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Department of Statistics
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Econometrics II

WRITTEN EXAMINATION

Wednesday January 10, 2018

Tools allowed: Pocket calculator

Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description.

For the maximum number of points on each problem detailed and clear solutions are required.

Observe:: If not indicated otherwise, the error terms ϵ_t in the models are assumed independent and $N(0, \sigma^2)$.

You may answer in Swedish.

1. (20p) Yearly values for some financial variable y_t are recorded for 11 years ($t = 1, \dots, 11$):

Year	y_t
1	7
2	9
3	5
4	9
5	13
6	8
7	12
8	13
9	9
10	11
11	7

- (a) Plot the data. By visual inspection would you consider y_t to be stationary? Why/why not?
- (b) Use an appropriate exponential smoothing method to compute a forecast (prediction) for year 12. Use the discount factor 0.2 and the whole given series of values for computation of the starting value.

(c) Another smoothing possibility is the use of the simple moving average with span N . Describe in short terms the main principal theoretical differences between the smoothing you used in (b) and the simple moving average smoothing.

2. (21p) True or false? Short motivation/comment also needed.

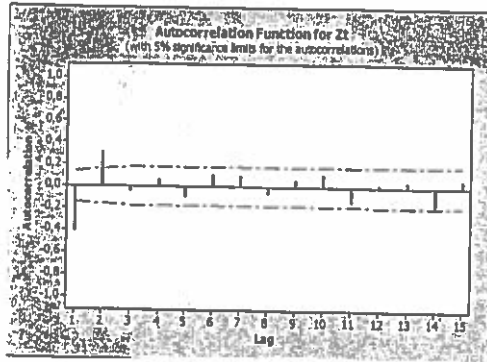
- (a) The partial adjustment (stock) model is constructed for special use on panel data.
- (b) The expectation of y_t in an MA-model is zero if the parameter μ is zero.
- (c) The test statistic in Durbins h -test is approximately F -distributed under H_0 .
- (d) In the Koyck model $Cov(v_t, y_{t-1}) = 0$, where v_t is the error term.
- (e) The null hypothesis in unit root tests means that the investigated process is stationary.
- (f) A natural choice of model for panel data is REM, if the data is assumed obtained as a sample from a larger population.
- (g) A GARCH model is used to account for autocorrelation in a time series model.

3. (16p) Consider the following estimated autocorrelation coefficients using 500 observations for some stationary process:

Lag	ACF
1	0.307
2	-0.013
3	0.086
4	0.031
5	-0.079

- (a) Test the null hypothesis that the autocorrelation functions for lag 1 to 5 are all zero at significance level 5%.
- (b) Which underlying time series model would you suggest (with motivation) given the information in this situation?
- (c) Derive an expression for ρ_1 , given your chosen model in (b).

4. (16p) Based on observed values of a time series y_t , an ARIMA(0,1,2) was fitted with results as below. ($z_t = y_t - y_{t-1}$)



ARIMA Model: X_t

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
MA 1	0,3565	0,0653	5,46	0,000
MA 2	-0,4202	0,0653	-6,44	0,000
Constant	0,20457	0,08959	2,28	0,023

Differencing: 1 regular difference

Number of observations: Original series 200, after differencing 199

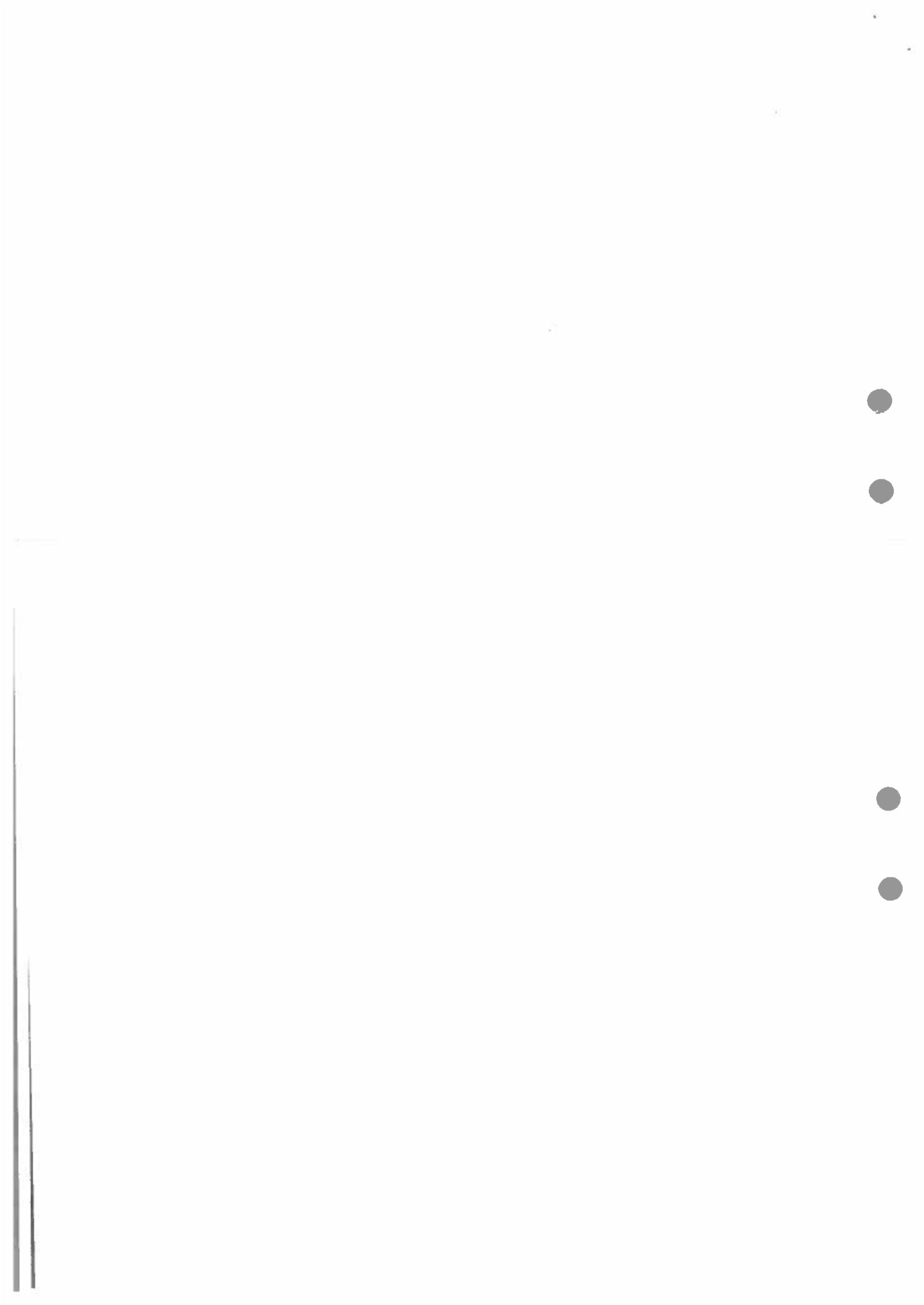
Residuals: SS = 276,173 (backforecasts excluded)

MS = 1,409 DF = 196

- (a) Do the results of the estimated ACF:s for z_t support the chosen fitted model? Why/why not?
- (b) Write out the estimated model of z_t as $\hat{z}_t = \dots$, with the parameter estimates inserted.
- Also, compute $E(\hat{z}_t)$ and $V(\hat{z}_t)$. (This means the same as computing $E(z_t)$ and $V(z_t)$ and then inserting the estimated parameters).
5. (14p) Let y_t be a stationary AR(2) process. From data we have obtained estimates of the autocorrelation functions: $\hat{\rho}_1 = 0.50$ and $\hat{\rho}_2 = 0.17$.
- Compute an estimate of ρ_3 .
6. (13p) Consider the stochastic processes below. For each process determine if it is stationary or nonstationary. If the latter case applies, determine a transformation to make it stationary.

1. $y_t = 1 + t + \epsilon_t$

2. $y_t = \epsilon_t \epsilon_{t-1}$



Formula sheet, Econometrics II, Spring 2017

Under the simple linear model $y_t = \beta_1 + \beta_2 x_t + u_t$, where $u_t \sim N(0, \sigma^2)$ and given independent pairs of observations $(y_1, x_1), \dots, (y_n, x_n)$, the OLS (and ML) estimators are:

$$\begin{aligned}\hat{\beta}_1 &= \bar{y} - \hat{\beta}_2 \bar{x} \\ \hat{\beta}_2 &= \frac{\sum (x_t - \bar{x})(y_t - \bar{y})}{\sum (x_t - \bar{x})^2} \\ \hat{\sigma}^2 &= \frac{RSS}{n-2} = \frac{\sum (y_t - \hat{y}_t)^2}{n-2}\end{aligned}$$

where $\hat{y}_t = \hat{\beta}_1 + \hat{\beta}_2 x_t$ and where $E(\hat{\beta}_1) = \beta_1$, $E(\hat{\beta}_2) = \beta_2$ and $E(\hat{\sigma}^2) = \sigma^2$

Comparing an "old" model with a "new" (larger):

$$\begin{aligned}F &= \frac{(ESS_{new} - ESS_{old})/\text{number of new regressors}}{RSS_{new}/(n - \text{number of parameters in the new model})} \\ &= \frac{(R_{new}^2 - R_{old}^2)/\text{number of new regressors}}{(1 - R_{new}^2)/(n - \text{number of parameters in the new model})}\end{aligned}$$

Comparing an "unrestricted" model with a "restricted":

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n - k)} = \frac{(R_{UR}^2 - R_R^2)/m}{(1 - R_{UR}^2)/(n - k)}$$

where m is the number of linear constraints and k is the number of parameters in the unrestricted model.

Dynamic models: $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_{t-1} + v_t$

Koyck: $y_t = \alpha(1 - \lambda) + \beta_0 x_t + \lambda y_{t-1} + v_t$

Adaptive expectations: $y_t = \gamma \beta_0 + \gamma \beta_1 x_t + (1 - \gamma) y_{t-1} + (u_t - (1 - \gamma) u_{t-1})$

Partial adjustment: $y_t = \delta \beta_0 + \delta \beta_1 x_t + (1 - \delta) y_{t-1} + \delta u_t$

The Durbin Watson d statistic:

$$d = \frac{\sum_{t=2}^n (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^n \hat{u}_t^2}$$

The Durbin h statistic:

$$h = \hat{\rho} \sqrt{\frac{n}{1 - n [\hat{V}(\hat{\alpha}_2)]}} \approx N(0, 1), \text{ if } \rho = 0$$

$$MSE = \frac{1}{n} \sum_{t=1}^n [e_t(1)]^2 = \frac{1}{n} \sum_{t=1}^n [y_t - \hat{y}_t(t-1)]^2$$

Autocorrelation function:

$$\rho_k = \frac{Cov(y_t, y_{t+k})}{V(y_t)}, \quad k = 0, 1, 2, \dots$$

Sample correlation function:

$$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k} (y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n-k} (y_t - \bar{y})^2}, \quad k = 0, 1, 2, \dots$$

Simple moving average:

$$M_T = \frac{1}{N} \sum_{t=T-N+1}^T y_t$$

First-order exponential smoothing:

$$\hat{y}_T = \lambda y_T + (1 - \lambda) \hat{y}_{T-1}$$

Second-order exponential smoothing:

$$\hat{y}_T^{(2)} = \lambda \hat{y}_T^{(1)} + (1 - \lambda) \hat{y}_{T-1}^{(2)},$$

where $\hat{y}_0^{(2)} = \hat{y}_1^{(1)}$

Holt's method:

$$L_t = \alpha y_t + (1 - \alpha) (L_{t-1} + T_{t-1})$$

$$T_t = \gamma (L_t - L_{t-1}) + (1 - \gamma) T_{t-1}$$

$$\hat{y}_{T+\tau}(T) = L_T + \tau T_T, \quad \tau = 1, 2, \dots$$

Forecast under a constant process:

$$\hat{y}_{T+\tau}(T) = \tilde{y}_T \quad \tau = 1, 2, \dots$$

Forecast under a linear trend:

$$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T}\tau,$$

$$\text{where } \hat{y}_T = \hat{\beta}_{0,T} + \hat{\beta}_{1,T}T = 2\hat{y}_T^{(1)} - \hat{y}_T^{(2)}$$

For white noise:

$$\hat{\rho}_k \approx N(0, 1/n), \quad k = 1, 2, \dots$$

The Q statistic:

$$Q = n \sum_{k=1}^K \hat{\rho}_k^2 \approx \chi^2(K)$$

The Ljung-Box statistic:

$$Q_{LB} = n(n+2) \sum_{k=1}^K \left(\frac{\hat{\rho}_k^2}{n-k} \right) \approx \chi^2(K)$$

ARMA(p,q):

$$y_t = \delta + \sum_{i=1}^p \phi_i y_{t-i} + \epsilon_t - \sum_{i=1}^q \theta_i \epsilon_{t-i}$$

Stationarity and invertibility conditions for some time series models:

Model	Stationarity conditions	Invertibility conditions
AR(1)	$ \phi_1 < 1$	None
AR(2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $ \phi_2 < 1$	None
MA(1)	None	$ \theta_1 < 1$
MA(2)	None	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $ \theta_2 < 1$
ARMA(1,1)	$ \phi_1 < 1$	$ \theta_1 < 1$
ARMA(2,2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $ \phi_2 < 1$	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $ \theta_2 < 1$

The Yule-Walker equations for AR(p):

$$\rho_k = \sum_{i=1}^p \phi_i \rho_{k-i}, \quad k = 1, 2, \dots$$

TABLE D.4
Upper Percentage
Points of the χ^2
Distribution

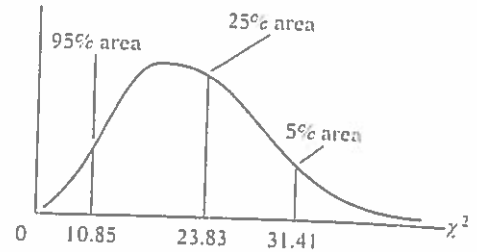
Example

$\Pr(\chi^2 > 10.85) = 0.95$

$\Pr(\chi^2 > 23.83) = 0.25$

$\Pr(\chi^2 > 31.41) = 0.05$

for $df = 20$



Degrees of freedom	.995	.990	.975	.950	.900
1	392704×10^{-10}	157088×10^{-9}	982069×10^{-9}	393214×10^{-8}	.0157908
2	.0100251	.0201007	.0506356	.102587	.210720
3	.0717212	.114832	.215795	.351846	.584375
4	.206990	.297110	.484419	.710721	1.063623
5	.411740	.554300	.831211	1.145476	1.61031
6	.675727	.872085	1.237347	1.63539	2.20413
7	.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100*	67.3276	70.0648	74.2219	77.9295	82.3581

*For df greater than 100 the expression $\sqrt{2\chi^2} - \sqrt{2k-1} = Z$ follows the standardized normal distribution, where k represents the degrees of freedom

χ^2 -table continued

.750	.500	.250	.100	.050	.025	.010	.005
.1015308	.454937	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
.575364	1.38629	2.77259	4.60517	5.99147	7.37776	9.21034	10.5966
1.212534	2.36597	4.10835	6.25139	7.81473	9.34840	11.3449	12.8381
1.92255	3.35670	5.38527	7.77944	9.48773	11.1433	13.2767	14.8602
2.67460	4.35146	6.62568	9.23635	11.0705	12.8325	15.0863	16.7496
3.45460	5.34812	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476
4.25485	6.34581	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777
5.07064	7.34412	10.2188	13.3616	15.5073	17.5346	20.0902	21.9550
5.89883	8.34283	11.3887	14.6837	16.9190	19.0228	21.6660	23.5893
6.73720	9.34182	12.5489	15.9871	18.3070	20.4831	23.2093	25.1882
7.58412	10.3410	13.7007	17.2750	19.6751	21.9200	24.7250	26.7569
8.43842	11.3403	14.8454	18.5494	21.0261	23.3367	26.2170	28.2995
9.29906	12.3398	15.9839	19.8119	22.3621	24.7356	27.6883	29.8194
10.1653	13.3393	17.1170	21.0642	23.6848	26.1190	29.1413	31.3193
11.0365	14.3389	18.2451	22.3072	24.9958	27.4884	30.5779	32.8013
11.9122	15.3385	19.3688	23.5418	26.2962	28.8454	31.9999	34.2672
12.7919	16.3381	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185
13.6753	17.3379	21.6049	25.9894	28.8693	31.5264	34.8053	37.1564
14.5620	18.3376	22.7178	27.2036	30.1435	32.8523	36.1908	38.5822
15.4518	19.3374	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968
16.3444	20.3372	24.9348	29.6151	32.6705	35.4789	38.9321	41.4010
17.2396	21.3370	26.0393	30.8133	33.9244	36.7807	40.2894	42.7956
18.1373	22.3369	27.1413	32.0069	35.1725	38.0757	41.6384	44.1813
19.0372	23.3367	28.2412	33.1963	36.4151	39.3641	42.9798	45.5585
19.9393	24.3366	29.3389	34.3816	37.6525	40.6465	44.3141	46.9278
20.8434	25.3364	30.4345	35.5631	38.8852	41.9232	45.6417	48.2899
21.7494	26.3363	31.5284	36.7412	40.1133	43.1944	46.9630	49.6449
22.6572	27.3363	32.6205	37.9159	41.3372	44.4607	48.2782	50.9933
23.5666	28.3362	33.7109	39.0875	42.5569	45.7222	49.5879	52.3356
24.4776	29.3360	34.7998	40.2560	43.7729	46.9792	50.8922	53.6720
25.3893	30.3359	35.8888	41.4212	44.8888	48.2323	52.1943	55.0000
26.3020	31.3358	36.9794	42.5718	45.9999	49.4816	53.4932	56.3000
27.2153	32.3357	38.0715	43.6949	47.1133	50.7263	54.7871	57.5933
28.1296	33.3356	39.1651	44.8175	48.2323	51.9663	56.0771	58.8800
29.0444	34.3355	40.2600	45.9396	49.3500	53.2000	57.3622	60.1600
30.0000	35.3354	41.3623	47.0612	50.4667	54.4289	58.6425	61.4333
31.0000	36.3353	42.4848	48.1823	51.5833	55.6422	59.9182	62.7000
32.0000	37.3352	43.6063	49.3029	52.7000	56.8111	61.1885	63.9600
33.0000	38.3351	44.7278	50.4230	53.8167	57.9800	62.4538	65.2133
34.0000	39.3350	45.8483	51.5426	54.9333	59.1489	63.7141	66.4600
35.0000	40.3349	46.9678	52.6617	56.0500	60.3178	64.9704	67.7033
36.0000	41.3348	48.0863	53.7803	57.1667	61.4867	66.2227	68.9433
37.0000	42.3347	49.2038	54.8984	58.2833	62.6556	67.4710	70.1800
38.0000	43.3346	50.3203	56.0160	59.4000	63.8245	68.6413	71.4133
39.0000	44.3345	51.4368	57.1331	60.5167	64.9734	69.8076	72.6433
40.0000	45.3344	52.5513	58.2507	61.6333	66.1167	70.9709	73.8700
41.0000	46.3343	53.6648	59.3678	62.7500	67.2556	72.1212	75.0933
42.0000	47.3342	54.7773	60.5024	63.8667	68.3889	73.2685	76.3133
43.0000	48.3341	55.8888	61.6375	64.9833	69.5167	74.4128	77.5300
44.0000	49.3340	56.9994	62.7721	66.1000	70.6389	75.5541	78.7433
45.0000	50.3339	58.1099	63.9062	67.2167	71.7556	76.6744	79.9533
46.0000	51.3338	59.2194	65.0408	68.2933	72.8667	77.7907	81.1600
47.0000	52.3337	60.3289	66.1749	69.3700	73.9734	78.9030	82.3633
48.0000	53.3336	61.4385	67.3085	70.4467	75.0767	80.0113	83.5633
49.0000	54.3335	62.5470	68.4416	71.5200	76.1767	81.1156	84.7600
50.0000	55.3334	63.6545	69.5742	72.5933	77.2734	82.2159	85.9533
51.0000	56.3333	64.7610	70.7063	73.6667	78.3667	83.3122	87.1433
52.0000	57.3332	65.8665	71.8389	74.7400	79.4567	84.4045	88.3300
53.0000	58.3331	66.9710	72.9710	75.8133	80.5434	85.4928	89.5133
54.0000	59.3330	68.0745	74.1026	76.8800	81.6267	86.5771	90.6933
55.0000	60.3329	69.1770	75.2337	77.9500	82.7067	87.6574	91.8700
56.0000	61.3328	70.2785	76.3643	79.0233	83.7834	88.7337	93.0433
57.0000	62.3327	71.3790	77.4944	80.2967	84.8567	89.8060	94.2133
58.0000	63.3326	72.4885	78.6239	81.5667	85.9267	90.8744	95.3800
59.0000	64.3325	73.5970	79.7530	82.8333	87.0000	91.9387	96.5433
60.0000	65.3324	74.7045	80.8816	84.0933	88.0667	92.9990	97.7033
61.0000	66.3323	75.8110	82.0097	85.1833	89.1334	94.0553	98.8600
62.0000	67.3322	76.9165	83.1373	86.2900	90.2000	95.1076	100.0133
63.0000	68.3321	78.0210	84.2644	87.3967	91.2667	96.1559	101.1633
64.0000	69.3320	79.1245	85.3910	88.5033	92.3300	97.2002	102.3100
65.0000	70.3319	80.2270	86.5171	89.6100	93.3900	98.2395	103.4533
66.0000	71.3318	81.3285	87.6427	90.7167	94.4467	99.2748	104.5933
67.0000	72.3317	82.4390	88.7678	91.8233	95.5000	100.3061	105.7300
68.0000	73.3316	83.5485	89.8924	92.9300	96.5500	101.3334	106.8633
69.0000	74.3315	84.6570	91.0165	94.0367	97.5967	102.3567	107.9933
70.0000	75.3314	85.7645	92.1401	95.1433	98.6400	103.3760	109.1200
71.0000	76.3313	86.8710	93.2632	96.2500	99.6800	104.3913	110.2433
72.0000	77.3312	87.9765	94.3858	97.3567	100.7167	105.4026	111.3633
73.0000	78.3311	89.0810	95.5079	98.4633	101.7500	106.4099	112.4800
74.0000	79.3310	90.1845	96.6285	99.5700	102.7800	107.4132	113.5933
75.0000	80.3309	91.2860	97.7476	100.6767	103.8067	108.4215	114.7033
76.0000	81.3308	92.3865	98.8661	101.7833	104.8300	109.4258	115.8100
77.0000	82.3307	93.4860	99.9841	102.8900	105.8500	110.4261	116.9133
78.0000	83.3306	94.5835	101.1016	104.0000	106.8667	111.4224	118.0133
79.0000	84.3305	95.6800	102.2187	105.1133	107.8800	112.4147	119.1100
80.0000	85.3304	96.7755	103.3353	106.2200	108.8900	113.4030	120.2033
81.0000	86.3303	97.8700	104.4514	107.3267	109.8967	114.3873	121.2933
82.0000	87.3302	98.9635	105.5670	108.4233	110.9000	115.3676	122.3800
83.0000	88.3301	100.0560	106.6821	109.5300	111.9000	116.3439	123.4633
84.0000	89.3300	101.1475	107.7967	110.6367	112.8967	117.3162	124.5433
85.0000	90.3299	102.2380	108.9108	111.7433	113.8900	118.2845	125.6200
86.0000	91.3298	103.3275	110.0244	112.8500	114.8800	119.2488	126.6933
87.0000	92.3297	104.4160	111.1375	113.9567	115.8667	120.2091	127.7633
88.0000	93.3296	105.5035	112.2501	115.0633	116.8500	121.1654	128.8300
89.0000	94.3295	106.5900	113.3622	116.2700	117.8300	122.1177	129.8933
90.0000	95.3294	107.6755	114.4738	117.4800	118.8000	123.0660	130.9533
91.0000	96.3293	108.7600	115.5849	118.6900	119.7600	124.0103	132.0100
92.0000	97.3292	109.8435	116.6955	119.9000	120.7167	124.9506	133.0633
93.0000	98.3291	110.9260	117.8056	121.0067	121.6700	125.8869	134.1133
94.0000	99.3290	112.0075	118.9152	122.1000	122.6200	126.8192	135.1600
95.0000		113.0880	120.0243	123.1967	123.5667	127.7475	136.2033

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3rd ed., table 8, Cambridge University Press, New York, 1966.
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Stockholms
universitet

Department of Statistics

Correction sheet

Date: 10/01/2018

Room: Ugglevikssalen

Course: Econometrics (eng)

Exam: Econometrics II (eng)

Anonymous code:

EK2-RJA-TAJ

I authorise the anonymous posting of my exam, in whole or in part, on the department homepage as a sample student answer.

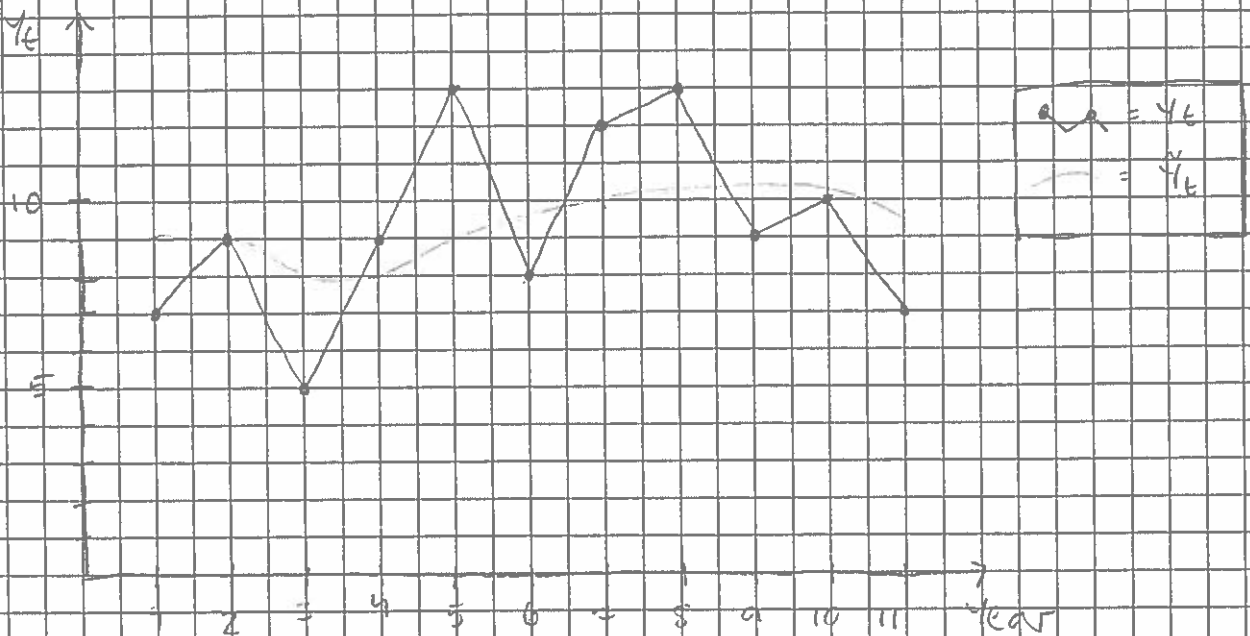
NOTE! ALSO WRITE ON THE BACK OF THE ANSWER SHEET

Mark answered questions

1	2	3	4	5	6	7	8	9	Total number of pages
X	X	X	X	X	X				9
Teacher's notes 16	21	15	14	14	11				

Points	Grade	Teacher's sign.
91	A	PGA

①



a) Från plotten verkar det som att y_t är stationärt då det inte finns tecken på någon trend eller förändring i varians. Men det verkar möjligt att korrelansen är konstant värdet $\rho = 0.2$.

b) Givet antagandet om stationaritet i a, kan simple exponential smoothing användas som en möjlig utvärderingsmetod.

De utvärderade variablerna ges av:

$$\hat{y}_T = \lambda y_T + (1-\lambda) \hat{y}_T \quad \text{där } \lambda = 0.2$$

och $\hat{y}_0 = \frac{\sum y_t}{n} = \frac{103}{11}$

För att göra prediktion för år 12 behövs det utvärderade värdet för år 11, och för det behövs det utvärderade värdet för år 10 osv. Vadför räknas $\hat{y}_1, \dots, \hat{y}_n$ ut \rightarrow

$$\hat{y}_1 = \lambda \cdot y_1 + (1-\lambda) \cdot y_0 = 0,2 \cdot 7 + 0,8 \cdot 103 = \frac{489}{55} \approx 8,8909$$

$$\hat{y}_2 = 0,2 \cdot 9 + 0,8 \cdot \frac{489}{55} = \frac{2451}{275} \approx 8,9127$$

$$\hat{y}_3 = 0,2 \cdot 5 + 0,8 \cdot \frac{2451}{275} = \frac{11174}{1375} \approx 8,1302$$

$$\hat{y}_4 = 0,2 \cdot 9 + 0,8 \cdot \frac{11174}{1375} = \frac{57091}{6875} \approx 8,3041$$

$$\hat{y}_5 = 0,2 \cdot 13 + 0,8 \cdot \frac{57091}{6875} \approx 9,2433$$

$$\hat{y}_6 = 0,2 \cdot 8 + 0,8 \cdot 9,2433 \approx 8,9947$$

$$\hat{y}_7 = 0,2 \cdot 12 + 0,8 \cdot 8,9947 \approx 9,5957$$

$$\hat{y}_8 = 0,2 \cdot 13 + 0,8 \cdot 9,5957 \approx 10,2766$$

$$\hat{y}_9 = 0,2 \cdot 9 + 0,8 \cdot 10,2766 \approx 10,0213$$

$$\hat{y}_{10} = 0,2 \cdot 11 + 0,8 \cdot 10,0213 \approx 10,2170$$

$$\hat{y}_{11} = 0,2 \cdot 7 + 0,8 \cdot 10,2170 \approx 9,5736$$

Prognos för år 12 ges av:

$$\hat{y}_{11+1}(11) = \hat{y}_{11} \Rightarrow \hat{y}_{12}(11) = \hat{y}_{11} \approx 9,5736$$

Svar: Det prediktade värdet av y år 12 (\hat{y}_{12}) är ca 9,5736

c) Den stora skillnaden mellan simple moving average och simple exponential smoothing är att den första endast använder N st observationer för att jämma ut datan, medan den senare använder all data tillgänglig fram till den tidpunkten max. t eller p . Det simple moving average "glömmer" datan som mer än $N-1$ tidpunkter bakom den man tittar på när simple exponential smoothing med alla observationer i minnet, även om äldre värden får mindre & mindre betydelse. Simple exponential smoothing är också mer förfärdig då λ används för att bestämma hur stor vikt den senaste obs ska få och hur stor vikt de äldre obs ska få. Med andra ord...

- ② a, FALSKT. IFA-modellen innehåller många tidsektionskomponenter, utan endast tidskomponenter, vilket gör den lämpad för tidsseriedata snarare än paneldata.
- b, SANT. $E(Y_t) = \mu$ i en MA-modell och när $\mu = 0 \Rightarrow E(Y_t) = 0$
- c, FALSKT. h -statistiken i Durbins h -test är approximativt $N(0, 1)$ -fördelad under H_0 .
- d, FALSKT. I Koyck-modellen är $y_t = \alpha y_{t-1} + \beta x_t + u_t$, vilket gör att $\text{cov}(y_t, y_{t-1}) \neq 0$.
- e, FALSKT. H_0 i unit root tests är att $d = 0$, dvs $\rho = 1$, dvs att processen är icke-stationär.
- f, SANT. REM lämpar sig för paneldata och om vi kan motivera att interceptet i modellen ges av $\beta_1 \epsilon = \beta_1 + \epsilon$, dvs populationsmodellvärdet & en slumpmässig felterm, tex pga att datan är ett slumpmässigt urval från en större population, kan REM passa. Ett Hausman-test kan dock även användas.

g, ~~Räskt~~. En GARCH-modell antogs varit heteroskedastisk, dvs. icke-konstant varians hos feltermen. /2/

(3) a, För att testa autokorrelationen för flera laggar kan tex ett Q-test eller ett Ljung-Box-test användas. Jag väljer att göra ett Ljung-Box-test.

$$H_0: \rho_1 = \dots = \rho_5 = 0$$

$$H_A: \text{någon } \rho_k \neq 0$$

$$\alpha = 0,05 \quad k = 5 \quad n = 200 \quad \hat{\rho}_k^2 = \text{skattad korrelation i lag } k \text{ i konstant}$$

$$\text{Förkast } H_0 \text{ om } Q_{LB} > \chi_{\alpha}^2(k) = \chi_{0,05}^2(5) = 11,0705$$

$$Q_{LB} = n(n+2) \sum_{k=1}^K \left(\frac{\hat{\rho}_k^2}{n-k} \right) =$$

$$= 200 \cdot 202 \cdot \left(\frac{0,307^2}{200-1} + \frac{-0,013^2}{200-2} + \frac{0,086^2}{200-3} + \frac{0,03^2}{200-4} + \frac{-0,079^2}{200-5} \right)$$

$$= 40400 \cdot 0,0005489179 \approx 22,17628467$$

SVN:
 $= 22,1763 > 11,0705 \Rightarrow H_0$ förkastas och /6/ i det fall att minst en $\rho_k \neq 0$ (vilket verkar stämma med ACF)

b, Då det verkar som att vi har en spik och resten brus i vår skattade ACF skulle jag föreslå en MA(1)-modell //

③ c)
$$\rho_1 = \frac{\text{COV}(Y_t, Y_{t-1})}{V(Y_t)}$$

för en MA(1) gäller:
$$Y_t = \mu + \varepsilon_t - \theta \varepsilon_{t-1}$$

$$V(Y_t) = V(\mu + \varepsilon_t - \theta \varepsilon_{t-1}) = V(\mu) + V(\varepsilon_t) + \theta^2 V(\varepsilon_{t-1})$$

$$= 0 + \sigma^2 + \theta^2 \cdot \sigma^2 \Rightarrow V(Y_t) = \sigma^2 + \theta^2 \sigma^2 \Rightarrow$$

$$V(Y_t) = \sigma^2(1 + \theta^2)$$

och
$$\text{COV}(Y_t, Y_{t-1}) = \text{COV}(\mu + \varepsilon_t - \theta \varepsilon_{t-1}, \mu + \varepsilon_{t-1} - \theta \varepsilon_{t-2})$$

$$= -\theta \cdot \text{COV}(\varepsilon_t, \varepsilon_{t-1}) = -\theta \cdot V(\varepsilon_{t-1}) = -\theta \cdot V(\varepsilon_t) =$$

$$= -\theta \cdot \sigma^2$$

$$\Rightarrow \rho_1 = \frac{\text{COV}(Y_t, Y_{t-1})}{V(Y_t)} = \frac{\sigma^2 \cdot -\theta}{\sigma^2(1 + \theta^2)} = \frac{-\theta}{1 + \theta^2} = \frac{-1}{5}$$

④ a) Ja det går den då $Y_t = \text{ARIMA}(0, 1, 2)$ innebär att första-differensen av Y_t (Z_t) blir en stationär ARMA(0, 2), dvs en MA(2), vilket stämmer överens med en seasonal ACF med 2 spikar som sedan finns för $k > q = 2$

b)
$$\hat{Z}_t = 0,20457 - 0,3565 \varepsilon_{t-1} + 0,4202 \varepsilon_{t-2} + \varepsilon_t$$

$$E(\hat{Z}_t) = E[0,20457 - 0,3565 \varepsilon_{t-1} + 0,4202 \varepsilon_{t-2} + \varepsilon_t]$$

$$= 0,20457 - 0,3565 \cdot E(\varepsilon_{t-1}) + 0,4202 \cdot E(\varepsilon_{t-2}) + E(\varepsilon_t)$$

$$= 0,20457 \quad (\text{dvs } \mu)$$

$$V(\hat{\beta}_t) = V[0,20457 - 0,35657 \varepsilon_{t-1} + 0,44202 \varepsilon_{t-2} + \varepsilon_t]$$

$$= 0 + (-0,35657)^2 \cdot V(\varepsilon_{t-1}) + 0,44202^2 \cdot V(\varepsilon_{t-2}) + V(\varepsilon_t) =$$

$$= 0,12709 \cdot \sigma^2 + 0,17657 \cdot \sigma^2 + \sigma^2 \Rightarrow$$

$$V(\hat{\beta}_t) = \sigma^2 (1 + 0,12709 + 0,17657) \quad (\text{dus } \sigma^2 (1 + \theta_1^2 + \theta_2^2))$$

$$\text{dus } \hat{\sigma}^2 = \frac{ESS}{n-2} = \frac{276,173}{106} = MS = 1,409 \quad \Rightarrow$$

$$V(\hat{\beta}_t) = 1,409 \cdot 1,3037 \approx 1,8369$$

SNW: $E(\hat{\beta}_t) = \hat{\mu} \approx 0,20457$;

$$V(\hat{\beta}_t) = \hat{\sigma}^2 (1 + \hat{\theta}_1^2 + \hat{\theta}_2^2) \approx 1,8369$$

/10

/14

⑤ $Y_t = \alpha + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t$
 där Y_t är en stationär AR(2)

$\hat{\rho}_1 = 0,5$; $\hat{\rho}_2 = 0,17$; $\hat{\rho}_3 ?$

För att ta reda på $\hat{\phi}_1$ & $\hat{\phi}_2$; Yule-Walker:

$\hat{\rho}_1 = 0,5 = \hat{\phi}_1 \cdot \underbrace{\rho(0)}_{=1} + \hat{\phi}_2 \cdot \underbrace{\rho(-1)}_{=\hat{\rho}_1}$ \Rightarrow

$0,5 = \hat{\phi}_1 + \hat{\phi}_2 \cdot 0,5 \Rightarrow \hat{\phi}_1 = 0,5 - 0,5 \hat{\phi}_2$
 $\hat{\phi}_2 = \frac{0,5 - \hat{\phi}_1}{0,5}$

$\hat{\rho}_2 = 0,17 = \hat{\phi}_1 \cdot \underbrace{\rho(1)}_{=0,5} + \hat{\phi}_2 \cdot \underbrace{\rho(0)}_{=1}$ \Rightarrow

$0,17 = \hat{\phi}_1 \cdot 0,5 + \hat{\phi}_2 \Rightarrow 0,17 = \hat{\phi}_1 \cdot 0,5 + \left(\frac{0,5 - \hat{\phi}_1}{0,5} \right)$

$\Rightarrow 0,17 - \hat{\phi}_1 \cdot 0,5 = \frac{0,5 - \hat{\phi}_1}{0,5} \Rightarrow 0,5(0,17 - \hat{\phi}_1 \cdot 0,5) = 0,5 - \hat{\phi}_1$

$\Rightarrow 0,085 - 0,25 \hat{\phi}_1 = 0,5 - \hat{\phi}_1 \Rightarrow 0,75 \hat{\phi}_1 = 0,415 \Rightarrow \hat{\phi}_1 \approx 0,5533$

$\Rightarrow \hat{\phi}_2 = \frac{0,5 - 0,5533}{0,5} \approx -0,1066$

$\hat{\phi}_1 \approx 0,5533$ & $\hat{\phi}_2 \approx -0,1066$ ger:

$\hat{\rho}_3 = \hat{\phi}_1 \cdot \hat{\rho}_2 + \hat{\phi}_2 \cdot \hat{\rho}_1 = 0,5533 \cdot 0,17 - 0,1066 \cdot 0,5 \approx 0,040761$

Svar: en skattning av ρ_k ($\hat{\rho}_3$) är ca 0,040761