

Econometrics I

WRITTEN EXAMINATION

Wednesday June 5 2018, 10 am – 3 pm

Tools allowed: Pocket calculator

Passing rate. 50% of overall total, which is 100 points. For detailed grading

Criteria, see the course description.

The exam will be handed back: not decided

For the maximum number of points on each problem detailed and clear solutions are required.

If not indicated otherwise, the disturbance term u_i in the models are assumed to fulfill the usual requirements of normality, homoscedasticity and independence.

Task 1 (20 points)

The following estimation results are given from performing an experiment twice. The X-values are the same in both experiments and the two experiments are independent.

Model: $Y_i = \beta_1 + \beta_2 X_i + u_i$. The error terms u_i is $\text{IN}(0, \sigma^2)$.

	Experiment 1	Experiment 2
Number of observations	$n_1 = 20$	$n_2 = 20$
Slope estimate	$b_1 = 1,2$	$b_2 = 1,5$
Estimated variance of the slope estimator	0,04	0,03

- A) Show that the slope estimate when using all 40 observations is equal to the mean of b_1 and b_2 . Hint: Use the OLS-formula for the slope in a simple linear regression model. (10)
- B) Use the results from both experiments to calculate a 95% confidence interval for β_2 . (10)

Task 2 (15 points)

A regression equation is estimated with the following results

$$\hat{Y} = 5 + 2.0X_2 + 5X_3 \quad R^2=0.69$$

(0.8) (3.0)

- A) If X_2 is multiplied by 4 how will the estimation results change if we re-estimate the model? Present a new version of the estimated equation including standard errors and R^2 . You don't need to justify your answer, just present the results. (7)
- B) If Y is divided by ten how will the estimation results change? Present a new version of the estimated equation including standard errors and R^2 . You don't need to justify your answer just present the results. (8)

Task 3 (15 points)

The table below shows the number of death penalty verdicts (DPV) for cases involving multiple murders (TOTALS) in Florida between 1976 and 1987, classified by the race of the defendant and the race of the victim.

The following model will be studied: $\ln \frac{P}{1-p} = \beta_1 + \beta_2 VR + \beta_3 DR$,

where P = probability that a multiple murder will lead to a death penalty, VR=1 if the victim is black, 0 if the victim is white and DR=1 if the defendant is black, DR = 0 if the defendant is white.

(See Jöreskog et al, Multivariate Analysis with LISREL, Springer Verlag, and references therein.)

- A) Present the model used for $P = P(Y=1)$. (3)
- B) Present the estimated model $\ln \left(\frac{\hat{P}}{1-\hat{P}} \right) = \dots$. (3)
- C) Present the estimated model $\frac{\hat{P}}{1-\hat{P}} = \dots$. (3)
- D) Use the estimated model to calculate an estimate of the relative risk of a death penalty between murders where the victim is black and defendant is white and murders where the victim is white and the defendant is white. Present the ODDS-ratio between the corresponding groups. As you notice the relative risk is not the same as the ODDS-ratio but is not so far from it or very approximately close to it. Give a short explanation why it is so in this case. (6)

Estimation results

Response Information

Variable	Value	Count
DPV	Event	68
	Non-event	606
TOTALS	Total	674

Logistic Regression Table

Predictor	Coef	SE Coef.	Z	P	Odds Ratio
Constant	-2,05946	0,145846	-14,12	0,000	
VR	-2,40444	0,600600	-4,00	0,000	0,09
DR	0,867797	0,367074	2,36	0,018	2,38

Log-Likelihood = -209,478

Test that all slopes are zero: G = 21,886, DF = 2, P-Value = 0,000

Task 4 (30 points)

Below you find twelve quarterly observations on Y_t = Total value of sales (millions of SEK) at time point t and X_t = Total value of orders (millions of SEK) received at time point t.

Data Display

Row	Quarter	t	Yt	Xt	Resids
1	1	1	125	102	*
2	2	2	120	110	*
3	3	3	122	105	4,2446
4	4	4	131	92	6,1635
5	1	5	110	104	-10,4108
6	2	6	124	102	15,0959
7	3	7	118	100	-1,5257
8	4	8	118	96	0,2446
9	1	9	118	88	2,0149
10	2	10	109	91	-3,4446
11	3	11	111	72	5,6365
12	4	12	90	93	-18,0189

- A) Present a linear model for the Sales at time point t as a function of orders received two quarters earlier. Estimate with least-squares the parameters of the model and interpret the estimate of the slope-parameter. (10)
- B) Use results for the estimated model to calculate a 90% prediction interval for the sales at time point 13. Residuals from the estimation in Task A) are presented in the table above. (10)
- C) Use the Durbin-Watson test-statistic to test for potential first order positive autocorrelation. (10)

Task 5 (20 points)

- A) Below you find a regression output from estimation of the model:

$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + u_t$ where the data in Task 4 has been used. Is this model to be preferred to the model in Task 4A? Perform a formal test on the 5% significance level. (10)

Regression Analysis: Yt versus Xt; Xt-1; Xt-2

The regression equation is
 $Y_t = 22,2 - 0,257 Xt + 0,959 Xt-1 + 0,253 Xt-2$

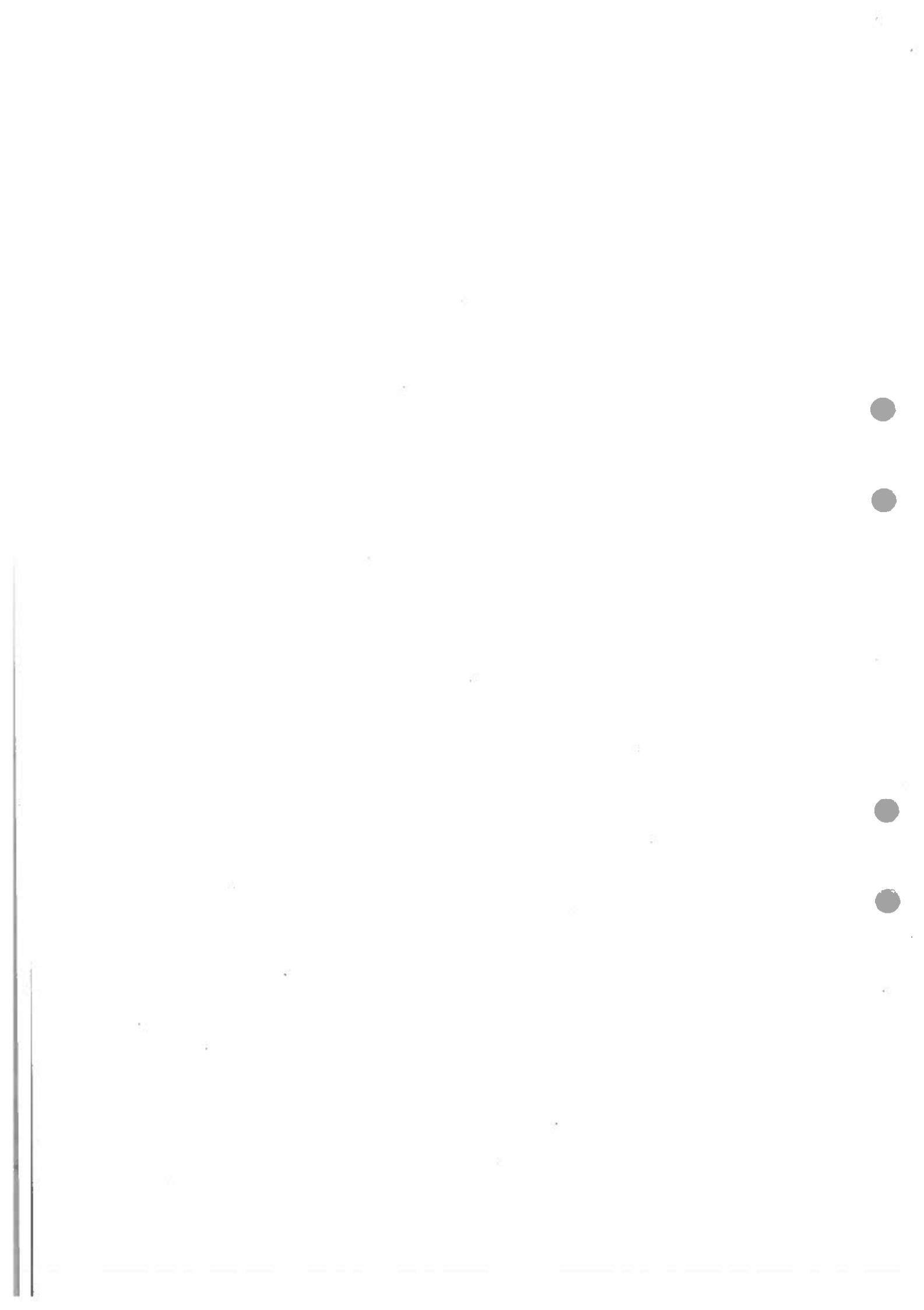
10 cases used, 2 cases contain missing values

Predictor	Coef	SE Coef	T	P
Constant	22,19	17,39	1,28	0,249
Xt	-0,2570	0,1428	-1,80	0,122
Xt-1	0,9594	0,1321	7,26	0,000
Xt-2	0,2529	0,2187	1,16	0,292

S = 3,59895 R-Sq = 93,0% R-Sq(adj) = 89,5%

Analysis of Variance					
Source	DF	SS	MS	F	P
Regression	3	1037,19	345,73	26,69	0,001
Residual Error	6	77,71	12,95		
Total	9	1114,90			

- B) If you want to test the restriction that the sum of the three slope parameters in the model in task A is equal to one, how would the restricted model look like? That is, impose the restriction into the model in task A and rewrite the model so it can be estimated. Also present the values on the variables in the restricted model for time-point t=12. (10)



Formula sheet, Econometrics I, Spring 2018

Under the simple linear model $Y_i = \beta_1 + \beta_2 X_i + u_i$, where $u_i \sim N(0, \sigma^2)$ and given independent pairs of observations $(Y_1, X_1), \dots, (Y_n, X_n)$, the OLS estimators are:

$$\begin{aligned}\hat{\beta}_1 &= \bar{Y} - \hat{\beta}_2 \bar{X} \\ \hat{\beta}_2 &= \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} \\ \hat{\sigma}^2 &= \frac{RSS}{n-2} = \frac{\sum (Y_i - \hat{Y}_i)^2}{n-2}\end{aligned}$$

where $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$ and where $E(\hat{\beta}_1) = \beta_1$, $E(\hat{\beta}_2) = \beta_2$ and $E(\hat{\sigma}^2) = \sigma^2$ and further

$$\begin{aligned}V(\hat{\beta}_1) &= \frac{\sum X_i^2}{n \sum (X_i - \bar{X})^2} \sigma^2 \\ V(\hat{\beta}_2) &= \frac{\sigma^2}{\sum (X_i - \bar{X})^2} \\ V(\hat{Y}_0) &= \sigma^2 \left(\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right) \\ V(Y_0 - \hat{Y}_0) &= \sigma^2 \left(1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right)\end{aligned}$$

Distributional results:

$$\begin{aligned}\frac{\hat{\beta}_i - \beta_i}{se(\hat{\beta}_i)} &\sim t(n-2), \quad i = 1, 2 \\ \frac{\hat{\sigma}^2 (n-2)}{\sigma^2} &\sim \chi^2(n-2)\end{aligned}$$

Coefficient of determination:

$$r^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum (Y_i - \hat{Y}_i)^2}{\sum (Y_i - \bar{Y})^2}$$

Coefficient of correlation:

$$r = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum (X_i - \bar{X})^2 \sum (Y_i - \bar{Y})^2}}$$

where $r = \pm\sqrt{r^2}$

If we let $Y_i^* = w_1 Y_i$ and $X_i^* = w_2 X_2$, then

$$\hat{\beta}_1^* = w_1 \hat{\beta}_1, \quad \hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right) \hat{\beta}_2, \quad \hat{\sigma}^{*2} = w_1^2 \hat{\sigma}^2$$

Under the multiple linear regression model $Y_i = \beta_1 + \beta_2 X_{2i} + \dots + \beta_k X_{ki} + u_i$, where $u_i \sim N(0, \sigma^2)$ and given independent vectors of observations $(Y_1, X_{21}, \dots, X_{k1}), \dots, (Y_n, X_{2n}, \dots, X_{kn})$, the following holds for the OLS estimators:

$$\hat{\sigma}^2 = \frac{RSS}{n-k} = \frac{\sum (Y_i - \hat{Y}_i)^2}{n-k}$$

$$\begin{aligned} \frac{\hat{\beta}_i - \beta_i}{se(\hat{\beta}_i)} &\sim t(n-k), \quad i = 1, \dots, k \\ \frac{\hat{\sigma}^2 (n-k)}{\sigma^2} &\sim \chi^2(n-k) \end{aligned}$$

The multiple coefficient of determination:

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum (Y_i - \hat{Y}_i)^2}{\sum (Y_i - \bar{Y})^2}$$

Adjusted:

$$\bar{R}^2 = 1 - \frac{RSS/(n-k)}{TSS/(n-1)}$$

Testing $H_0 : \beta_2 = \dots = \beta_k = 0$:

$$F = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{\sum (\hat{Y}_i - \bar{Y})^2 / (k-1)}{\sum (Y_i - \hat{Y}_i)^2 / (n-k)}$$

Comparing an "old" model with a "new" (larger):

$$\begin{aligned} F &= \frac{(ESS_{new} - ESS_{old})/\text{number of new regressors}}{RSS_{new}/(n - \text{number of parameters in the new model})} \\ &= \frac{(R^2_{new} - R^2_{old})/\text{number of new regressors}}{(1 - R^2_{new})/(n - \text{number of parameters in the new model})} \end{aligned}$$

Comparing an "unrestricted" model with a "restricted":

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} = \frac{(R_{UR}^2 - R_R^2)/m}{(1 - R_{UR}^2)/(n-k)}$$

where m is the number of linear constraints and k is the number of parameters in the unrestricted model.

Variance inflation factor:

$$VIF_j = \frac{1}{1 - R_j^2}$$

Auxiliary regression:

$$F_j = \frac{R_j^2/(k-2)}{(1 - R_j^2)/(n-k+1)}$$

where $R_j^2 = R^2$ in the regression of x_j on the remaining $(k-2)$ regressors.

Tests of heteroscedasticity: (all test statistics are evaluated under the null hypothesis of no heteroscedasticity)

White's test: Regress \hat{u}_i^2 against the $k-1$ regressors and the squares of these.
Test statistic: $n R^2 \underset{\text{appr}}{\sim} \chi^2(2(k-1))$

Glejser test: Regress $|\hat{u}_i|$ against the regressor X_j (one regressor at a time)
Test statistic: t -test of the slope

Park test: Regress $\ln \hat{u}_i^2$ against the regressor $\ln X_j$, (one regressor at a time)
Test statistic: t -test of the slope

Goldefeld Quandt test of equal variances in two separate regressions:
Test statistic: $\frac{S_1^2}{S_2^2} \sim F(n_1 - k_1, n_2 - k_2)$

Tests of autocorrelation:

The Runs test: For R = number of runs, where $N = N_1 + N_2$ total number of observations:

$$\begin{aligned} E(R) &= \frac{2N_1 N_2}{N} + 1 \\ V(R) &= \frac{2N_1 N_2 (2N_1 N_2 - N)}{N^2(N-1)} \end{aligned}$$

The Durbin Watson d statistic:

$$d = \frac{\sum_{t=2}^n (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^n \hat{u}_t^2}$$

Breusch Godfrey test: Null hypothesis: $H_0: \rho_1 = \rho_2 = \dots = \rho_K = 0$

Test statistic: nR^2 from the regression of \hat{u}_t on the regressors which have produced \hat{u}_t plus lagged \hat{u}_t up to lag K .

n = the number of observations used in this regression.

The test statistic is approximately $\chi^2(K)$

Akaike's information criterion:

$$AIC = \frac{e^{2k/n} RSS}{n}$$

Schwartz's information criterion:

$$SIC = \frac{n^{k/n} RSS}{n}$$

Mallow's C_p criterion:

$$C_p = \frac{RSS_p}{\hat{\sigma}^2} - (n - 2p)$$

Logistic regression (logit model):

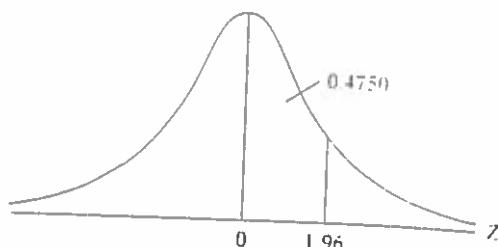
$$P(Y = 1) = \frac{1}{1 + e^{-(\beta_1 + \beta_2 X_2 + \dots + \beta_k X_k)}}, \quad \ln\left(\frac{P(Y = 1)}{1 - P(Y = 1)}\right) = \beta_1 + \beta_2 X_2 + \dots + \beta_k X_k$$

TABLE D.1
Areas Under the
Standardized Normal
Distribution

Example

$$\Pr(0 \leq Z \leq 1.96) = 0.4750$$

$$\Pr(Z \geq 1.96) = 0.5 - 0.4750 = 0.025$$



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4454	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Note: This table gives the area in the right-hand tail of the distribution (i.e., $Z \geq 0$). But since the normal distribution is symmetrical about $Z = 0$, the area in the left-hand tail is the same as the area in the corresponding right-hand tail. For example, $\Pr(-1.96 \leq Z \leq 0) = 0.4750$. Therefore, $\Pr(-1.96 \leq Z \leq 1.96) = 2(0.4750) = 0.95$.

TABLE D.2
Percentage Points of
the t Distribution

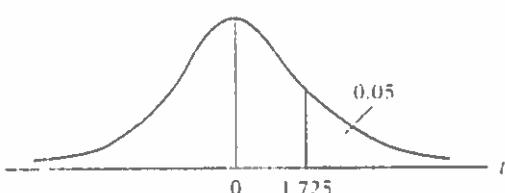
Source: From E. S. Pearson and
H. O. Hartley, eds., *Biometrika Tables
for Statisticians*, vol. 1,
3d ed., table 12, Cambridge
University Press, New York,
1968. Reproduced by
permission of the editors and
trustees of *Biometrika*.

Example

$$\Pr(t > 2.086) = 0.025$$

$$\Pr(t > 1.725) = 0.05 \quad \text{for } df = 20$$

$$\Pr(|t| > 1.725) = 0.10$$



\Pr	0.25	0.10	0.05	0.025	0.01	0.005	0.001
df	0.50	0.20	0.10	0.05	0.02	0.010	0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

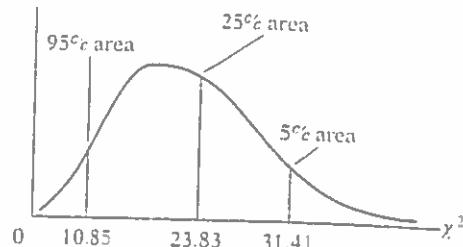
Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

TABLE D.4
Upper Percentage Points of the χ^2 Distribution

Example

$\Pr(\chi^2 > 10.85) = 0.95$
 $\Pr(\chi^2 > 23.83) = 0.25$
 $\Pr(\chi^2 > 31.41) = 0.05$

for df = 20



Degrees of freedom	.995	.990	.975	.950	.900
1	392704×10^{-10}	157088×10^{-9}	982069×10^{-9}	393214×10^{-8}	.0157908
2	.0100251	.0201007	.0506356	.102587	.210720
3	.0717212	.114832	.215795	.351846	.584375
4	.206990	.297110	.484419	.710721	1.063623
5	.411740	.554300	.831211	1.145476	1.61031
6	.675727	.872085	1.237347	1.63539	2.20413
7	.989265	1.239043	1.68987	2.16735	2.83311
8	1.344419	1.646482	2.17973	2.73264	3.48954
9	1.734926	2.087912	2.70039	3.32511	4.16816
10	2.15585	2.55821	3.24697	3.94030	4.86518
11	2.60321	3.05347	3.81575	4.57481	5.57779
12	3.07382	3.57056	4.40379	5.22603	6.30380
13	3.56503	4.10691	5.00874	5.89186	7.04150
14	4.07468	4.66043	5.62872	6.57063	7.78953
15	4.60094	5.22935	6.26214	7.26094	8.54675
16	5.14224	5.81221	6.90766	7.96164	9.31223
17	5.69724	6.40776	7.56418	8.67176	10.0852
18	6.26481	7.01491	8.23075	9.39046	10.8649
19	6.84398	7.63273	8.90655	10.1170	11.6509
20	7.43386	8.26040	9.59083	10.8508	12.4426
21	8.03366	8.89720	10.28293	11.5913	13.2396
22	8.64272	9.54249	10.9823	12.3380	14.0415
23	9.26042	10.19567	11.6885	13.0905	14.8479
24	9.88623	10.8564	12.4011	13.8484	15.6587
25	10.5197	11.5240	13.1197	14.6114	16.4734
26	11.1603	12.1981	13.8439	15.3791	17.2919
27	11.8076	12.8786	14.5733	16.1513	18.1138
28	12.4613	13.5648	15.3079	16.9279	18.9392
29	13.1211	14.2565	16.0471	17.7083	19.7677
30	13.7867	14.9535	16.7908	18.4926	20.5992
40	20.7065	22.1643	24.4331	26.5093	29.0505
50	27.9907	29.7067	32.3574	34.7642	37.6886
60	35.5346	37.4848	40.4817	43.1879	46.4589
70	43.2752	45.4418	48.7576	51.7393	55.3290
80	51.1720	53.5400	57.1532	60.3915	64.2778
90	59.1963	61.7541	65.6466	69.1260	73.2912
100*	67.3276	70.0648	74.2219	77.9295	82.3581

*For df greater than 100 the expression $\sqrt{2\chi^2} - \sqrt{(2k-1)} = Z$ follows the standardized normal distribution, where k represents the degrees of freedom.



χ^2 -table continued

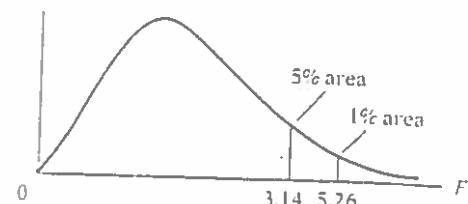
.750	.500	.250	.100	.050	.025	.010	.005
1.015308	1.454937	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
1.575364	1.38629	2.77259	4.60517	5.99147	7.37776	9.21034	10.5966
1.212534	2.36597	4.10835	6.25139	7.81473	9.34840	11.3449	12.8381
1.92255	3.35670	5.38527	7.77944	9.48773	11.1433	13.2767	14.8602
1.67460	4.35146	6.62568	9.23635	11.0705	12.8325	15.0863	16.7496
3.45460	5.34812	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476
4.25485	6.34581	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777
5.07064	7.34412	10.2188	13.3616	15.5073	17.5346	20.0902	21.9550
5.89883	8.34283	11.3887	14.6837	16.9190	19.0228	21.6660	23.5893
7.73720	9.34182	12.5489	15.9871	18.3070	20.4831	23.2093	25.1882
1.58412	10.3410	13.7007	17.2750	19.6751	21.9200	24.7250	26.7569
8.43842	11.3403	14.8454	18.5494	21.0261	23.3367	26.2170	28.2995
9.29906	12.3398	15.9839	19.8119	22.3621	24.7356	27.6883	29.8194
10.1653	13.3393	17.1170	21.0642	23.6848	26.1190	29.1413	31.3193
11.0365	14.3389	18.2451	22.3072	24.9958	27.4884	30.5779	32.8013
11.9122	15.3385	19.3688	23.5418	26.2962	28.8454	31.9999	34.2672
12.7919	16.3381	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185
13.6753	17.3379	21.6049	25.9894	28.8693	31.5264	34.8053	37.1564
14.5620	18.3376	22.7178	27.2036	30.1435	32.8523	36.1908	38.5822
15.4518	19.3374	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968
16.3444	20.3372	24.9348	29.6151	32.6705	35.4789	38.9321	41.4010
17.2396	21.3370	26.0393	30.8133	33.9244	36.7807	40.2894	42.7956
18.1373	22.3369	27.1413	32.0069	35.1725	38.0757	41.6384	44.1813
19.0372	23.3367	28.2412	33.1963	36.4151	39.3641	42.9798	45.5585
19.9393	24.3366	29.3389	34.3816	37.6525	40.6465	44.3141	46.9278
20.8434	25.3364	30.4345	35.5631	38.8852	41.9232	45.6417	48.2899
21.7494	26.3363	31.5284	36.7412	40.1133	43.1944	46.9630	49.6449
22.6572	27.3363	32.6205	37.9159	41.3372	44.4607	48.2782	50.9933
23.5666	28.3362	33.7109	39.0875	42.5569	45.7222	49.5879	52.3356
24.4776	29.3360	34.7998	40.2560	43.7729	46.9792	50.8922	53.6720
33.6603	39.3354	45.6160	51.8050	55.7585	59.3417	63.6907	66.7659
42.9421	49.3349	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900
52.2938	59.3347	66.9814	74.3970	79.0819	83.2976	88.3794	91.9517
61.6983	69.3344	77.5766	85.5271	90.5312	95.0231	100.425	104.215
71.1445	79.3343	88.1303	96.5782	101.879	106.629	112.329	116.321
80.6247	89.3342	98.6499	107.565	113.145	118.136	124.116	128.299
90.1332	99.3341	109.141	118.498	124.342	129.561	135.807	140.169

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., Table 8, Cambridge University Press, New York, 1966.
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TABLE D-3 Upper Percentage Points of the F Distribution
Example

$\Pr(F > 1.59) = 0.25$
 $\Pr(F > 2.42) = 0.10$ for df $N_1 = 10$
 $\Pr(F > 3.14) = 0.05$ and $N_2 = 9$
 $\Pr(F > 5.26) = 0.01$



df for denominator N_2	Pr	df for numerator N_1											
		1	2	3	4	5	6	7	8	9	10	11	12
1	.25	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.36	9.41
	.10	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7
	.05	161	200	216	225	230	234	237	239	241	242	243	244
2	.25	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.39
	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41
	.05	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4
	.01	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4
3	.25	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.45
	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22
	.05	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74
	.01	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1
4	.25	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91
	.01	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.4
5	.25	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.71	4.68
	.01	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89
6	.25	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.77
	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00
	.01	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72
7	.25	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.69	1.69	1.69	1.68
	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57
	.01	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47
8	.25	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.63	1.62
	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28
	.01	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67
9	.25	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07
	.01	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 18, Cambridge University Press, New York, 1966.
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F-table (continued)

df for numerator N_1												df for denominator N_2
15	20	24	30	40	50	60	100	120	200	500	∞	Pr
9.49	9.58	9.63	9.67	9.71	9.74	9.76	9.78	9.80	9.82	9.84	9.85	.25
61.2	61.7	62.0	62.3	62.5	62.7	62.8	63.0	63.1	63.2	63.3	63.3	.10
246	248	249	250	251	252	252	253	253	254	254	254	.05
3.41	3.43	3.43	3.44	3.45	3.45	3.46	3.47	3.47	3.48	3.48	3.48	.25
9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.48	9.49	9.49	9.49	.10
19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	.05
99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	99.5	.01
2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	.25
5.20	5.18	5.18	5.17	5.16	5.15	5.15	5.14	5.14	5.14	5.14	5.13	.10
8.70	8.66	8.64	8.62	8.59	8.58	8.57	8.55	8.55	8.54	8.53	8.53	.05
26.9	26.7	26.6	26.5	26.4	26.4	26.3	26.2	26.2	26.2	26.1	26.1	.01
2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	.25
3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.78	3.77	3.76	3.76	.10
5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.66	5.65	5.64	5.63	.05
14.2	14.0	13.9	13.8	13.7	13.7	13.7	13.6	13.6	13.5	13.5	13.5	.01
1.89	1.88	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.87	1.87	.25
3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.10	.10
4.62	4.56	4.53	4.50	4.46	4.44	4.43	4.41	4.40	4.39	4.37	4.36	.05
9.72	9.55	9.47	9.38	9.29	9.24	9.20	9.13	9.11	9.08	9.04	9.02	.01
1.76	1.76	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	.25
2.87	2.84	2.82	2.80	2.78	2.77	2.76	2.75	2.74	2.73	2.73	2.72	.10
3.94	3.87	3.84	3.81	3.77	3.75	3.74	3.71	3.70	3.69	3.68	3.67	.05
7.56	7.40	7.31	7.23	7.14	7.09	7.06	6.99	6.97	6.93	6.90	6.88	.01
1.68	1.67	1.67	1.66	1.66	1.66	1.65	1.65	1.65	1.65	1.65	1.65	.25
2.63	2.59	2.58	2.56	2.54	2.52	2.51	2.50	2.49	2.48	2.48	2.47	.10
3.51	3.44	3.41	3.38	3.34	3.32	3.30	3.27	3.27	3.25	3.24	3.23	.05
6.31	6.16	6.07	5.99	5.91	5.86	5.82	5.75	5.74	5.70	5.67	5.65	.01
1.62	1.61	1.60	1.60	1.59	1.59	1.59	1.58	1.58	1.58	1.58	1.58	.25
2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.32	2.31	2.30	2.29	.10
3.22	3.15	3.12	3.08	3.04	3.02	3.01	2.97	2.97	2.95	2.94	2.93	.05
5.52	5.36	5.28	5.20	5.12	5.07	5.03	4.96	4.95	4.91	4.88	4.86	.01
1.57	1.56	1.56	1.55	1.55	1.54	1.54	1.53	1.53	1.53	1.53	1.53	.25
2.34	2.30	2.28	2.25	2.23	2.22	2.21	2.19	2.18	2.17	2.17	2.16	.10
3.01	2.94	2.90	2.86	2.83	2.80	2.79	2.76	2.75	2.73	2.72	2.71	.05
4.96	4.81	4.73	4.65	4.57	4.52	4.48	4.42	4.40	4.36	4.33	4.31	.01

(Continued)

TABLE D.3 Upper Percentage Points of the *F* Distribution (Continued)

df for denominator N_2	Pr	df for numerator N_1											
		1	2	3	4	5	6	7	8	9	10	11	12
10	.25	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.55	1.54
	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91
	.01	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71
11	.25	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.52	1.51
	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40
12	.25	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.50	1.49
	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16
13	.25	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47
	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96
14	.25	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.46	1.45
	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.08	2.05
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80
15	.25	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.44
	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67
16	.25	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.44	1.43
	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55
17	.25	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.42	1.41
	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38
	.01	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46
18	.25	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.96	1.93
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37
19	.25	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.40
	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.94	1.91
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30
20	.25	1.40	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.39
	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.92	1.89
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23

F-table (continued)

	df for numerator N ₁								df for denominator N ₂			
15	20	24	30	40	50	60	100	120	200	500	∞	Pr
1.53	1.52	1.52	1.51	1.51	1.50	1.50	1.49	1.49	1.49	1.48	1.48	.25
2.24	2.20	2.18	2.16	2.13	2.12	2.11	2.09	2.08	2.07	2.06	2.06	.10
2.85	2.77	2.74	2.70	2.66	2.64	2.62	2.59	2.58	2.56	2.55	2.54	.05
4.56	4.41	4.33	4.25	4.17	4.12	4.08	4.01	4.00	3.96	3.93	3.91	.01
1.50	1.49	1.49	1.48	1.47	1.47	1.47	1.46	1.46	1.46	1.45	1.45	.25
2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	2.00	1.99	1.98	1.97	.10
2.72	2.65	2.61	2.57	2.53	2.51	2.49	2.46	2.45	2.43	2.42	2.40	.05
4.25	4.10	4.02	3.94	3.86	3.81	3.78	3.71	3.69	3.66	3.62	3.60	.01
1.48	1.47	1.46	1.45	1.45	1.44	1.44	1.43	1.43	1.43	1.42	1.42	.25
2.10	2.06	2.04	2.01	1.99	1.97	1.96	1.94	1.93	1.92	1.91	1.90	.10
2.62	2.54	2.51	2.47	2.43	2.40	2.38	2.35	2.34	2.32	2.31	2.30	.05
4.01	3.86	3.78	3.70	3.62	3.57	3.54	3.47	3.45	3.41	3.38	3.36	.01
1.46	1.45	1.44	1.43	1.42	1.42	1.42	1.41	1.41	1.40	1.40	1.40	.25
2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.88	1.86	1.85	1.85	.10
2.53	2.46	2.42	2.38	2.34	2.31	2.30	2.26	2.25	2.23	2.22	2.21	.05
3.82	3.66	3.59	3.51	3.43	3.38	3.34	3.27	3.25	3.22	3.19	3.17	.01
1.44	1.43	1.42	1.41	1.41	1.40	1.40	1.39	1.39	1.39	1.38	1.38	.25
2.01	1.96	1.94	1.91	1.89	1.87	1.86	1.83	1.83	1.82	1.80	1.80	.10
2.46	2.39	2.35	2.31	2.27	2.24	2.22	2.19	2.18	2.16	2.14	2.13	.05
3.66	3.51	3.43	3.35	3.27	3.22	3.18	3.11	3.09	3.06	3.03	3.00	.01
1.43	1.41	1.41	1.40	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.36	.25
1.97	1.92	1.90	1.87	1.85	1.83	1.82	1.79	1.79	1.77	1.76	1.76	.10
2.40	2.33	2.29	2.25	2.20	2.18	2.16	2.12	2.11	2.10	2.08	2.07	.05
3.52	3.37	3.29	3.21	3.13	3.08	3.05	2.98	2.96	2.92	2.89	2.87	.01
1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.36	1.35	1.35	1.34	1.34	.25
1.94	1.89	1.87	1.84	1.81	1.79	1.78	1.76	1.75	1.74	1.73	1.72	.10
2.35	2.28	2.24	2.19	2.15	2.12	2.11	2.07	2.06	2.04	2.02	2.01	.05
3.41	3.26	3.18	3.10	3.02	2.97	2.93	2.86	2.84	2.81	2.78	2.75	.01
1.40	1.39	1.38	1.37	1.36	1.35	1.35	1.34	1.34	1.34	1.33	1.33	.25
1.91	1.86	1.84	1.81	1.78	1.76	1.75	1.73	1.72	1.71	1.69	1.69	.10
2.31	2.23	2.19	2.15	2.10	2.08	2.06	2.02	2.01	1.99	1.97	1.96	.05
3.31	3.16	3.08	3.00	2.92	2.87	2.83	2.76	2.75	2.71	2.68	2.65	.01
1.39	1.38	1.37	1.36	1.35	1.34	1.34	1.33	1.33	1.32	1.32	1.32	.25
1.89	1.84	1.81	1.78	1.75	1.74	1.72	1.70	1.69	1.68	1.67	1.66	.10
2.27	2.19	2.15	2.11	2.06	2.04	2.02	1.98	1.97	1.95	1.93	1.92	.05
3.23	3.08	3.00	2.92	2.84	2.78	2.75	2.68	2.66	2.62	2.59	2.57	.01
1.38	1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.32	1.31	1.31	1.30	.25
1.86	1.81	1.79	1.76	1.73	1.71	1.70	1.67	1.67	1.65	1.64	1.63	.10
2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.94	1.93	1.91	1.89	1.88	.05
3.15	3.00	2.92	2.84	2.76	2.71	2.67	2.60	2.58	2.55	2.51	2.49	.01
1.37	1.36	1.35	1.34	1.33	1.33	1.32	1.31	1.31	1.30	1.30	1.29	.25
1.84	1.79	1.77	1.74	1.71	1.69	1.68	1.65	1.64	1.63	1.62	1.61	.10
2.20	2.12	2.08	2.04	1.99	1.97	1.95	1.91	1.90	1.88	1.86	1.84	.05
3.09	2.94	2.86	2.78	2.69	2.64	2.61	2.54	2.52	2.48	2.44	2.42	.01

(Continued)

TABLE D.3 Upper Percentage Points of the F Distribution (Continued)

df for denominator N_2	Pr	df for numerator N_1										
		1	2	3	4	5	6	7	8	9	10	11
22	.25	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.38
	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18
24	.25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.37
	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.21
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09
26	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.84
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02
28	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96
30	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91
40	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.73
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73
60	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56
120	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.62
	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40
200	.25	1.33	1.39	1.38	1.36	1.34	1.32	1.31	1.29	1.28	1.27	1.26
	.10	2.73	2.33	2.11	1.97	1.88	1.80	1.75	1.70	1.66	1.63	1.60
	.05	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84
	.01	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34
∞	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57
	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25

F-table (continued)

	df for numerator N_1										df for denominator		
	20	24	30	40	50	60	100	120	200	500	∞	Pr	N_2
15													
1.36	1.34	1.33	1.32	1.31	1.31	1.30	1.30	1.30	1.29	1.29	1.28	.25	
1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.61	1.60	1.59	1.58	1.57	.10	22
2.15	2.07	2.03	1.98	1.94	1.91	1.89	1.85	1.84	1.82	1.80	1.78	.05	
2.98	2.83	2.75	2.67	2.58	2.53	2.50	2.42	2.40	2.36	2.33	2.31	.01	
1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.28	1.28	1.27	1.27	1.26	.25	
1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.58	1.57	1.56	1.54	1.53	.10	24
2.11	2.03	1.98	1.94	1.89	1.86	1.84	1.80	1.79	1.77	1.75	1.73	.05	
2.89	2.74	2.66	2.58	2.49	2.44	2.40	2.33	2.31	2.27	2.24	2.21	.01	
1.34	1.32	1.31	1.30	1.29	1.28	1.28	1.26	1.26	1.26	1.25	1.25	.25	
1.76	1.71	1.68	1.65	1.61	1.59	1.58	1.55	1.54	1.53	1.51	1.50	.10	26
2.07	1.99	1.95	1.90	1.85	1.82	1.80	1.76	1.75	1.73	1.71	1.69	.05	
2.81	2.66	2.58	2.50	2.42	2.36	2.33	2.25	2.23	2.19	2.16	2.13	.01	
1.33	1.31	1.30	1.29	1.28	1.27	1.27	1.26	1.26	1.25	1.25	1.24	.25	
1.74	1.69	1.66	1.63	1.59	1.57	1.56	1.53	1.52	1.50	1.49	1.48	.10	28
2.04	1.96	1.91	1.87	1.82	1.79	1.77	1.73	1.71	1.69	1.67	1.65	.05	
2.75	2.60	2.52	2.44	2.35	2.30	2.26	2.19	2.17	2.13	2.09	2.06	.01	
1.32	1.30	1.29	1.28	1.27	1.26	1.26	1.25	1.24	1.24	1.23	1.23	.25	
1.72	1.67	1.64	1.61	1.57	1.55	1.54	1.51	1.50	1.48	1.47	1.46	.10	30
2.01	1.93	1.89	1.84	1.79	1.76	1.74	1.70	1.68	1.66	1.64	1.62	.05	
2.70	2.55	2.47	2.39	2.30	2.25	2.21	2.13	2.11	2.07	2.03	2.01	.01	
1.30	1.28	1.26	1.25	1.24	1.23	1.22	1.21	1.21	1.20	1.19	1.19	.25	
1.66	1.61	1.57	1.54	1.51	1.48	1.47	1.43	1.42	1.41	1.39	1.38	.10	40
1.92	1.84	1.79	1.74	1.69	1.66	1.64	1.59	1.58	1.55	1.53	1.51	.05	
2.52	2.37	2.29	2.20	2.11	2.06	2.02	1.94	1.92	1.87	1.83	1.80	.01	
1.27	1.25	1.24	1.22	1.21	1.20	1.19	1.17	1.17	1.16	1.15	1.15	.25	
1.60	1.54	1.51	1.48	1.44	1.41	1.40	1.36	1.35	1.33	1.31	1.29	.10	60
1.84	1.75	1.70	1.65	1.59	1.56	1.53	1.48	1.47	1.44	1.41	1.39	.05	
2.35	2.20	2.12	2.03	1.94	1.88	1.84	1.75	1.73	1.68	1.63	1.60	.01	
1.24	1.22	1.21	1.19	1.18	1.17	1.16	1.14	1.13	1.12	1.11	1.10	.25	
1.55	1.48	1.45	1.41	1.37	1.34	1.32	1.27	1.26	1.24	1.21	1.19	.10	120
1.75	1.66	1.61	1.55	1.50	1.46	1.43	1.37	1.35	1.32	1.28	1.25	.05	
2.19	2.03	1.95	1.86	1.76	1.70	1.66	1.56	1.53	1.48	1.42	1.38	.01	
1.23	1.21	1.20	1.18	1.16	1.14	1.12	1.11	1.10	1.09	1.08	1.06	.25	
1.52	1.46	1.42	1.38	1.34	1.31	1.28	1.24	1.22	1.20	1.17	1.14	.10	200
1.72	1.62	1.57	1.52	1.46	1.41	1.39	1.32	1.29	1.26	1.22	1.19	.05	
2.13	1.97	1.89	1.79	1.69	1.63	1.58	1.48	1.44	1.39	1.33	1.28	.01	
1.22	1.19	1.18	1.16	1.14	1.13	1.12	1.09	1.08	1.07	1.04	1.00	.25	
1.49	1.42	1.38	1.34	1.30	1.26	1.24	1.18	1.17	1.13	1.08	1.00	.10	
1.67	1.57	1.52	1.46	1.39	1.35	1.32	1.24	1.22	1.17	1.11	1.00	.05	
2.04	1.88	1.79	1.70	1.59	1.52	1.47	1.36	1.32	1.25	1.15	1.00	.01	

TABLE D.5A Durbin-Watson d Statistic: Significance Points of d_L and d_U at 0.05 Level of Significance

n	$k' = 1$		$k' = 2$		$k' = 3$		$k' = 4$		$k' = 5$		$k' = 6$		$k' = 7$		$k' = 8$		$k' = 9$		$k' = 10$		
	d_L	d_U	d_L	d_U																	
6	0.610	1.400	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	0.700	1.356	0.467	1.896	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	0.763	1.332	0.559	1.777	0.368	2.287	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.824	1.320	0.629	1.699	0.455	2.128	0.296	2.588	—	—	—	—	—	—	—	—	—	—	—	—	—
10	0.879	1.320	0.697	1.641	0.525	2.016	0.376	2.414	0.243	2.822	—	—	—	—	—	—	—	—	—	—	—
11	0.927	1.324	0.658	1.603	0.595	1.928	0.444	2.283	0.316	2.645	0.203	3.005	—	—	—	—	—	—	—	—	—
12	0.971	1.331	0.812	1.579	0.658	1.864	0.512	2.177	0.379	2.506	0.268	2.832	0.171	3.149	—	—	—	—	—	—	—
13	1.010	1.340	0.861	1.562	0.715	1.816	0.574	2.094	0.415	2.390	0.328	2.692	0.230	2.985	0.147	3.266	—	—	—	—	—
14	1.045	1.350	0.905	1.551	0.767	1.779	0.632	2.030	0.505	2.296	0.389	2.572	0.286	2.848	0.200	3.111	0.127	3.360	—	—	—
15	1.077	1.361	0.946	1.543	0.814	1.750	0.685	1.977	0.562	2.220	0.447	2.472	0.343	2.727	0.251	2.979	0.175	3.216	0.111	3.438	—
16	1.106	1.371	0.982	1.539	0.857	1.729	0.734	1.935	0.615	2.157	0.502	2.388	0.398	2.624	0.304	2.860	0.222	3.090	0.155	3.304	—
17	1.133	1.381	1.015	1.536	0.897	1.710	0.779	1.900	0.664	2.104	0.554	2.318	0.451	2.537	0.356	2.757	0.272	2.975	0.198	3.184	—
18	1.158	1.391	1.046	1.535	0.933	1.696	0.820	1.872	0.710	2.060	0.603	2.257	0.502	2.461	0.407	2.667	0.321	2.873	0.244	3.073	—
19	1.180	1.401	1.074	1.536	0.967	1.685	0.859	1.849	0.752	2.023	0.649	2.206	0.549	2.396	0.456	2.589	0.369	2.783	0.290	2.974	—
20	1.201	1.411	1.100	1.537	0.998	1.676	0.894	1.828	0.792	1.991	0.692	2.162	0.595	2.339	0.502	2.521	0.416	2.704	0.336	2.885	—
21	1.221	1.420	1.125	1.538	1.026	1.669	0.927	1.812	0.829	1.964	0.732	2.124	0.637	2.290	0.547	2.460	0.461	2.633	0.380	2.806	—
22	1.239	1.429	1.147	1.541	1.053	1.664	0.958	1.797	0.863	1.940	0.769	2.090	0.677	2.246	0.588	2.407	0.504	2.571	0.424	2.734	—
23	1.257	1.437	1.168	1.543	1.078	1.660	0.986	1.785	0.895	1.920	0.804	2.061	0.715	2.208	0.628	2.360	0.545	2.514	0.465	2.670	—
24	1.273	1.446	1.188	1.546	1.101	1.656	1.013	1.775	0.925	1.902	0.837	2.035	0.751	2.174	0.666	2.318	0.584	2.464	0.506	2.613	—
25	1.288	1.454	1.206	1.550	1.123	1.654	1.038	1.767	0.953	1.836	0.868	2.012	0.784	2.144	0.702	2.280	0.621	2.419	0.544	2.560	—
26	1.302	1.461	1.224	1.553	1.143	1.652	1.062	1.759	0.979	1.873	0.897	1.992	0.816	2.117	0.735	2.246	0.657	2.379	0.581	2.513	—
27	1.316	1.469	1.240	1.556	1.162	1.651	1.084	1.753	1.004	1.861	0.925	1.974	0.845	2.093	0.767	2.216	0.691	2.342	0.616	2.470	—
28	1.328	1.476	1.255	1.560	1.181	1.650	1.104	1.747	1.028	1.850	0.951	1.958	0.874	2.071	0.798	2.188	0.723	2.309	0.650	2.431	—
29	1.341	1.483	1.270	1.563	1.198	1.650	1.124	1.743	1.050	1.841	0.975	1.944	0.900	2.052	0.826	2.164	0.753	2.278	0.682	2.396	—
30	1.352	1.489	1.284	1.567	1.214	1.650	1.143	1.739	1.071	1.833	0.998	1.931	0.926	2.034	0.854	2.141	0.782	2.251	0.712	2.363	—
31	1.363	1.496	1.297	1.570	1.229	1.650	1.160	1.735	1.090	1.825	1.020	1.920	0.950	2.018	0.879	2.120	0.810	2.226	0.741	2.333	—
32	1.373	1.502	1.309	1.574	1.244	1.650	1.177	1.732	1.109	1.819	1.041	1.909	0.972	2.004	0.904	2.102	0.836	2.203	0.769	2.306	—
33	1.383	1.508	1.321	1.577	1.258	1.651	1.193	1.730	1.127	1.813	1.061	1.900	0.994	1.991	0.927	2.085	0.861	2.181	0.795	2.281	—
34	1.393	1.514	1.333	1.580	1.271	1.652	1.208	1.728	1.144	1.808	1.080	1.891	1.015	1.979	0.950	2.069	0.885	2.162	0.821	2.257	—
35	1.402	1.519	1.343	1.584	1.283	1.653	1.222	1.726	1.160	1.803	1.097	1.884	1.034	1.967	0.971	2.054	0.908	2.144	0.845	2.235	—
36	1.411	1.525	1.354	1.587	1.295	1.654	1.236	1.724	1.175	1.799	1.114	1.877	1.053	1.957	0.991	2.041	0.930	2.127	0.868	2.216	—
37	1.419	1.530	1.364	1.590	1.307	1.655	1.249	1.723	1.190	1.795	1.131	1.870	1.071	1.948	1.011	2.029	0.951	2.112	0.891	2.198	—
38	1.427	1.535	1.373	1.594	1.318	1.656	1.261	1.722	1.204	1.792	1.146	1.864	1.088	1.939	1.029	2.017	0.970	2.098	0.912	2.180	—
39	1.435	1.540	1.382	1.597	1.328	1.658	1.273	1.722	1.219	1.789	1.161	1.859	1.104	1.932	1.047	2.007	0.990	2.085	0.932	2.164	—
40	1.442	1.544	1.391	1.600	1.338	1.659	1.285	1.721	1.230	1.786	1.175	1.854	1.120	1.924	1.064	1.997	1.008	2.072	0.952	2.149	—
45	1.475	1.566	1.430	1.615	1.383	1.666	1.336	1.720	1.287	1.776	1.238	1.835	1.189	1.895	1.139	1.958	1.089	2.022	1.038	2.088	—
50	1.503	1.585	1.462	1.628	1.421	1.674	1.378	1.721	1.335	1.771	1.291	1.822	1.246	1.875	1.201	1.930	1.156	1.986	1.110	2.044	—
55	1.528	1.601	1.490	1.641	1.452	1.681	1.414	1.724	1.374	1.768	1.334	1.814	1.294	1.861	1.253	1.909	1.212	1.959	1.170	2.010	—
60	1.549	1.616	1.514	1.652	1.480	1.689	1.444	1.727	1.408	1.767	1.372	1.808	1.335	1.850	1.298	1.894	1.260	1.939	1.222	1.984	—
65	1.567	1.629	1.536	1.662	1.503	1.696	1.471	1.731	1.438	1.767	1.404	1.805	1.370	1.843	1.336	1.882	1.301	1.923	1.266	1.964	—
70	1.583	1.641	1.554	1.672	1.525	1.703	1.494	1.735	1.464	1.768	1.433	1.802	1.401	1.837	1.369	1.873	1.337	1.910	1.305	1.948	—
75	1.598	1.652	1.571	1.680	1.543	1.709	1.515	1.739	1.487	1.770	1.458	1.801	1.428	1.834	1.399	1.867	1.369	1.901	1.339	1.935	—
80	1.611	1.662	1.586	1.688	1.560	1.715	1.534	1.743	1.507	1.772	1.480	1.801	1.453	1.831	1.425	1.861	1.397	1.893	1.369	1.925	—
85	1.624	1.671	1.600	1.696	1.575	1.721	1.550	1.747	1.525	1.774	1.500	1.801	1.474	1.829	1.448	1.857	1.422	1.886	1.396	1.916	—
90	1.635	1.679	1.612	1.703	1.589	1.726	1.566	1.751	1.542	1.776	1.518	1.801	1.494	1.827	1.469	1.854	1.445	1.881	1.420	1.909	—
95	1.645	1.687	1.623	1.709	1.602	1.732	1.579	1.755	1.557	1.778	1.535	1.802	1.512	1.827	1.499	1.852	1.465	1.877	1.442	1.903	—
100	1.654	1.694	1.634	1.715	1.613	1.736	1.592	1.758	1.571	1.780	1.550	1.803	1.528	1.826	1.506	1.850	1.484	1.874	1.462	1.898	—
150	1.720	1.746	1.706	1.760	1.693	1.774															

n	$k' = 11$		$k' = 12$		$k' = 13$		$k' = 14$		$k' = 15$		$k' = 16$		$k' = 17$		$k' = 18$		$k' = 19$		$k' = 20$		
	d_L	d_U																			
16	0.098	3.503	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	0.138	3.378	0.087	3.557	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	0.177	3.265	0.123	3.441	0.078	3.603	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	0.220	3.159	0.160	3.335	0.111	3.496	0.020	3.642	—	—	—	—	—	—	—	—	—	—	—	—	—
20	0.263	3.063	0.200	3.231	0.145	3.395	0.100	3.542	0.063	3.676	—	—	—	—	—	—	—	—	—	—	—
21	0.307	2.976	0.240	3.131	0.182	3.300	0.112	3.448	0.091	3.583	0.053	3.705	—	—	—	—	—	—	—	—	—
22	0.349	2.897	0.281	3.057	0.220	3.211	0.166	3.358	0.120	3.493	0.083	3.619	0.032	3.731	—	—	—	—	—	—	—
23	0.391	2.826	0.322	2.979	0.259	3.128	0.202	3.272	0.153	3.409	0.110	3.535	0.076	3.650	0.018	3.753	—	—	—	—	—
24	0.431	2.761	0.362	2.908	0.297	3.053	0.239	3.193	0.186	3.327	0.141	3.454	0.101	3.572	0.070	3.678	0.044	3.773	—	—	
25	0.470	2.702	0.400	2.844	0.335	2.993	0.275	3.119	0.221	3.251	0.172	3.376	0.130	3.494	0.094	3.604	0.065	3.702	0.041	3.790	
26	0.509	2.649	0.418	2.781	0.373	2.919	0.312	3.051	0.236	3.179	0.205	3.303	0.160	3.420	0.120	3.531	0.087	3.632	0.060	3.724	
27	0.544	2.600	0.475	2.730	0.409	2.859	0.348	2.987	0.291	3.112	0.238	3.233	0.191	3.349	0.149	3.460	0.112	3.563	0.081	3.658	
28	0.578	2.555	0.510	2.680	0.445	2.805	0.383	2.928	0.325	3.050	0.271	3.168	0.222	3.283	0.178	3.392	0.144	3.495	0.104	3.592	
29	0.612	2.515	0.541	2.631	0.477	2.755	0.418	2.874	0.359	2.992	0.305	3.107	0.254	3.219	0.208	3.327	0.166	3.431	0.129	3.528	
30	0.643	2.477	0.577	2.592	0.512	2.708	0.451	2.823	0.392	2.937	0.337	3.050	0.286	3.160	0.248	3.266	0.193	3.368	0.156	3.465	
31	0.674	2.443	0.608	2.553	0.543	2.665	0.484	2.776	0.425	2.887	0.370	2.996	0.317	3.103	0.269	3.258	0.224	3.309	0.183	3.406	
32	0.703	2.411	0.638	2.517	0.576	2.625	0.515	2.733	0.457	2.840	0.401	2.946	0.347	3.050	0.299	3.153	0.253	3.252	0.211	3.348	
33	0.731	2.382	0.668	2.434	0.606	2.588	0.546	2.692	0.488	2.796	0.432	2.899	0.379	3.000	0.329	3.100	0.283	3.198	0.239	3.293	
34	0.758	2.355	0.695	2.454	0.634	2.554	0.575	2.654	0.513	2.751	0.462	2.851	0.409	2.954	0.359	3.051	0.312	3.147	0.267	3.240	
35	0.783	2.330	0.722	2.425	0.662	2.521	0.604	2.619	0.547	2.716	0.492	2.813	0.433	2.910	0.388	3.005	0.340	3.099	0.295	3.190	
36	0.808	2.306	0.748	2.398	0.689	2.492	0.631	2.586	0.575	2.680	0.520	2.771	0.467	2.868	0.417	2.961	0.369	3.053	0.323	3.142	
37	0.831	2.285	0.772	2.374	0.714	2.464	0.657	2.555	0.602	2.646	0.548	2.739	0.495	2.829	0.445	2.920	0.397	3.009	0.351	3.097	
38	0.854	2.265	0.796	2.351	0.739	2.439	0.683	2.526	0.628	2.614	0.573	2.703	0.522	2.792	0.472	2.880	0.424	2.968	0.378	3.054	
39	0.875	2.246	0.819	2.329	0.763	2.413	0.707	2.499	0.653	2.593	0.600	2.671	0.549	2.757	0.499	2.843	0.451	2.929	0.404	3.013	
40	0.896	2.228	0.810	2.309	0.785	2.391	0.731	2.473	0.678	2.557	0.626	2.641	0.573	2.721	0.525	2.808	0.477	2.892	0.410	2.974	
45	0.988	2.156	0.918	2.225	0.887	2.296	0.838	2.367	0.788	2.439	0.740	2.512	0.692	2.586	0.644	2.659	0.598	2.733	0.553	2.807	
50	1.064	2.103	1.019	2.163	0.973	2.225	0.927	2.297	0.882	2.350	0.836	2.414	0.792	2.479	0.747	2.544	0.703	2.610	0.660	2.675	
55	1.129	2.062	1.087	2.116	1.043	2.170	1.003	2.225	0.961	2.281	0.912	2.333	0.877	2.396	0.836	2.454	0.793	2.512	0.751	2.571	
60	1.184	2.031	1.145	2.079	1.106	2.127	1.068	2.177	1.029	2.227	0.990	2.278	0.951	2.330	0.913	2.382	0.874	2.444	0.836	2.487	
65	1.231	2.006	1.195	2.049	1.160	2.093	1.124	2.138	1.088	2.181	1.052	2.229	1.016	2.276	0.980	2.323	0.944	2.371	0.908	2.419	
70	1.272	1.986	1.239	2.026	1.206	2.066	1.172	2.106	1.139	2.148	1.103	2.189	1.072	2.232	1.038	2.275	1.005	2.318	0.971	2.362	
75	1.308	1.970	1.277	2.006	1.247	2.043	1.215	2.080	1.184	2.118	1.153	2.156	1.121	2.193	1.090	2.235	1.058	2.275	1.027	2.315	
80	1.340	1.957	1.311	1.991	1.283	2.024	1.253	2.059	1.224	2.091	1.195	2.129	1.165	2.165	1.136	2.201	1.106	2.238	1.076	2.275	
85	1.369	1.946	1.342	1.977	1.315	2.009	1.287	2.040	1.260	2.073	1.232	2.105	1.203	2.139	1.177	2.172	1.149	2.206	1.121	2.241	
90	1.393	1.937	1.369	1.966	1.344	1.925	1.318	2.025	1.292	2.055	1.266	2.085	1.240	2.116	1.213	2.148	1.137	2.179	1.160	2.211	
95	1.418	1.929	1.391	1.956	1.320	1.934	1.343	2.012	1.321	2.040	1.296	2.068	1.271	2.097	1.247	2.126	1.222	2.156	1.197	2.186	
100	1.439	1.923	1.416	1.948	1.393	1.974	1.371	2.000	1.347	2.026	1.324	2.053	1.301	2.080	1.277	2.108	1.253	2.135	1.229	2.161	
150	1.579	1.892	1.561	1.908	1.550	1.924	1.513	1.910	1.512	1.956	1.504	1.972	1.499	1.989	1.474	2.006	1.458	2.023	1.443	2.040	
200	1.654	1.885	1.643	1.896	1.612	1.908	1.621	1.919	1.610	1.931	1.599	1.943	1.553	1.958	1.526	1.967	1.565	1.979	1.554	1.991	

Note: n = number of observations, k' = number of explanatory variables excluding the constant term.

Source: This table is an extension of the original Durbin-Watson table and is reproduced from S. E. Savin and K. F. White, "The Durbin-Watson Test for Serial Correlation with Extreme Small Samples or Many Regressors," *Econometrica*, vol. 43, November 1975, pp. 195-206, and as corrected by R. W. Farebrother, *Econometrica*, vol. 48, September 1980, p. 1354. Reprinted by permission of the Econometric Society.

EXAMPLE 1

If $n = 40$ and $k' = 4$, $d_L = 1.285$ and $d_U = 1.721$. If a computed d value is less than 1.285, there is evidence of positive first-order serial correlation; if it is greater than 1.721, there is no evidence of positive first-order serial correlation; but if d lies between the lower and the upper limit, there is inconclusive evidence regarding the presence or absence of positive first-order serial correlation.

Appendix D Statistical Tables

Durbin-Watson d Statistic: Significance Points of d_L and d_U at 0.01 Level of Significance

n	$K=1$		$K=2$		$K=3$		$K=4$		$K=5$		$K=6$		$K=7$		$K=8$		$K=9$		$K=10$		
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U									
6	0.390	1.142	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
7	0.435	1.036	0.294	1.676	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
8	0.497	1.003	0.345	1.489	0.229	2.102	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
9	0.554	0.998	0.408	1.389	0.279	1.875	0.183	2.433	—	—	—	—	—	—	—	—	—	—	—	—	—
10	0.604	1.001	0.466	1.333	0.340	1.733	0.230	2.193	0.150	2.690	—	—	—	—	—	—	—	—	—	—	—
11	0.653	1.010	0.519	1.297	0.396	1.640	0.286	2.030	0.193	2.453	0.124	2.892	—	—	—	—	—	—	—	—	—
12	0.697	1.023	0.569	1.274	0.449	1.575	0.339	1.913	0.244	2.280	0.164	2.665	0.105	3.053	—	—	—	—	—	—	—
13	0.738	1.038	0.616	1.261	0.499	1.526	0.391	1.924	0.214	2.150	0.211	2.490	0.140	2.838	0.090	3.182	—	—	—	—	—
14	0.776	1.054	0.660	1.254	0.547	1.490	0.411	1.757	0.343	2.049	0.2354	2.354	0.183	2.667	0.122	2.981	0.078	3.287	—	—	
15	0.811	1.070	0.700	1.252	0.591	1.464	0.488	1.704	0.391	1.967	0.303	2.244	0.226	2.530	0.161	2.917	0.107	3.101	0.068	3.374	
16	0.844	1.086	0.737	1.252	0.633	1.446	0.532	1.663	0.437	1.900	0.349	2.153	0.269	2.416	0.200	2.681	0.142	2.944	0.094	3.201	
17	0.874	1.102	0.772	1.255	0.672	1.432	0.574	1.630	0.480	1.847	0.393	2.078	0.313	2.319	0.241	2.566	0.179	2.811	0.127	3.053	
18	0.902	1.118	0.805	1.259	0.709	1.422	0.613	1.604	0.522	1.803	0.435	2.015	0.355	2.238	0.252	2.467	0.216	2.697	0.160	2.925	
19	0.928	1.132	0.835	1.265	0.742	1.415	0.650	1.584	0.561	1.767	0.476	1.963	0.396	2.169	0.322	2.381	0.255	2.597	0.196	2.813	
20	0.952	1.147	0.863	1.271	0.773	1.411	0.685	1.567	0.598	1.737	0.515	1.918	0.436	2.110	0.362	2.308	0.294	2.510	0.232	2.714	
21	0.975	1.161	0.890	1.277	0.803	1.408	0.718	1.554	0.633	1.712	0.552	1.881	0.474	2.059	0.400	2.244	0.331	2.434	0.268	2.625	
22	0.997	1.174	0.914	1.284	0.831	1.407	0.748	1.543	0.667	1.691	0.587	1.849	0.510	2.015	0.437	2.188	0.368	2.367	0.304	2.548	
23	1.018	1.197	0.938	1.291	0.858	1.407	0.777	1.534	0.698	1.673	0.620	1.821	0.545	1.977	0.473	2.140	0.404	2.308	0.340	2.479	
24	1.037	1.199	0.960	1.298	0.882	1.407	0.805	1.528	0.728	1.658	0.652	1.797	0.578	1.944	0.507	2.097	0.439	2.255	0.375	2.417	
25	1.055	1.211	0.981	1.305	0.906	1.409	0.831	1.523	0.756	1.645	0.682	1.776	0.610	1.915	0.540	2.059	0.473	2.209	0.409	2.362	
26	1.072	1.222	1.001	1.312	0.928	1.411	0.855	1.518	0.783	1.635	0.711	1.759	0.640	1.839	0.572	2.026	0.505	2.168	0.441	2.313	
27	1.089	1.233	1.019	1.319	0.949	1.413	0.873	1.515	0.808	1.626	0.738	1.743	0.669	1.867	0.602	1.997	0.536	2.131	0.473	2.269	
28	1.104	1.244	1.037	1.325	0.969	1.415	0.900	1.513	0.832	1.618	0.764	1.729	0.696	1.847	0.630	1.970	0.566	2.098	0.504	2.229	
29	1.119	1.254	1.054	1.332	0.988	1.418	0.921	1.512	0.855	1.611	0.788	1.718	0.723	1.830	0.658	1.947	0.595	2.068	0.533	2.193	
30	1.133	1.263	1.070	1.339	1.006	1.421	0.941	1.511	0.877	1.606	0.812	1.707	0.748	1.814	0.684	1.925	0.622	2.041	0.562	2.160	
31	1.147	1.273	1.085	1.345	1.023	1.425	0.960	1.510	0.907	1.601	0.834	1.698	0.772	1.800	0.710	1.908	0.649	2.017	0.589	2.131	
32	1.160	1.282	1.100	1.352	1.040	1.428	0.979	1.510	0.917	1.597	0.856	1.690	0.794	1.788	0.734	1.889	0.674	1.995	0.615	2.104	
33	1.172	1.291	1.114	1.358	1.055	1.432	0.996	1.510	0.936	1.594	0.876	1.683	0.816	1.776	0.757	1.874	0.698	1.975	0.641	2.080	
34	1.184	1.299	1.128	1.364	1.070	1.435	1.012	1.511	0.954	1.591	0.896	1.677	0.837	1.766	0.779	1.860	0.722	1.957	0.665	2.057	
35	1.195	1.307	1.140	1.370	1.085	1.439	1.028	1.512	0.971	1.589	0.914	1.671	0.857	1.757	0.800	1.847	0.744	1.940	0.689	2.037	
36	1.206	1.315	1.153	1.376	1.098	1.442	1.043	1.513	0.988	1.588	0.932	1.666	0.877	1.749	0.821	1.836	0.766	1.925	0.711	2.018	
37	1.217	1.323	1.165	1.382	1.112	1.446	1.058	1.514	1.004	1.586	0.950	1.662	0.895	1.742	0.841	1.825	0.787	1.911	0.733	2.001	
38	1.227	1.330	1.176	1.388	1.124	1.449	1.072	1.515	1.019	1.585	0.966	1.658	0.913	1.735	0.860	1.816	0.807	1.899	0.754	1.985	
39	1.237	1.337	1.187	1.393	1.137	1.453	1.085	1.517	1.034	1.584	0.992	1.655	0.930	1.729	0.878	1.807	0.826	1.887	0.774	1.970	
40	1.246	1.344	1.198	1.398	1.148	1.457	1.098	1.518	1.048	1.584	0.997	1.652	0.946	1.724	0.895	1.799	0.844	1.876	0.749	1.956	
45	1.288	1.376	1.245	1.423	1.201	1.474	1.156	1.528	1.111	1.584	1.065	1.643	1.019	1.704	0.974	1.768	0.927	1.834	0.881	1.902	
50	1.324	1.403	1.285	1.446	1.245	1.491	1.205	1.538	1.164	1.587	1.123	1.639	1.081	1.692	1.039	1.748	0.997	1.805	0.955	1.864	
55	1.356	1.427	1.320	1.466	1.284	1.506	1.247	1.548	1.209	1.592	1.172	1.638	1.134	1.683	1.095	1.734	1.057	1.785	1.018	1.837	
60	1.383	1.449	1.350	1.484	1.317	1.520	1.283	1.558	1.249	1.598	1.214	1.639	1.179	1.682	1.144	1.726	1.108	1.771	1.072	1.817	
65	1.407	1.463	1.377	1.500	1.346	1.534	1.315	1.568	1.283	1.604	1.251	1.642	1.218	1.680	1.186	1.720	1.153	1.761	1.120	1.802	
70	1.429	1.485	1.400	1.515	1.372	1.546	1.343	1.578	1.313	1.611	1.283	1.645	1.253	1.680	1.223	1.716	1.192	1.754	1.162	1.792	
75	1.448	1.501	1.422	1.529	1.395	1.557	1.368	1.587	1.340	1.617	1.313	1.649	1.284	1.682	1.256	1.714	1.227	1.748	1.199	1.783	
80	1.466	1.515	1.441	1.541	1.416	1.568	1.390	1.595	1.364	1.624	1.338	1.653	1.312	1.683	1.295	1.714	1.245	1.732	1.277	1.777	
85	1.482	1.528	1.458	1.553	1.435	1.578	1.411	1.603	1.386	1.630	1.362	1.657	1.337	1.685	1.312	1.714	1.287	1.743	1.262	1.773	
90	1.495	1.540	1.474	1.563	1.452	1.587	1.429	1.611	1.406	1.636	1.383	1.661	1.360	1.687	1.336	1.714	1.312	1.741	1.288	1.769	
95	1.510	1.552	1.489	1.573	1.468	1.596	1.446	1.618	1.425	1.642	1.403	1.666	1.381	1.690	1.358	1.715	1.336	1.741	1.313	1.767	
100	1.522	1.562	1.503	1.583	1.482	1.604	1.462	1.625	1.441	1.647	1.421	1.670	1.400	1.693	1.378	1.717	1.357	1.741	1.335	1.765	
150	1.611	1.637	1.598	1.651	1.584	1.665	1.571	1.679	1.557	1.693	1.543	1.708	1.530	1.722	1.515	1.737	1.501	1.752	1.486	1.767	
200	1.664	1.684	1.653	1.693	1.643	1.704	1.633	1.715	1.623	1.725	1.613	1.735	1.603	1.746</							

n	k' = 11		k' = 12		k' = 13		k' = 14		k' = 15		k' = 16		k' = 17		k' = 18		k' = 19		k' = 20		
	d ₁	d ₂																			
16	0.360	3.446	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	0.084	3.236	0.053	3.506	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
18	0.113	3.146	0.073	3.358	0.047	3.357	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	0.145	3.023	0.102	3.227	0.067	3.420	0.043	3.601	—	—	—	—	—	—	—	—	—	—	—	—	—
20	0.178	2.914	0.131	3.109	0.092	3.297	0.061	3.474	0.038	3.639	—	—	—	—	—	—	—	—	—	—	—
21	0.212	2.817	0.162	3.004	0.119	3.185	0.084	3.358	0.055	3.521	0.035	3.671	—	—	—	—	—	—	—	—	—
22	0.246	2.729	0.194	2.909	0.149	3.084	0.109	3.252	0.077	3.412	0.050	3.562	0.032	3.703	—	—	—	—	—	—	—
23	0.281	2.651	0.227	2.822	0.178	2.991	0.136	3.155	0.100	3.311	0.070	3.459	0.046	3.597	0.029	3.723	—	—	—	—	—
24	0.315	2.580	0.260	2.741	0.209	2.906	0.165	3.063	0.125	3.213	0.092	3.363	0.065	3.501	0.043	3.629	0.027	3.747	—	—	
25	0.343	2.517	0.292	2.674	0.241	2.829	0.194	2.982	0.152	3.131	0.115	3.274	0.085	3.410	0.060	3.538	0.037	3.657	0.025	3.766	
26	0.381	2.460	0.324	2.610	0.272	2.758	0.224	2.906	0.183	3.050	0.141	3.191	0.107	3.320	0.079	3.452	0.055	3.572	0.036	3.682	
27	0.413	2.429	0.336	2.552	0.303	2.691	0.253	2.830	0.208	2.976	0.167	3.113	0.131	3.245	0.100	3.371	0.073	3.490	0.051	3.602	
28	0.444	2.363	0.387	2.499	0.333	2.615	0.283	2.772	0.237	2.907	0.194	3.042	0.156	3.169	0.122	3.294	0.093	3.412	0.068	3.524	
29	0.474	2.321	0.417	2.451	0.363	2.582	0.313	2.713	0.266	2.813	0.222	2.972	0.182	3.099	0.146	3.220	0.114	3.338	0.087	3.450	
30	0.503	2.243	0.447	2.407	0.393	2.533	0.342	2.659	0.294	2.783	0.249	3.007	0.208	3.032	0.171	3.152	0.137	3.267	0.107	3.379	
31	0.531	2.243	0.475	2.367	0.422	2.487	0.371	2.609	0.322	2.730	0.277	2.831	0.234	2.972	0.195	3.087	0.160	3.201	0.128	3.311	
32	0.558	2.216	0.503	2.330	0.450	2.446	0.399	2.563	0.350	2.680	0.304	2.797	0.261	2.912	0.221	3.026	0.184	3.137	0.151	3.246	
33	0.585	2.187	0.530	2.296	0.477	2.408	0.426	2.520	0.377	2.633	0.331	2.746	0.287	2.858	0.246	2.959	0.209	3.078	0.174	3.184	
34	0.610	2.150	0.556	2.266	0.503	2.373	0.452	2.481	0.404	2.577	0.357	2.699	0.313	2.808	0.272	2.915	0.233	3.022	0.197	3.126	
35	0.634	2.136	0.581	2.237	0.529	2.340	0.479	2.444	0.430	2.550	0.383	2.655	0.339	2.761	0.297	2.865	0.257	2.969	0.221	3.071	
36	0.658	2.113	0.605	2.210	0.554	2.310	0.504	2.413	0.455	2.512	0.409	2.614	0.364	2.717	0.322	2.813	0.282	2.919	0.244	3.019	
37	0.680	2.092	0.628	2.186	0.578	2.282	0.528	2.379	0.483	2.477	0.434	2.576	0.389	2.673	0.347	2.774	0.306	2.872	0.268	2.961	
38	0.702	2.073	0.651	2.164	0.601	2.256	0.552	2.350	0.504	2.443	0.458	2.540	0.414	2.617	0.371	2.733	0.333	2.823	0.291	2.923	
39	0.723	2.055	0.673	2.143	0.623	2.232	0.575	2.323	0.523	2.411	0.482	2.507	0.438	2.600	0.395	2.694	0.354	2.787	0.315	2.879	
40	0.741	2.039	0.694	2.123	0.615	2.210	0.597	2.297	0.555	2.386	0.505	2.476	0.461	2.566	0.418	2.657	0.377	2.749	0.338	2.833	
45	0.835	1.972	0.790	2.044	0.714	2.113	0.700	2.193	0.655	2.267	0.612	2.346	0.570	2.421	0.539	2.503	0.493	2.582	0.448	2.661	
50	0.913	1.925	0.871	1.937	0.829	2.051	0.787	2.116	0.746	2.132	0.705	2.230	0.665	2.313	0.625	2.337	0.586	2.456	0.548	2.526	
55	0.979	1.891	0.940	1.945	0.902	2.002	0.863	2.059	0.825	2.117	0.786	2.176	0.748	2.237	0.711	2.298	0.574	2.359	0.535	2.421	
60	1.037	1.863	1.001	1.914	0.965	1.964	0.929	2.015	0.893	2.067	0.857	2.123	0.822	2.173	0.789	2.227	0.751	2.293	0.716	2.339	
65	1.087	1.845	1.053	1.889	1.020	1.934	0.986	1.930	0.953	2.027	0.919	2.075	0.886	2.123	0.852	2.172	0.819	2.221	0.786	2.272	
70	1.131	1.831	1.099	1.870	1.068	1.911	1.037	1.951	1.003	1.935	0.974	2.034	0.943	2.082	0.911	2.127	0.880	2.172	0.849	2.217	
75	1.170	1.819	1.141	1.856	1.111	1.893	1.082	1.931	1.052	1.970	1.023	2.009	0.993	2.049	0.964	2.090	0.934	2.131	0.905	2.172	
80	1.205	1.810	1.177	1.844	1.150	1.878	1.122	1.913	1.094	1.949	1.066	1.934	1.049	2.022	1.013	2.059	0.983	2.097	0.955	2.113	
85	1.236	1.803	1.210	1.834	1.184	1.866	1.158	1.893	1.132	1.931	1.106	1.965	1.080	1.929	1.053	2.033	1.027	2.068	1.000	2.104	
90	1.264	1.798	1.240	1.827	1.215	1.856	1.191	1.886	1.168	1.917	1.141	1.948	1.116	1.972	1.091	2.012	1.066	2.044	1.041	2.077	
95	1.290	1.793	1.267	1.821	1.214	1.843	1.221	1.876	1.197	1.935	1.171	1.934	1.150	1.953	1.126	1.993	1.102	2.023	1.079	2.054	
100	1.314	1.790	1.292	1.816	1.270	1.841	1.248	1.868	1.223	1.875	1.203	1.922	1.181	1.949	1.158	1.977	1.136	2.006	1.113	2.034	
150	1.473	1.783	1.458	1.799	1.444	1.814	1.429	1.830	1.424	1.847	1.400	1.863	1.385	1.880	1.370	1.897	1.355	1.913	1.340	1.931	
200	1.561	1.791	1.550	1.801	1.539	1.813	1.528	1.824	1.518	1.836	1.507	1.847	1.495	1.860	1.484	1.871	1.474	1.883	1.462	1.896	

Note: n = number of observations.

k' = number of explanatory variables excluding the constant term.

Source: Savin and White, op. cit., by permission of the Econometric Society.