Stockholm University
Department of Statistics
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Econometrics II

WRITTEN EXAMINATION

Friday June 1, 2018

Tools allowed: Pocket calculator

Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description.

For the maximum number of points on each problem detailed and clear solutions are required.

Observe:: If not indicated otherwise, the error terms ϵ_t in the models are assumed independent and $N(0, \sigma^2)$.

You may answer in Swedish.

1. (20p) Consider the following data consisting of annual lumber production in USA from 1947 through 1976 (the years being coded $1, \ldots, 30$) (unit: millions of board feet):

1–6	7–12	13–18	19–24	25–30
35 404	36 762	32 901	38 902	37 515
37 462	36 742	36 356	37 858	38 629
32 901	33 285	37 166	32 926	32 019
33 178	34 171	35 733	35 697	35 710
24 449	36 124	35 791	34 548	36 693
38 044	38 658	34 592	32 087	37 153

(a) We are first worried about possible autocorrelation, so we compute estimated autocorrelations for lag 1 up to lag 6:

Lag k	1	2	3	4	5	6
$\hat{\rho}_k$	0.20	-0.05	0.13	0.14	0.04	-0.17

Test if we should reject the hypothesis that $\rho_1 = \cdots = \rho_6 = 0$ at approximate significance level 5%.

(b) We now want to use smoothing methods in order to extract the "signal" from this data set. A simple smoother to start with is

$$\hat{\mu}_T = \frac{1}{T}\sum_{t=1}^{T} y_t$$

Compute $\hat{\mu}_T$ for the first five time points and the last five time points. ($\sum_{i=1}^{30} y_t = 1\ 076\ 556$)

Another possibility is the simple moving average

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} y_t$$

Compute M_T for the last five time points using the span $N = 5$.

(c) The next step is to use exponential smoothing. Use with motivation expential smoothing of appropriate order.

Compute $\tilde{y}_T$ for the first five time points and the last five time points. ($\lambda = 0.5$) Use $\tilde{y}_0 = \bar{y}$ and $\tilde{y}_{25} = 35\ 000$.

Actually, $\hat{\mu}_T$ can be seen as a special case of $\tilde{y}_T$. In what way?

(d) Finally we want to use our "smoothers" to make forecasts. Assuming (perhaps not very realistic) that the pattern of observations are similar up to the year 2016, what are the predicted values of y_{70} using the three different "smoothers "$\hat{\mu}_{30}$, M_{30} and $\tilde{y}_{30}$?

2. (20p) Below we have a process, which is essentially is a regression model, but here we will look at it from a times series model perspective.

$$y_t = 3 - 2t + \epsilon_t,$$

where $t = 0, 1, 2\ldots$, $\epsilon_t \sim N(0, 1)$ and $Cov(\epsilon_t, \epsilon_{t-k}) = \tau(k)$.

(Observe that the error terms are here not assumed to be independent.)

(a) In the model, what is the part $2t$ called?

(b) Compute for the process y_t (derive expressions of) $E(y_t)$, $V(y_t)$ and $Cov(y_t, y_{t-k})$

(c) Is y_t stationary? Why/why not? Is ϵ_t stationary? Why/why not?

3. (20p) Consider the following situation: We have five similar companies and for each company we have observed values of four variables Y, X_1, X_2 and X_3 and each variable is observed during the time points $t = 1, \ldots 10$. We want to formulate a model with the X-variables as regressors and Y as the dependent variable.

(a) Even if we suspect that the companies may show different patterns, we first try the pooled OLS regression model. Formulate this model using appropriate notation.

(b) As an alternative we would also like to try a fixed effects regression model using dummy variables. Formulate this model using appropriate notation.

(c) As it turned out, the R^2-values for the models in (a) and (b), were 0.946 and 0.971, respectively.

Compute the value of a suitable test statistic for deciding if we should prefer using the fixed effects model in this case. (You do not have to perform the test since corresponding tables are not provided here.)

(d) In general, besides the risk of lower R^2-values using the pooled model instead of the fixed effects model, what other negative consequence can we get if we use the pooled model in cases where the fixed effects model is (theoretically) more appropriate?

4. (20p) Consider the following time series process:

$$w_t = 4 + \epsilon_t - 0.65\epsilon_{t-1} - 0.24\epsilon_{t-2},$$

where $w_t = (1 - B)y_t = y_t - y_{t-1}$.

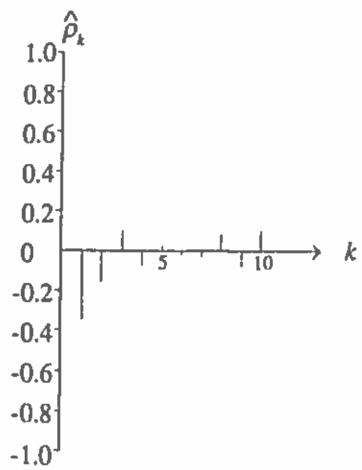
(a) What would you call the process w_t?

(b) Is this model invertible? Why/why not?

(c) Is the process y_t stationary? Why/why not? What would you call the process y_t?

(d) Compute $E(w_t)$, $V(w_t)$ and the ACF:s for lag $k = 1, 2, \ldots$.

(e) Compare your results for the ACF:s with the results of the sample ACF:s from a simulation based on the model above for w_t in the figure below. Do the simulation results results agree with your theoretical results?



5. (20p) Some random walk problems:

(a) Let us first consider the most simple random walk model:

$$y_t = y_{t-1} + \epsilon_t \tag{1}$$

This means that, for example, $y_1 = y_0 + \epsilon_1$. Rewrite (1) so that it is a sum of the starting value y_0 and a stochastic trend. Use this to show that y_t is not stationary.

If we realize values of this process over time, what is the effect of the violation against stationarity? (You can illustrate it graphically if you prefer that.)

(b) Now we add a constant δ to our model:

$$y_t = \delta + y_{t-1} + \epsilon_t \tag{2}$$

What is δ ususally called? Rewrite (2) in the same way as you did for (1), so that the result now is that y_t is the sum of the starting value y_0, some function of δ and a stochastic trend.

Show that y_t is nonstationary.

What is δ usually called? What effect has this function on the process? Also here you can illustrate it graphically.)

Formula sheet, Econometrics II, Spring 2018

Under the simple linear model $y_t = \beta_1 + \beta_2 x_t + u_t$, where $u_t \sim N(0, \sigma^2)$ and given independent pairs of observations $(y_1, x_1), \ldots, (y_n, x_n)$, the OLS (and ML) estimators are:

$$\begin{aligned}
\widehat{\beta}_1 &= \overline{y} - \widehat{\beta}_2 \overline{x} \\
\widehat{\beta}_2 &= \frac{\sum (x_t - \overline{x})(y_t - \overline{y})}{\sum (x_t - \overline{x})^2} \\
\widehat{\sigma}^2 &= \frac{RSS}{n-2} = \frac{\sum (y_t - \widehat{y}_t)^2}{n-2}
\end{aligned}$$

where $\widehat{y}_t = \widehat{\beta}_1 + \widehat{\beta}_2 x_t$ and where $E(\widehat{\beta_1}) = \beta_1$, $E(\widehat{\beta_2}) = \beta_2$ and $E(\widehat{\sigma}^2) = \sigma^2$

Comparing an "old" model with a "new" (larger):

$$\begin{aligned}
F &= \frac{(ESS_{new} - ESS_{old})/\text{number of new regressors}}{RSS_{new}/(n - \text{number of parameters in the new model})} \\
&= \frac{(R^2_{new} - R^2_{old})/\text{number of new regressors}}{(1 - R^2_{new})/(n - \text{number of parameters in the new model})}
\end{aligned}$$

Comparing an "unrestricted" model with a "restricted":

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} = \frac{(R^2_{UR} - R^2_R)/m}{(1 - R^2_{UR})/(n-k)}$$

where m is the number of linear constraints and k is the number of parameters in the unrestricted model.

Dynamic models: $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_{t-1} + v_t$

Koyck: $y_t = \alpha(1-\lambda) + \beta_0 x_t + \lambda y_{t-1} + v_t$
Adaptive expectations: $y_t = \gamma\beta_0 + \gamma\beta_1 x_t + (1-\gamma)y_{t-1} + (u_t - (1-\gamma)u_{t-1})$
Partial adjustment: $y_t = \delta\beta_0 + \delta\beta_1 x_t + (1-\delta)y_{t-1} + \delta u_t$

The Durbin Watson d statistic:

$$d = \frac{\sum_{t=2}^{n} (\widehat{u}_t - \widehat{u}_{t-1})^2}{\sum_{t=1}^{n} \widehat{u}_t^2}$$

The Durbin h statistic:

$$h = \widehat{\rho}\sqrt{\frac{n}{1 - n\left[\hat{V}(\widehat{\alpha}_2)\right]}} \approx N(0,1), \text{ if } \rho = 0$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}[e_t(1)]^2 = \frac{1}{n}\sum_{t=1}^{n}[y_t - \widehat{y}_t(t-1)]^2$$

Autocorrelation function:

$$\rho_k = \frac{Cov(y_t, y_{t+k})}{V(y_t)}, \quad k = 0, 1, 2, \ldots$$

Sample correlation function:

$$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{n-k}(y_t - \bar{y})^2}, \quad k = 0, 1, 2, \ldots$$

Simple moving average:

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} y_t$$

First-order exponential smoothing:

$$\tilde{y}_T = \lambda y_T + (1-\lambda)\,\tilde{y}_{T-1}$$

Second-order exponential smoothing:

$$\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)},$$

where $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$

Holt's method:

$$\begin{aligned} L_t &= \alpha y_t + (1-\alpha)(L_{t-1} + T_{t-1}) \\ T_t &= \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1} \end{aligned}$$

$$\widehat{y}_{T+\tau}(T) = L_T + \tau T_T, \quad \tau = 1, 2, \ldots$$

Forecast under a constant process:

$$\hat{y}_{T+\tau}(T) = \tilde{y}_T \quad \tau = 1, 2, ...$$

Forecast under a linear trend:

$$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T}\tau.$$

where $\hat{y}_T = \hat{\beta}_{0,T} + \hat{\beta}_{1,T}T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$

For white noise:

$$\hat{\rho}_k \approx N(0, 1/n)\ , k = 1, 2, \ldots$$

The Q statistic:

$$Q = n \sum_{k=1}^{K} \hat{\rho}_k^2 \approx \chi^2(K)$$

The Ljung-Box statistic:

$$Q_{LB} = n\,(n+2) \sum_{k=1}^{K} \left(\frac{\hat{\rho}_k^2}{n-k} \right) \approx \chi^2(K)$$

ARMA(p,q):

$$y_t = \delta + \sum_{i=1}^{p} \phi_i y_{t-i} + \epsilon_t - \sum_{i=1}^{q} \theta_i \epsilon_{t-i}$$

Stationarity and invertibility conditions for some time series models:

Model	Stationarity conditions	Invertibility conditions
AR(1)	$\|\phi_1\| < 1$	None
AR(2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $\|\phi_2\| < 1$	None
MA(1)	None	$\|\theta_1\| < 1$
MA(2)	None	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\|\theta_2\| < 1$
ARMA(1,1)	$\|\phi_1\| < 1$	$\|\theta_1\| < 1$
ARMA(2,2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $\|\phi_2\| < 1$	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\|\theta_2\| < 1$

The Yule-Walker equations for AR(p):

$$\rho_k = \sum_{i=1}^{p} \phi_i \rho_{k-i}, \quad k = 1, 2, \ldots$$

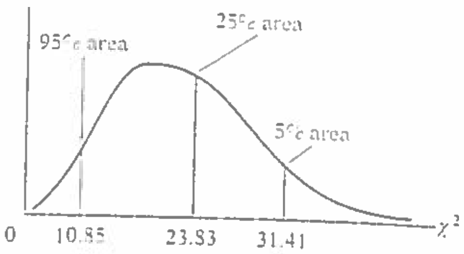
TABLE D.4
Upper Percentage Points of the χ^2 Distribution

Example

$Pr(\chi^2 > 10.85) = 0.95$

$Pr(\chi^2 > 23.83) = 0.25$ for df = 20

$Pr(\chi^2 > 31.41) = 0.05$



Degrees of freedom \ Pr	.995	.990	.975	.950	.900
1	392704 × 10^{-10}	157088 × 10^{-9}	982069 × 10^{-9}	393214 × 10^{-8}	.0157908
2	.0100251	.0201007	.0506356	.102587	.210720
3	.0717212	.114832	.215795	.351846	.584375
4	.206990	.297110	.484419	.710721	1.063623
5	.411740	.554300	.831211	1.145476	1.61031
6	.675727	.872085	1.237347	1.63539	2.20413
7	.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100*	67.3276	70.0648	74.2219	77.9295	82.3581

*For df greater than 100 the expression $\sqrt{2\chi^2} - \sqrt{(2k-1)} = Z$ follows the standardized normal distribution, where k represents the degrees of freedom.

χ^2-table continued

.750	.500	.250	.100	.050	.025	.010	.005
.1015308	.454937	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
.575364	1.38629	2.77259	4.60517	5.99147	7.37776	9.21034	10.5966
1.212534	2.36597	4.10835	6.25139	7.81473	9.34840	11.3449	12.8381
1.92255	3.35670	5.38527	7.77944	9.48773	11.1433	13.2767	14.8602
[illegible].67460	4.35146	6.62568	9.23635	11.0705	12.8325	15.0863	16.7496
3.45460	5.34812	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476
4.25485	6.34581	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777
5.07064	7.34412	10.2188	13.3616	15.5073	17.5346	20.0902	21.9550
5.89883	8.34283	11.3887	14.6837	16.9190	19.0228	21.6660	23.5893
[illegible].73720	9.34182	12.5489	15.9871	18.3070	20.4831	23.2093	25.1882
7.58412	10.3410	13.7007	17.2750	19.6751	21.9200	24.7250	26.7569
8.43842	11.3403	14.8454	18.5494	21.0261	23.3367	26.2170	28.2995
9.29906	12.3398	15.9839	19.8119	22.3621	24.7356	27.6883	29.8194
10.1653	13.3393	17.1170	21.0642	23.6848	26.1190	29.1413	31.3193
11.0365	14.3389	18.2451	22.3072	24.9958	27.4884	30.5779	32.8013
11.9122	15.3385	19.3688	23.5418	26.2962	28.8454	31.9999	34.2672
12.7919	16.3381	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185
13.6753	17.3379	21.6049	25.9894	28.8693	31.5264	34.8053	37.1564
14.5620	18.3376	22.7178	27.2036	30.1435	32.8523	36.1908	38.5822
15.4518	19.3374	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968
16.3444	20.3372	24.9348	29.6151	32.6705	35.4789	38.9321	41.4010
17.2396	21.3370	26.0393	30.8133	33.9244	36.7807	40.2894	42.7956
18.1373	22.3369	27.1413	32.0069	35.1725	38.0757	41.6384	44.1813
19.0372	23.3367	28.2412	33.1963	36.4151	39.3641	42.9798	45.5585
19.9393	24.3366	29.3389	34.3816	37.6525	40.6465	44.3141	46.9278
20.8434	25.3364	30.4345	35.5631	38.8852	41.9232	45.6417	48.2899
21.7494	26.3363	31.5284	36.7412	40.1133	43.1944	46.9630	49.6449
22.6572	27.3363	32.6205	37.9159	41.3372	44.4607	48.2782	50.9933
[illegible]3.5666	28.3362	33.7109	39.0875	42.5569	45.7222	49.5879	52.3356
24.4776	29.3360	34.7998	40.2560	43.7729	46.9792	50.8922	53.6720
33.6603	39.3354	45.6160	51.8050	55.7585	59.3417	63.6907	66.7659
42.9421	49.3349	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900
52.2938	59.3347	66.9814	74.3970	79.0819	83.2976	88.3794	91.9517
[illegible]1.6983	69.3344	77.5766	85.5271	90.5312	95.0231	100.425	104.215
[illegible]1.1445	79.3343	88.1303	96.5782	101.879	106.629	112.329	116.321
80.6247	89.3342	98.6499	107.565	113.145	118.136	124.116	128.299
90.1332	99.3341	109.141	118.498	124.342	129.561	135.807	140.169

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3rd ed., table 8, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

Department of Statistics

Stockholms universitet

Correction sheet

Date: 1/06/2018

Room: Brunnsvikssalen

Course: Econometrics (eng)

Exam: Econometrics II (eng)

Anonymous code: 0030-WHK

☑ I authorise the anonymous posting of my exam, in whole or in part, on the department homepage as a sample student answer.

NOTE! ALSO WRITE ON THE BACK OF THE ANSWER SHEET

Mark answered questions

1	2	3	4	5	6	7	8	9	Total number of pages
✓	✓	✓	✓	✓					6
Teacher's notes 17	15	19	20	20					

IK

Points	Grade	Teacher's sign.
91	A	PGA

SU, DEPARTMENT OF STATISTICS

Room: BR Anonymous code: 0030-WHK Sheet number: 1

1

(a) $H_0: \rho_1 = \cdots = \rho_6 = 0$

H_a: At least on of ρ_k $(k=1,2,\cdots,6)$ is not equal to 0.

Test statistic: we use the Ljung-Box statistic to determine if we should reject H_0.

OBS: $Q_{LB} = n(n+2)\sum_{k=1}^{K}\left(\frac{\rho_k^2}{n-k}\right) \approx \chi^2(k)$

$\Rightarrow Q_{LB} = 30\cdot(30+2)\sum_{k=1}^{6}\left(\frac{\rho_k^2}{30-k}\right)$

$= 30\cdot 32\left(\frac{(0.20)^2}{29} + \frac{(-0.05)^2}{28} + \frac{(0.13)^2}{27} + \frac{(0.14)^2}{26} + \frac{(0.04)^2}{25} + \frac{(-0.17)^2}{24}\right)$

$\doteq 3.9518$

Decision rule: we reject H_0 if $\chi^2_{obs} > \chi^2_{.05}(6)$

$\chi^2_{obs} = 3.9518 < \cancel{14.4494}\ 12.59 = \chi^2_{.05}(6)$

Since $\chi^2_{obs} < \chi^2_{.05}(6)$, we fail to reject H_0

and conclude that we don't have enough evidence to assume that there is a severe autocorrelation.

/4

1

(b)

$\hat{\mu}_T = \frac{1}{T} \sum_{t=1}^{T} y_t$.

$\hat{\mu}_{30} = \frac{1}{30} \sum_{t=1}^{T} y_t$

$= \frac{1076556}{30}$

$= \boxed{35885.2}$

$M_T = \frac{1}{N} \sum_{t=T-N+1}^{T} y_t$.

$T=26$. $M_{26} = \frac{1}{5} \sum_{t=22}^{26} y_t$

$= \boxed{35695.2}$

$T=27$. $M_{27} = \frac{1}{5} \sum_{t=23}^{27} y_t$

$= \boxed{34959.6}$

$T=28$ $M_{28} = \frac{1}{5} \sum_{t=24}^{28} y_t$

$= \boxed{35192}$

$T=29$. $M_{29} = \frac{1}{5} \sum_{t=25}^{29} y_t$

$= \boxed{36113.2}$

$T=30$. $M_{30} = \frac{1}{5} \sum_{t=26}^{30} y_t$

$= \boxed{36040.8}$

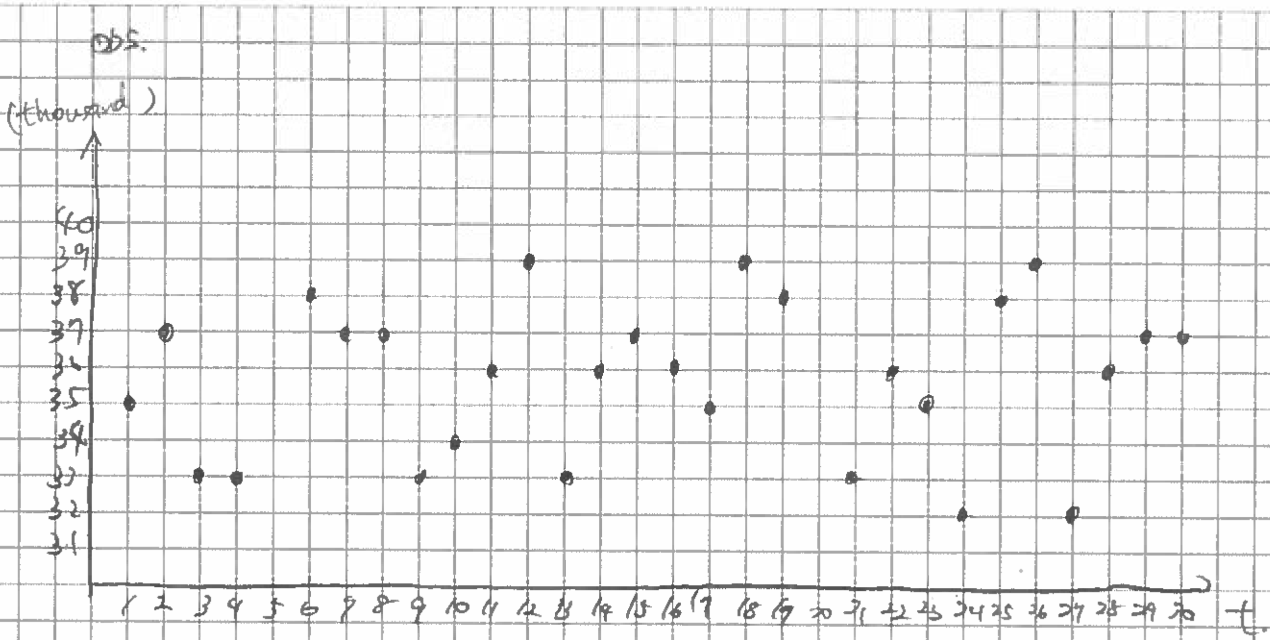
/3

SU, DEPARTMENT OF STATISTICS

Room: BR Anonymous code: 0030-WHK Sheet number: 2

4

(C)



The scatter plot of the observation would be like above.
Since I cannot detect any trend, change in variance and also we can assume that $cov(Y_t, Y_{t-k})$ is independent of t, I would like to use the simple exponential smoothing.

$$\hat{Y}_t = \lambda Y_t + (1-\lambda)\hat{Y}_{t-1}$$

$$\hat{y}_0 = \bar{y} = \frac{1}{30}\sum_{t=1}^{T} y_t = 35885.2$$

$$\hat{y}_1 = 0.5 \cdot y_1 + 0.5\,\hat{y}_0 = \boxed{35644.6}$$

$$\hat{y}_2 = 0.5 \cdot y_2 + 0.5\,\hat{y}_1 = \boxed{36553.3}$$

$$\hat{y}_3 = 0.5\, y_3 + 0.5\,\hat{y}_2 = \boxed{34727.15}$$

$$\hat{y}_4 = 0.5\, y_4 + 0.5\,\hat{y}_3 = \boxed{33952.575}$$

$$\hat{y}_5 = 0.5\, y_5 + 0.5\,\hat{y}_4 = \boxed{29200.7875}$$

continue →

1

(c) cont.

$$\tilde{y}_{25} = 35000$$

$$\tilde{y}_{26} = 0.5\, y_{26} + 0.5\, \tilde{y}_{25} = \boxed{36814.5}$$

$$\tilde{y}_{27} = 0.5\, y_{27} + 0.5\, \tilde{y}_{26} = \boxed{34916.75}$$

$$\tilde{y}_{28} = 0.5\, y_{28} + 0.5\, \tilde{y}_{27} = \boxed{35063.375}$$

$$\tilde{y}_{29} = 0.5\, y_{29} + 0.5\, \tilde{y}_{28} = \boxed{35878.1875}$$

$$\tilde{y}_{30} = 0.5\, y_{30} + 0.5\, \tilde{y}_{29} = \boxed{36515.59375}$$

when $\lambda = 1$, $\tilde{y}_T = \tilde{y}_{T-1} = \cdots = \tilde{y}_0 = \bar{y} = \hat{\mu}_T$ OK

So, $\hat{\mu}_T$ can be seen as a special case of $\tilde{y}_T$, if $\lambda = 1$.

/5

(d). (1) $\hat{\mu}_{30}$, Since we assume stationarity

$$\hat{y}_{70}(30) = \hat{\mu}_{30} = 35885.2$$

(2). M_{30}, Also, since we assume stationarity

$$\hat{y}_{70}(30) = M_{30} = 36049.8$$

(3) $\tilde{y}_{30}$, Again, since we assume stationarity

$$\hat{y}_{70}(30) = \tilde{y}_{30} = 36515.59375$$

OK /5

/17

SU, DEPARTMENT OF STATISTICS

Room: BR **Anonymous code:** 0030-WHK **Sheet number:** 3

(2)

$$y_t = 3 - 2t + \varepsilon_t$$

where $t = 0, 1, 2, \cdots$, $\varepsilon_t \sim N(0,1)$

(a) The part $2t$ is called deterministic trend.

/2

(b) $E(y_t) = E(3 - 2t + \varepsilon_t)$

$= E(3-2t) + E(\varepsilon_t)$

$= \underline{3 - 2t}$ //

$V(y_t) = V(3 - 2t + \varepsilon_t)$

$= V(3-2t) + V(\varepsilon_t)$

$= V(\varepsilon_t) = \underline{1}$ //

$Cov(y_t, y_{t-k}) = cov(3-2t+\varepsilon_t, 3-2(t-k) + \varepsilon_{t-k})$

$= cov(3-2t, 3-2(t-k)) + cov(3-2t, \varepsilon_{t-k}) + cov(\varepsilon_t, 3-2(t-k))$

$+ cov(\varepsilon_t, \varepsilon_{t-k})$

$= cov(\varepsilon_t, \varepsilon_{t-k}) = \underline{\gamma(k)}$ //

OK /10

四

(c)

$$y_t = 3 - 2t + \varepsilon_t$$

$$y_{t-1} = 3 - 2(t-1) + \varepsilon_{t-1}$$

$$= 5 - 2t + \varepsilon_{t-1}$$

$$\Rightarrow y_t - y_{t-1} = -2 + \varepsilon_t - \varepsilon_{t-1}$$

$$\Rightarrow y_t = -2 + y_{t-1} + \varepsilon_t - \varepsilon_{t-1}$$

y_t can be transformed in this way. This can be seen as ARMA(1,1). Since $\phi = 1$, y_t is not stationary.

ε_t is stationary because.

$E(\varepsilon_t) = 0$, $\Rightarrow$ constant

$V(\varepsilon_t) = 1$ $\Rightarrow$ constant

$Cov(\varepsilon_t, \varepsilon_{t-k}) = \gamma(k) \Rightarrow$ independent of t.

/3

/15

SU, DEPARTMENT OF STATISTICS

Room: BR Anonymous code: 0030-WHK Sheet number: 4

$\boxed{3}$

(a) $Y_{it} = \beta_0 + \beta_1 \cdot X_{1it} + \beta_2 \cdot X_{2it} + \beta_3 \cdot X_{3it} + \varepsilon_{it}$ /4

(b) $Y_{it} = \alpha_1 + \alpha_2 \cdot D_{2i} + \alpha_3 \cdot D_{3i} + \alpha_4 \cdot D_{4i} + \alpha_5 \cdot D_{5i}$

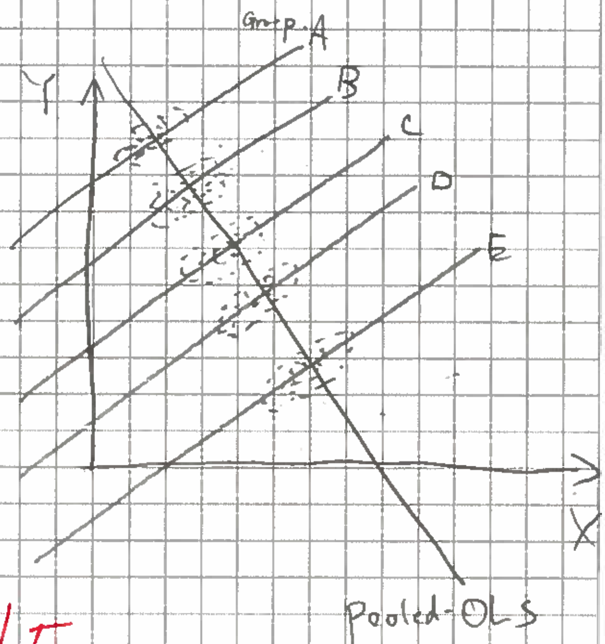
$+ \beta_1 \cdot X_{1it} + \beta_2 \cdot X_{2it} + \beta_3 X_{3it} + \varepsilon_{it}$ /4

(c) $$F = \frac{(R^2_{new} - R^2_{old}) / \text{number of new regressors}}{(1 - R^2_{new}) / (n - \text{number of parameters in the new model})}$$

$$= \frac{(0.971 - 0.946) / 4}{(1 - 0.971) / (50 - 8)}$$

$$= 9.0517$$ /5

(d) As can be seen from the graph on the right-hand side, the pooled OLS model may suffer from biased estimation.



/5

/19

$\boxed{4}$ $W_t = 4 + \varepsilon_t - 0.65\,\varepsilon_{t-1} - 0.24\,\varepsilon_{t-2}$.

where $W_t = (1-B)\,y_t$.

(a). MA(2) OK /3

(b).

$$\theta_1 + \theta_2 = 0.65 + 0.24 = 0.89 < 1$$

$$\theta_2 - \theta_1 = 0.24 - 0.65 = -0.41 < 1$$

$$|\theta_2| = 0.24 < 1.$$

OK /3

Thus, this model is invertible.

(c)

$$(1-B)\,y_t = y_t - y_{t-1} = W_t.$$

$$\Rightarrow y_t - y_{t-1} = 4 + \varepsilon_t - 0.65\,\varepsilon_{t-1} - 0.24\,\varepsilon_{t-2}$$

$$\Rightarrow y_t = 4 + y_{t-1} + \varepsilon_t - 0.65\,\varepsilon_{t-1} - 0.24\,\varepsilon_{t-2}.$$

This can be called ARMA(1,2), However, since $|\phi| \not< 1$.
y_t is not stationary.
I would call y_t ARIMA(0,1,2) OK /4

(d).

$$E(W_t) = E(4 + \varepsilon_t - 0.65\,\varepsilon_{t-1} - 0.24\,\varepsilon_{t-2})$$

$$= E(4) + E(\varepsilon_t) - 0.65\,E(\varepsilon_{t-1}) - 0.24\,E(\varepsilon_{t-2})$$

$$= \underline{4}\,//$$

$$V(W_t) = V(4 + \varepsilon_t - 0.65\,\varepsilon_{t-1} - 0.24\,\varepsilon_{t-2})$$

$$= V(\varepsilon_t) + 0.65^2\,V(\varepsilon_{t-1}) + 0.24^2\,V(\varepsilon_{t-2})$$

$$= \sigma^2(1 + 0.65^2 + 0.24^2)$$

$$= \underline{1.4801\,\sigma^2}\,//$$

OK

continue →

SU, DEPARTMENT OF STATISTICS

Room: BR Anonymous code: 0030-WHK Sheet number: 5

4.

(d). cont.

$\gamma(0) = V(W_t) = 1.4801\sigma^2.$

$\gamma(1) = cov(W_t, W_{t-1})$

$= Cov(4+\varepsilon_t - 0.65\varepsilon_{t-1} - 0.24\varepsilon_{t-2}, 4+\varepsilon_{t-1} - 0.65\varepsilon_{t-2} - 0.24\varepsilon_{t-3})$

$= -0.65 V(\varepsilon_{t-1}) + (0.24)(0.65) V(\varepsilon_{t-2})$

$= -0.494\sigma^2.$

$\gamma(2) = cov(W_t, W_{t-2})$

$= Cov(4+\varepsilon_t - 0.65\varepsilon_{t-1} - 0.24\varepsilon_{t-2}, 4+\varepsilon_{t-2} - 0.65\varepsilon_{t-3} - 0.24\varepsilon_{t-4})$

$= -0.24 V(\varepsilon_{t-2})$

$= -0.24\sigma^2.$

$\gamma(k) = 0 \quad k > 2.$

$\rho(0) = \frac{\gamma(0)}{\gamma(0)} = 1$

$\rho(1) = \frac{\gamma(1)}{\gamma(0)} = \frac{-0.494\sigma^2}{1.4801\sigma^2} \approx \underline{-0.33376}.$

$\rho(2) = \frac{\gamma(2)}{\gamma(0)} = \frac{-0.24\sigma^2}{1.4801\sigma^2} \approx \underline{-0.16215}.$

$\rho(k) = \underline{0} \quad k > 2.$

OK

/7

4.

(e)

$\hat{\rho}(1) \approx -0.38$

$\hat{\rho}(2) \approx -0.17$

$|\hat{\rho}(k)| < 0.2$

I would say the simulation results agree with the theoretical results.

OK /3

/20

SU, DEPARTMENT OF STATISTICS

Room: BR Anonymous code: 0030-WHK Sheet number: 6

5

(a).

$$y_t = y_{t-1} + \varepsilon_t \qquad \cdots (1)$$

From (1).

$$y_t = y_{t-1} + \varepsilon_t$$

$$= y_{t-2} + \varepsilon_t + \varepsilon_{t-1}$$

$$= \cdots\cdots$$

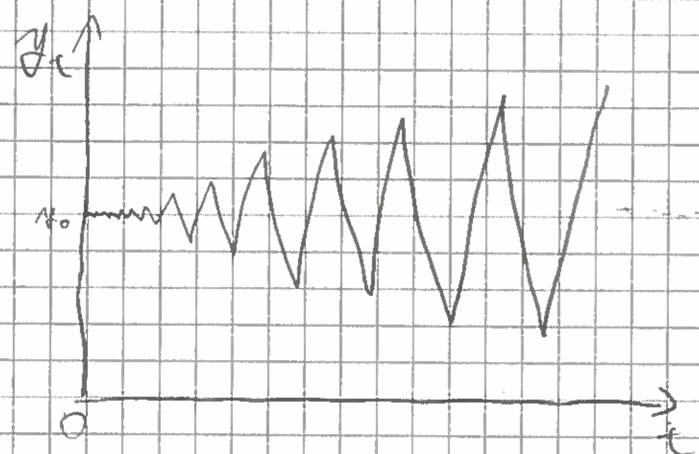
$$\boxed{y_t = y_0 + \sum_{i=0}^{t-1} \varepsilon_{t-i}}$$

$$V(y_t) = V\left(y_0 + \sum_{i=0}^{t-1} \varepsilon_{t-i}\right)$$

$$= \sum_{i=0}^{t-1} V(\varepsilon_{t-i})$$

$$= t \cdot \sigma^2.$$

Since the variance of y_t is proportional to the value of t. y_t is not stationary.



OK

1/10

5

(b) $$y_t = \delta + y_{t-1} + \varepsilon_t \quad \cdots (2)$$

From (2) $$y_t = \delta + y_{t-1} + \varepsilon_t$$

$$= \delta + (\delta + y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t$$

$$= \cdots$$

$$= y_0 + \delta t + \sum_{i=0}^{t-1} \varepsilon_{t-i}$$

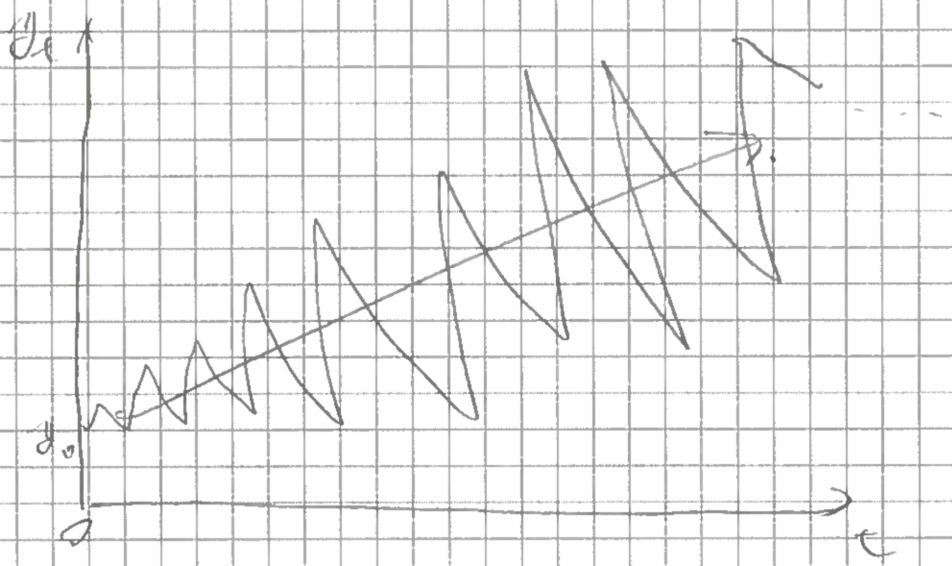
$$E(y_t) = E\left(y_0 + \delta t + \sum_{i=0}^{t-1} \varepsilon_{t-i}\right)$$

$$= y_0 + \delta t$$

Since the expected value of y_t is no longer constant, y_t is not stationary.

δ is called drift.

With drift, the random walk process has a trend.



OK /18

/20