Room:** Uggleviksssalen

**Course:** Econometrics (eng)

**Exam:** Econometrics II (eng)

**Anonymous code:** EK2-RJA-TAJ

☒ I authorise the anonymous posting of my exam, in whole or in part, on the department homepage as a sample student answer.

**NOTE! ALSO WRITE ON THE BACK OF THE ANSWER SHEET**

**Mark answered questions**

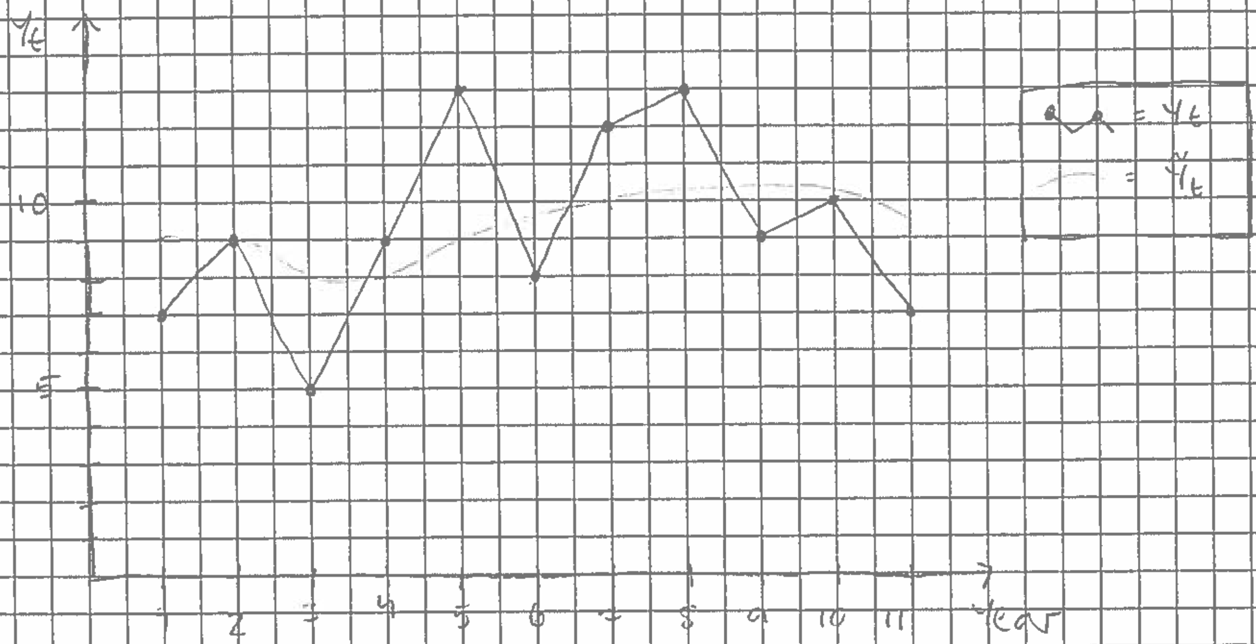
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total number of pages |
|---|---|---|---|---|---|---|---|---|---|
| X | X | X | X | X | X | | | | 4 |
| Teacher's notes 16 | 21 | 15 | 14 | 14 | 11 | | | | |

| Points | Grade | Teacher's sign. |
|---|---|---|
| 91 | A | PGA |

**SU, DEPARTMENT OF STATISTICS**

Room: Ugglevik  Anonymous code: EK2-RJA-TAJ  Sheet number: 1

(1)




a) Från plotten verkar det som att $Y_t$ är stationär då det inte finns tecken på någon trend eller förändring i varians dvs det verkar rimligt att kunna anta konstant väntevärde & varians. /3

b) Givet antagandet om stationaritet i a) kan simple exponential smoothing antas vara en rimlig uppskattningsmetod.

De utjämnade värdena ges av:

$\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$  där $\lambda = 0{,}2$

och $\tilde{y}_0 = \frac{\sum y_t}{11} = \frac{103}{11}$.

För att göra prediktion för år 12 behövs det utjämnade värdet för år 11, och för det behövs det utjämnade värdet för år 10 osv. Därför räknas $\tilde{y}_1, \dots, \tilde{y}_{11}$ ut →

$\tilde{Y}_1 = \lambda \cdot Y_1 + (1-\lambda) \cdot \tilde{Y}_0 = 0{,}2 \cdot 7 + 0{,}8 \cdot \frac{103}{11} = \frac{489}{55} \approx 8{,}8909$

$\tilde{Y}_2 = 0{,}2 \cdot 9 + 0{,}8 \cdot \frac{489}{55} = \frac{2451}{275} \approx 8{,}9127$

$\tilde{Y}_3 = 0{,}2 \cdot 5 + 0{,}8 \cdot \frac{2451}{275} = \frac{11179}{1375} \approx 8{,}1302$

$\tilde{Y}_4 = 0{,}2 \cdot 9 + 0{,}8 \cdot \frac{11179}{1375} = \frac{57091}{6875} \approx 8{,}3041$

$\tilde{Y}_5 = 0{,}2 \cdot 13 + 0{,}8 \cdot \frac{57091}{6875} \approx 9{,}2433$

$\tilde{Y}_6 = 0{,}2 \cdot 8 + 0{,}8 \cdot 9{,}2433 \approx 8{,}9947$

$\tilde{Y}_7 = 0{,}2 \cdot 12 + 0{,}8 \cdot 8{,}9947 \approx 9{,}5957$

$\tilde{Y}_8 = 0{,}2 \cdot 13 + 0{,}8 \cdot 9{,}5957 \approx 10{,}2766$

$\tilde{Y}_9 = 0{,}2 \cdot 9 + 0{,}8 \cdot 10{,}2766 \approx 10{,}0213$

$\tilde{Y}_{10} = 0{,}2 \cdot 11 + 0{,}8 \cdot 10{,}0213 \approx 10{,}2170$

$\tilde{Y}_{11} = 0{,}2 \cdot 7 + 0{,}8 \cdot 10{,}2170 \approx 9{,}5736$

Prognos för år 12 ges av:

$\hat{Y}_{11+1}(11) = \tilde{Y}_{11} \Rightarrow \hat{Y}_{12}(11) = \tilde{Y}_{11} \approx 9{,}5736$ //

Svar: Det predikterade värdet av Y år 12 ($\hat{Y}_{12}$) är ca 9,5736

c) Den stora skillnaden mellan simple moving average och simple exponential smoothing är att den första endast använder N st observationer för att jämna ut datan, medan den senare använder all data tillgänglig fram till den tidpunkten man tittar på. Där simple moving average "glömmer" datan som mer än N-1 tidpunkter bakom den man tittar på har simple exponential smoothing med alla observationer i minnet, även om äldre värden får mindre & mindre betydelse. Simple exponential smoothing är också mer förfinad då λ används för att bestämma hur stor vikt den senaste obs ska få och hur stor vikt de äldre obs ska få/medan simple moving average lägger lika stor vikt vid alla N obs //

**SU, DEPARTMENT OF STATISTICS**

Room: Ugglevik  Anonymous code: EK2-RJA-DA  Sheet number: 2

(2) a, FALSKT. RPA-modellen innehåller inga tvärsektionskomponenter, utan endast tidskomponenter, vilket gör den lämpad för tidsseriedata snarare än paneldata.

b, SANT. $E(Y_t)=\mu$ i en MA-modell och när $\mu=0 \Rightarrow E(Y_t)=0$

c, FALSKT. h-statistikan i Durbins h-test är approximativt $N(0,1)$-fördelad under $H_0$

d, FALSKT. Koyck-modellen är $v_t = u_t - \lambda u_{t-1}$ vilket gör att $Cov(v_t, v_{t-1}) \neq 0$

e, FALSKT. $H_0$ i unit root tests är att $\delta=0$, dvs $\rho=1$, dvs att processen är icke-stationär.

f, SANT. REM lämpar sig för paneldata och om vi kan motivera att intercepten i modellen ges av $B_{1i} = B_1 + \varepsilon_i$, dvs populationsmodell vilket är en slumpmässig felterm, t ex pga att datan är ett slumpmässigt urval från en större population, kan REM passa. Ett Hausman-test bör dock även genomföras.

g) Rätt. En GARCH-modell används när heteroskedasticitet råder, dvs icke-konstant varians hos feltermen. /2

(3) a) För att testa autokorrelationen för flera laggar kan t.ex. ett Q-test eller ett Ljung-Box-test användas. Jag väljer att göra ett Ljung-Box-test.

$H_0: \rho_1 = \dots = \rho_5 = 0$

$H_A$: någon $\rho_k \neq 0$

$\alpha = 0{,}05 \quad K = 5 \quad n = 200 \quad \hat{\rho}_k^2$ = skattad korrelation i lagg k i kvadrat

Förkasta $H_0$ om $Q_{LB} > \chi^2_\alpha(K) = \chi^2_{0,05}(5) = 11{,}0705$

$$Q_{LB} = n(n+2)\sum_{k=1}^{K}\left(\frac{\hat{\rho}_k^2}{n-k}\right) =$$

$$= 200 \cdot 202 \cdot \left(\frac{0{,}307^2}{200-1} + \frac{-0{,}013^2}{200-2} + \frac{0{,}086^2}{200-3} + \frac{0{,}03^2}{200-4} + \frac{-0{,}074^2}{200-5}\right)$$

$= 40400 \cdot 0{,}0005489179 \approx 22{,}17628467$

Svar: $22{,}1763 > 11{,}0705 \Rightarrow H_0$ förkastas och vi tror på att minst en $\rho_k \neq 0$ (vilket verkar stämma med ACF) /6

b) Då det verkar som att vi har en spik och resten brus i vår skattade ACF skulle jag föreslå en MA(1)-modell /1

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(3) c) $\rho_1 = \frac{Cov(Y_t, Y_{t-1})}{V(Y_t)}$

För en MA(1) gäller: $Y_t = \mu + \varepsilon_t - \theta\varepsilon_{t-1}$ (=0 pga konstant) (pga konstant minus $=V(\varepsilon_t)$)

$V(Y_t) = V(\mu + \varepsilon_t - \theta\varepsilon_{t-1}) = V(\mu) + V(\varepsilon_t) + \theta^2 \cdot V(\varepsilon_{t-1})$

$= 0 + \sigma^2 + \theta^2 \cdot \sigma^2 \Rightarrow V(Y_t) = \sigma^2 + \theta^2\sigma^2 \Rightarrow$

$V(Y_t) = \sigma^2(1+\theta^2)$

och $Cov(Y_t, Y_{t-1}) = Cov(\mu + \varepsilon_t - \theta\varepsilon_{t-1}, \mu + \varepsilon_{t-1} - \theta\varepsilon_{t-2})$

$= -\theta \cdot Cov(\varepsilon_{t-1}, \varepsilon_{t-1}) = -\theta \cdot V(\varepsilon_{t-1}) = -\theta \cdot V(\varepsilon_t) =$ ($=V(\varepsilon_t)$)

$= -\theta \cdot \sigma^2$

$\Rightarrow \rho_1 = \frac{Cov(Y_t, Y_{t-1})}{V(Y_t)} = \frac{\sigma^2 \cdot -\theta}{\sigma^2(1+\theta^2)} = \frac{-\theta}{1+\theta^2}$ /5

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(4)

a) Ja det gör den då $Y_t = ARIMA(0,1,2)$ innebär att första-differensen av $Y_t$ ($Z_t$) bör bli en stationär ARMA(0,2), dvs en MA(2), vilket stämmer överens med en skattad ACF med 2 spikar och sedan avslut för $k > q = 2$ /4

b) $\hat{Z}_t = 0{,}20457 - 0{,}3565\varepsilon_{t-1} + 0{,}4202\varepsilon_{t-2} + \varepsilon_t$

$E(Z_t) = E[0{,}20457 - 0{,}3565\varepsilon_{t-1} + 0{,}4202\varepsilon_{t-2} + \varepsilon_t]$

$= 0{,}20457 - 0{,}3565 \cdot E(\varepsilon_{t-1}) - 0{,}4202 \cdot E(\varepsilon_{t-2}) \cdot E(\varepsilon_t)$ (=0) (=0) (=0)

$= 0{,}20457$ (dvs $\mu$)

$$V(\hat{Z}_t) = V\left[0{,}20457 - 0{,}35655\,\varepsilon_{t-1} + 0{,}4202\,\varepsilon_{t-2} + \varepsilon_t\right]$$

$$= 0 + (-0{,}3565^2)\cdot V(\varepsilon_{t-1}) + 0{,}4202^2 \cdot V(\varepsilon_{t-2}) + V(\varepsilon_t) =$$

$$= 0{,}12709 \cdot \sigma^2 + 0{,}17657 \cdot \sigma^2 + \sigma^2 \Rightarrow$$

$$V(\hat{Z}_t) = \sigma^2(1 + 0{,}12709 + 0{,}17657) \quad \left(\text{dus } \sigma^2(1+\theta_1^2+\theta_2^2)\right)$$

dus $\hat{\sigma}^2 = \frac{RSS}{n-2} = \frac{276{,}123}{196} = MS = 1{,}409 \Rightarrow$

$$V(\hat{Z}_t) = 1{,}409 \cdot 1{,}3037 \approx 1{,}8369$$

<u>svar:</u> $E(\hat{Z}_t) = \hat{\mu} \approx 0{,}20457$ ;

$$V(\hat{Z}_t) = \hat{\sigma}^2(1+\hat{\theta}_1^2+\hat{\theta}_2^2) \approx 1{,}8369$$

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(4) $Y_t = \delta + \phi_1 Y_{t-1} + \phi Y_{t-2} + \varepsilon_t$
där $Y_t$ är en stationär AR(2)

$\hat{\rho}_1 = 0,5$ ; $\hat{\rho}_2 = 0,17$ ; $\hat{\rho}_3$ ?

För att ta reda på $\hat{\phi}_1$ & $\hat{\phi}_2$ ; Yule-Walker:

$\hat{\rho}_1 = 0,5 = \hat{\phi}_1 \cdot \underbrace{\rho(0)}_{=1} + \hat{\phi}_2 \cdot \underbrace{\hat{\rho}(-1)}_{=\hat{\rho}(1)} \Rightarrow$

$0,5 = \hat{\phi}_1 + \hat{\phi}_2 0,5 \Rightarrow \hat{\phi}_1 = 0,5 - 0,5\hat{\phi}_2$

$\hat{\phi}_2 = \frac{0,5 - \hat{\phi}_1}{0,5}$

$\hat{\rho}_2 = 0,17 = \hat{\phi}_1 \cdot \underbrace{\hat{\rho}_1}_{=0,5} + \hat{\phi}_2 \cdot \underbrace{\rho(0)}_{=1} \Rightarrow$

$0,17 = \hat{\phi}_1 \cdot 0,5 + \hat{\phi}_2 \Rightarrow 0,17 = \hat{\phi}_1 \cdot 0,5 + \left(\frac{0,5 - \hat{\phi}_1}{0,5}\right)$

$\Rightarrow 0,17 - \hat{\phi}_1 0,5 = \frac{0,5 - \hat{\phi}_1}{0,5} \Rightarrow 0,5(0,17 - \hat{\phi}_1 \cdot 0,5) = 0,5 - \hat{\phi}_1$

$\Rightarrow 0,085 - 0,25\hat{\phi}_1 = 0,5 - \hat{\phi}_1 \Rightarrow 0,75\hat{\phi}_1 = 0,415 \Rightarrow \hat{\phi}_1 \approx 0,5533$

$\Rightarrow \hat{\phi}_2 = \frac{0,5 - 0,5533}{0,5} \approx -0,1066$

$\hat{\phi}_1 \approx 0,5533$ & $\hat{\phi}_2 \approx -0,1066$ ger :

$\rho_3 = \hat{\phi}_1 \cdot \hat{\rho}_2 + \hat{\phi}_2 \cdot \hat{\rho}_1 = 0,5533 \cdot 0,17 - 0,1066 \cdot 0,5 \approx 0,040761$

Svar: en skattning av $\rho_k$ $(\hat{\rho}_3)$ är ca 0,040761

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