Stockholm University

STOCKHOLM UNIVERSITY
Department of Statistics
Spring 2019, period A-B
Andriy Andreev (examiner)
Ulf Högnäs

FINANCIAL STATISTICS

2019-04-25

Time:	15.00 - 20.00
Place:	*Värtasalen*
Approved aid:	Hand-held calculator with no stored text, data or formulas
Provided aid:	*Formula Sheet and Probability Distribution Tables*, returned after the exam

- **Problems 1 – 4: MULTIPLE CHOICE QUESTIONS – max 32 points**
  - A total of three multiple choice questions with five alternative answers per question one of which is the correct answer. Mark your answers on the attached **answer form**.
  - Marking more than one alternative will result in zero points for that question.
  - Written solutions are <u>not</u> required to be submitted but if submitted, they might be used to evaluate the extent of the mistake in the final answer: that is done on case-by-case basis and decided by the examiner; only your answers on the answer form are guaranteed to be considered in the assessment and final grading.
- **Problems 5 – 6: COMPLETE WRITTEN SOLUTIONS – max 28 points**
  - Use only the provided **answer sheets** when submitting your solutions and answers.
  - For full marks, clear, comprehensive and well-motivated solutions are required. Unclear and un-explained solutions may result in point deductions even if the final answer is correct.
  - Check your calculations and solutions before submitting. Careless mistakes may result in unnecessary point deductions.
- The maximum number of points is stated for each question. The maximum total number of points is 32 + 28 = 60. At least 30 points is required to pass (grades A-E). The grading scale is as follows:

  A: 54 – 60 points
  B: 48 – 53 points
  C: 42 – 47 points
  D: 36 – 41 points
  E: 30 – 35 points
  Fx: 24 – 29 points
  F: 0 – 23 points
- NOTE! Fx and F are failing grades that require re-examination. Students who receive the grade Fx or F <u>cannot</u> supplement for a higher grade.
- Outlines of solutions will be posted on Mondo within several days after the exam.

GOOD LUCK!

1. (Essay Type Question) (2p + 3p + 2p +3p = 10 points) (Correlation, mean and variance)

You may choose between two corporate bonds (issued by the companies A and B), which have the same maturity, five years. Each company runs the risk of bankruptcy. If the company goes bankrupt before the end of five years, the bond is worthless and you lose your money. Let

X = net return on investment, company A's bond
Y = net return on investment, company B's bond

If the company does not go bankrupt, A pays the investment amount plus 10% interest, B pays the investment plus 20% interest. The risk that company A goes bankrupt is 3% and the corresponding risk of company B is 5%. The risk that *both* A and B go bankrupt before the end of five years is 2%.

We summarize the the joint distribution of A and B in a table:

	Y = -1	Y = 0,20	
X = -1	0,02	0,01	0,03
X = 0,10	0,03	0,94	0,97
	0,05	0,95	1

1. (2 points) What is the expected return on investment from A and B (what is $\mathbf{E}[X]$ and $\mathbf{E}[Y]$)? Report the expected profit in per cents.

2. (3 points) Calculate the correlation between the two investments (calculate $\mathbf{Corr}(X, Y)$).

3. (2 points) If you invest 5 000 SEK in each bond, what is the expected value and variance of your portfolio after five years? Tip: Let W = 5000*X+5000*Y and find $\mathbf{E}[W]$ and $\mathbf{Var}(W)$.

4. (3 points) Suppose that you invest a total of 10 000 SEK in a portfolio consisting of the two bonds. How much should you invest in each bond if you want to minimize the variance? No short selling is allowed.

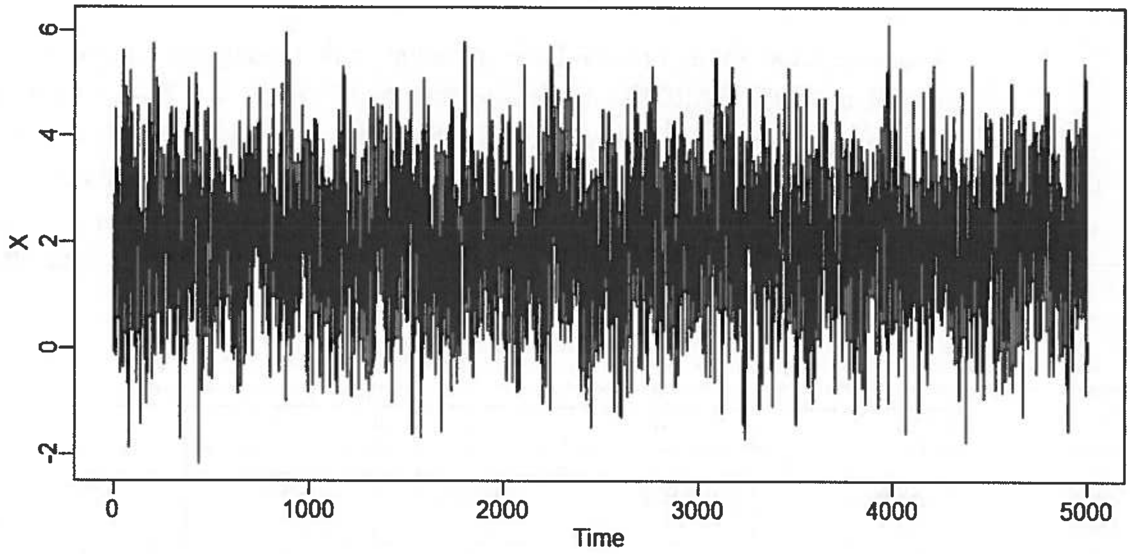
2. (Essay type Question) (6p +2p +2p = 10 points) (MA)

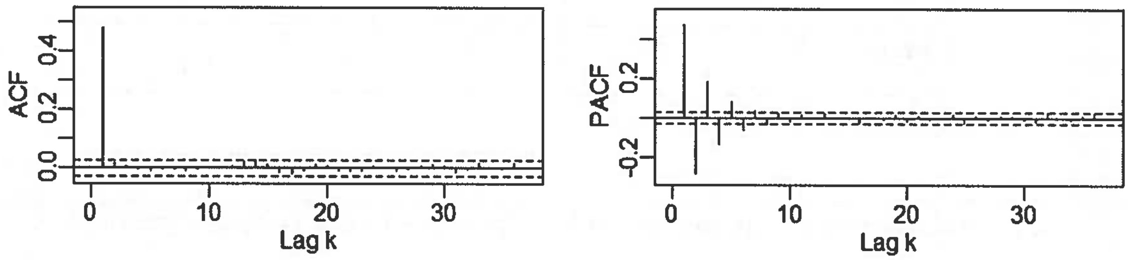
1. (6 points) Derive the autocorrelation function for the following MA(2) process $Z_t = \varepsilon_t - 1.4\varepsilon_{t-1} + 0.6\varepsilon_{t-2}$. Assume that Z_t is stationary.

2. (2 points)What is the correlation between Z_t and Z_{t-1} for the model $Z_t = 0.4 * Z_{t-1} + \varepsilon_t$? Repeat and report your calculations for the model $Z_t = (-0.4) * Z_{t-1} + \varepsilon_t$.

3. (2 points) Calculate the correlation between Z_t and Z_{t-2} for both models stated above in sub-question 2.

3. (Multiple Choice type question) (4p + 6p =10 points) (ARMA + random walk)





1. (4 points) The picture above shows a stationary time series with 5000 observations its ACF and PACF plots. According to lecture and the readings, which of the following models is most likely to fit the data?

   a) $X_t = 2 + 0.7\,X_{t-1} + \varepsilon_t$
   b) $X_t = 2 - 0.7\,X_{t-1} + \varepsilon_t$
   c) $X_t = 2 + \varepsilon_t + 0.7\,\varepsilon_{t-1}$
   d) $X_t = 2 + \varepsilon_t - 0.7\,\varepsilon_{t-1}$
   e) $X_t = 2 + 0.7\,X_{t-1} + \varepsilon_t - 0.7\,\varepsilon_{t-1}$

2. (6 points) Let us assume that we choose a random walk with trend a_0=11 and an error term that is normal with mean zero to model an exchange rate. Let us further assume that we have an accurate forecast of variances of error terms at time *(t+1)* and *(t+2),* that are *9* and *7* respectively. Assuming that the exchange rate at time *t* is X_t *=200,* calculate the probability that X_{t+2} increases by *15%* compared to X_t.

a) 0.022

b) 0.019

c) 0.015

d) 0.025

e) 0.023

4. (Essay Type Question) (8 Points) (GARCH)

A time series X_t for an index follows a random walk model with a trend. The variance is modelled using a GARCH(1,1) model with parameters $\alpha_0 = 0.2$, $\alpha_1 = 0.8$, and $\beta_1 = 0.3$. The time series starts at time $t = 0$ when the index is set to $X_0 = 100$. (i.e. the variance at $t = 0$ is $h_0 = 0$, since the index is fixed in the beginning. We assume that the variance at time $t = 1$ is $h_1 = 1$. The index values at times $t = 1, 2, 3$ can be found in the table below. **Please, fill the missing places in the table for the ε_t and for the h_t.** Note, that you are expected to calculate h_4 but not ε_0, ε_4.

Time, t	0	1	2	3	4
Index, X_t	100	101.2	98.3	98.2	
Change, ε_t		?	?	?	
Variance, h_t	0	1	?	?	?

5. (Multiple Choice type question) (3p + 4p + 4p = 11 points) (log-regression)

A (new) group of researchers studied factors related to Low Birth Weight (LBW), defined as less than 2500 g at birth, among newborn babies. They proposed the following logistic regression model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 \cdot x_1 \cdot x_2$$

y = The log-odds of a baby being LBW
x_1 = Mother smoked during pregnancy (1 = yes, 0 = no)
x_2 = Baby is a twin (1 = yes, 0 = no)

The third term is an interaction term. Notice that $x_1 \cdot x_2 = 1$ if and only if the mother smoked during pregnancy and the baby is a twin. The scientists collected data from hospitals, from a total of 150 000 newborn babies. You can see the output of their analysis below.

```
Call:
glm(formula = low ~ smoker + twins + twins.smoker, family = binomial(link = "lo
git"),
    data = twins.data)

Deviance Residuals:
   Min       1Q   Median       3Q      Max
-1.456   -0.408   -0.408   -0.408   2.248

Coefficients:
             Estimate Std. Error  z value Pr(>|z|)
(Intercept)  -2.44428    0.01040 -235.109   <2e-16 ***
smoker        0.68477    0.02597   26.365   <2e-16 ***
twins         2.68587    0.03181   84.446   <2e-16 ***
```

```
twins.smoker -0.29172    0.10135   -2.878    0.004 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 96331  on 144999  degrees of freedom
Residual deviance: 88687  on 144996  degrees of freedom
AIC: 88695

Number of Fisher Scoring iterations: 5
```

1) (3 points) Find the odds-ratio of the variable "smoker." Choose the value closest to your answer.

   a) 0.17
   b) 0.40
   c) 0.68
   d) 2.0
   e) 2.2

2) (4 points) Find the **probability** that a randomly selected baby is born with LBW, given that the mother **did not** smoke during pregnancy and given that the baby is **not** a twin. Choose the value closest to your answer.

   a) 0.070
   b) 0.076
   c) 0.080
   d) 0.087
   e) 0.089

3) (4 points) Find the **probability** that a randomly selected baby is born with LBW, given that the mother **did** smoke during pregnancy and given that the baby **is** a twin. Choose the value closest to your answer.

   a) 0.59
   b) 0.63
   c) 0.65
   d) 0.72
   e) 0.89

6. (Multiple Choice type question) (4p + 4p + 3p = 11 points) (log-regression)

The time series $\boldsymbol{X_t}$ describes Swedish GDP 1975-2016 (Source: Statistics Sweden); $\boldsymbol{Y_t}$ is the logarithm of Swedish GDP at time t, so $\boldsymbol{Y_t} = \mathbf{ln}\,(\boldsymbol{X_t})$ where ln is the natural logarithm. The series Y_t has been **differentiated two times** and the result is denoted $\boldsymbol{Z_t}$. The series Z_t has been analyzed as an ARIMA(2,2,1) model and you can find the output of that analysis on the next page, along with data for the years 2012-2016.

1. (4 points) The forecast of $\boldsymbol{Z}$ for 2017 is $\hat{Z}_{2017} = -0.02670$. **Find the forecast of $\boldsymbol{Z}$ for 2018**, i.e. find $\hat{Z}_{2018}$ according to the model. Choose the value that is closest to your answer.

   a) -0.083 b) -0.016 c) -0.0001 d) 0.0083 e) 0.016

2. (4 points) Which of the following describes the relationship between $\boldsymbol{Y}$ and $\boldsymbol{Z}$?

   a) $Z_t = Y_{t-1} - Y_{t-2}$

   b) $Z_t = Y_t - Y_{t-1} + Y_{t-2}$

   c) $Z_t = Y_t - Y_{t-1}$

   d) $Z_t = Y_t + Y_{t-1} - 2\,Y_{t-2}$

   e) $Z_t = Y_t - 2\,Y_{t-1} + Y_{t-2}$

3. (3 points) We want to formally test whether a time series is stationary, using a 5% level of significance. Which of the following statements is correct?

   a) We will conclude that the time series **is** stationary if the p-value from a suitable (Augmented) Dickey-Fuller test is less than 0.05.
   b) We will conclude that the time series is **not** stationary if the p-value from a suitable (Augmented) Dickey-Fuller test is less than 0.05.
   c) We will conclude that the time series is **not** stationary if the **unit-root** of the p-value from a suitable (Augmented) Dickey-Fuller test is less than 0.05.
   d) We will conlude that the time series **is** stationary if the p-value from a suitable Box-Ljung test is less than 0.05.
   e) We will conlude that the time series is **not** stationary if the p-value from a suitable Box-Ljung test is less than 0.05.

[Output and data on the next page]

ARIMA(2,2,1)-modell for ln(GDP):

```
Sample:  1975 - 2016                            Number of obs      =        42
                                                Wald chi2(3)       =     30.79
Log likelihood =   97.4724                      Prob > chi2        =    0.0000
------------------------------------------------------------------------------
             |                 OPG
  D2.logGDP  |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
logGDP       |
       _cons |  -.0021798   .0010395    -2.10   0.036    -.0042171   -.0001425
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |   .2186694   .1661418     1.32   0.188    -.1069625    .5443014
         L2. |  -.4658909    .173241    -2.69   0.007     -.805437   -.1263448
             |
          ma |
         L1. |  -.7230982   .1971705    -3.67   0.000    -1.109545   -.3366512
-------------+----------------------------------------------------------------
      /sigma |   .0233636   .0022514    10.38   0.000     .0189509    .0277762
------------------------------------------------------------------------------
Note: The test of the variance against zero is one sided, and the two-sided
      confidence interval is truncated at zero.
```

Year	GDP X_t	ln(GDP) Y_t	Z_t
2012	3684800	15,1197	-0,03038
2013	3769909	15,1426	0,015146
2014	3936840	15,1859	0,020493
2015	4199860	15,2506	0,021345
2016	4404802	15,2982	-0,01703

Department of Statistics

Stockholms universitet

Correction sheet

Date: 25/4 - 2019

Room: Värtasalen

Exam: Financial Statistics

Course: Financial Statistics

Anonymous code: 0005-Jsk

☒ I authorise the anonymous posting of my exam, in whole or in part, on the department homepage as a sample student answer.

NOTE! ALSO WRITE ON THE BACK OF THE ANSWER SHEET

Mark answered questions

1	2	3	4	5	6	7	8	9	Total number of pages
X	X	X	X	X	X				9
Teacher's notes									

BnKg

Points	Grade	Teacher's sign.

STOCKHOLM UNIVERSITY
Department of Statistics

Spring 2019 A-B

ANSWER FORM Exam – Financial Statistics 2019-04-25

Anonymous code: 0005 – JSK *(write clearly!)*

Mark your answers with a clear cross (X) in the corresponding boxes below.

NOTE! Only one cross per question. If more than one alternative has been marked, zero points will be awarded for that question.

NOTE! If, after checking your calculations properly, you are convinced that the correct answer is not included among the given alternatives, write your answer in the margin to the right and explain you reasoning on the back.

	A	B	C	D	E			
3.1				X	X	(4)	0	
3.2					X	(6)	6	/6
5.1				X		(3)	3	
5.2			X			(4)	4	
5.3			X			(4)	4	/11
6.1			X			(4)	4	
6.2					X	(4)	4	
6.3	X					(3)	3	/11

SU, DEPARTMENT OF STATISTICS

Room: VÄ   Anonymous code: 0005-JSK   Sheet number: 1

1.a) $E[X] = (-1 \cdot 0{,}03) + (0{,}1 \cdot 0{,}97) = 6{,}7\%$ or $0{,}067$

$E[Y] = (-1 \cdot 0{,}05) + (0{,}2 \cdot 0{,}95) = 14\%$ or $0{,}14$

Company A expected return: 6,7%

Company B expected return: 14%

(2)

1.2 $\text{Corr}(X,Y) = ?$

$$\rho_{xy} = \text{Corr}(X,Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}}$$

$\text{Var}(X) = (-1^2 \cdot 0{,}03) + (0{,}1^2 \cdot 0{,}97) - 0{,}067^2 = 0{,}035211$

test formula for $\text{Var}(X) = ((-1-0{,}067)^2 \cdot 0{,}03) + (0{,}1-0{,}067)^2 \cdot 0{,}97 = 0{,}035211$

$\text{Var}(Y) = (-1^2 \cdot 0{,}05) + (0{,}2^2 \cdot 0{,}95) - 0{,}14^2 = 0{,}0684$

test formula to verify: $((-1-0{,}14)^2 \cdot 0{,}05) + (0{,}2-0{,}14)^2 \cdot 0{,}95 = 0{,}0684$

$\text{Cov}(X,Y) = xy\,P(x,y) - M_x \cdot M_y$ → four possible outcomes, will split this in 4 steps then add them up

Case 1 $(-1,-1) = -1 \cdot -1 \cdot 0{,}02 = 0{,}02$

Case 2 $(0{,}1,-1) = -1 \cdot 0{,}1 \cdot 0{,}03 = -0{,}003$

Case 3 $(-1, 0{,}2) = -1 \cdot 0{,}2 \cdot 0{,}01 = -0{,}002$

Case 4 $(0{,}1, 0{,}2) = 0{,}1 \cdot 0{,}2 \cdot 0{,}94 = 0{,}0188$

$\mu_x \cdot \mu_y = 0{,}067 \cdot 0{,}14 = 0{,}00938$

$\text{Cov}(X,Y) = 0{,}02 - 0{,}003 - 0{,}002 + 0{,}0188 - 0{,}00938 = 0{,}02442$

SU, DEPARTMENT OF STATISTICS 0005-JSK

Room: VÄ Anonymous code: Sheet number: 2

1.2 Continue $Corr(x,y) = ?$

$$Corr(x,y) = \frac{0{,}02442}{\sqrt{0{,}035211 \cdot 0{,}0684}} = 0{,}4975 \approx 0{,}498$$

$Corr(x,y) = 0{,}498$

(3)

SU, DEPARTMENT OF STATISTICS 0005 – JSK

Room: VÄ Anonymous code: ______ Sheet number: 3

1.3 5000 SEK each bond

$E[W] = 5000 \cdot 0{,}067 + 5000 \cdot 0{,}14 = \underline{1035 \text{ SEK}}$

$Var[W] = 5000^2 \cdot 0{,}035211 + 5000^2 \cdot 0{,}0684 + 2 \cdot 5000 \cdot 5000 \cdot 0{,}02442$

$Var[W] = 880275 + 1710000 + 1221000$

$Var[W] = \underline{3811275}$

Expected Return: 1035 SEK

Variance: 3811275

2

Formulas used:

$E[W] = a \cdot E[X] + b \cdot E[Y]$

$Var[W] = a^2 \cdot Var(X) + b^2 \cdot var(Y) + 2ab\, cov(X,Y)$

SU, DEPARTMENT OF STATISTICS 0005-JSK

Room: VÄ **Anonymous code:** ______ **Sheet number:** 4

1.4 10 000 SEK invested

$a+b = 1$ (100%)

minimize: $a = \dfrac{Var(y) - cov(x,y)}{Var(x) + Var(y) - 2\,cov(x,y)}$

$a = \dfrac{0{,}0684 - 0{,}02442}{0{,}035211 + 0{,}0684 - 2 \cdot 0{,}02442} = \dfrac{0{,}04398}{0{,}103611 - 0{,}04884}$

$a \approx 0{,}80297 = 0{,}80$ (r). a

$a = 10\,000 \cdot 0{,}80(r) = 8029{,}79 \approx 8030$ in bond A

$b = 10\,000 - 8030 = 1970$ in Bond B

Invest: 8030 sek in bond A

1970 sek in bond B

3

/10

SU, DEPARTMENT OF STATISTICS

Room: VÄ Anonymous code: 0005-JSH Sheet number: 5

2.1 $Z_t = \varepsilon_t - 1{,}4\,\varepsilon_{t-1} + 0{,}6\,\varepsilon_{t-2}$ MA(2)

will start to use Z_{t-1} and Z_t as example

$$Corr(Z_t, Z_{t-1}) = \frac{cov(Z_t, Z_{t-1})}{\sqrt{Var(Z_t) \cdot Var(Z_{t-1})}}$$

an arima model that is stationary shows signs of homoscedastiсity which gives the following $Var(Z_t) = Var(Z_{t-1})$, puting this in the equation above gives =>

$$Corr(Z_t, Z_{t-1}) = \frac{cov(Z_t, Z_{t-1})}{Var(Z_t)}$$

since there are no covariance between different error-terms such as ε_t and ε_{t-1} we can neglect cov when calculating the variance. The assumption behind this is the iid of error terms and we assume that there are no autocorrelation

* $Var(Z_t) = a^2 + b^2 + c^2 \Rightarrow 1^2 + (-1{,}4^2) + 0{,}6^2 = \underline{3{,}32}$

since the assumption above about iid the error terms don't effect each other

$Z_t = \varepsilon_t - 1{,}4\,\varepsilon_{t-1} + 0{,}6\,\varepsilon_{t-2}$

$Z_{t-1} = \varepsilon_{t-1} - 1{,}4\,\varepsilon_{t-2} + 0{,}6\,\varepsilon_{t-3}$

We neglect the error terms which doesn't appear in both models

$cov(Z_t, Z_{t-1}) = 1 \cdot (-1{,}4)\,\varepsilon_{t-1} + 0{,}6 \cdot -1{,}4\,\varepsilon_{t-2} = -2{,}24$

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Room: VÄ Anonymous code: 0005-JSK Sheet number: 6

The variance doesnt change when we look at different correlations, for example (Z_t, Z_{t-1}) and (Z_t, Z_{t-2}) and the reason is due to the constant variance, the thing that changes and effect the correlation is the covariance

$Cov(Z_t, Z_{t-1}) = \varepsilon_t + (-1{,}4 \cdot 1\ \varepsilon_{t-1}) + (0{,}6 \cdot -1{,}4\ \varepsilon_{t-2}) + 0{,}6\varepsilon_{t-3}$

$Cov(Z_t, Z_{t-2}) = \varepsilon_t - 1{,}4\varepsilon_{t-1} + (0{,}6 \cdot 1\ \varepsilon_{t-2}) - 1{,}4\varepsilon_{t-3} + 0{,}6\varepsilon_{t-4}$

$Cov(Z_t, Z_{t-3}) = \varepsilon_t - 1{,}4\varepsilon_{t-1} + 0{,}6\varepsilon_{t-2} + \varepsilon_{t-3} - 1{,}4\varepsilon_{t-4} + 0{,}6\varepsilon_{t-5}$

In the last covariance, between Z_t and Z_{t-3} there are no error terms that are in the same time as any else, this leads to a covariance between Z_t and $Z_{t-3} = 0$ hence also results in a correlation between these two Z_t and $Z_{t-3} = 0$

$$Corr(Z_t, Z_{t-1}) = \frac{-1{,}4 \cdot 1 + 0{,}6 \cdot -1{,}4}{3{,}32} = -0{,}674$$

$$Corr(Z_t, Z_{t-2}) = \frac{0{,}6 \cdot 1}{3{,}32} = 0{,}18$$

$$corr(Z_t, Z_{t-3}) = \frac{0}{3{,}32} = 0$$

6

~~So~~ In a MA(2) we can see that $\rho_k = 0$ if $k > 2$

In a MA(1) it is $\rho_k = 0$ if $k > 1$

SU, DEPARTMENT OF STATISTICS

Room: VÄ Anonymous code: 0005-JSK Sheet number: 7

2.2 $Z_t = 0{,}4 \cdot Z_{t-1} + \varepsilon_t$ AR(1) correlation: $\rho_k = a^k$

$\rho_1 = 0{,}4^1 = 0{,}4$, in the formula above the correlation between Z_t and $Z_{t-1} = 0{,}4$ $\rho_{Z_t, Z_{t-1}} = 0{,}4$

$Z_t = -0{,}4 Z_{t-1} + \varepsilon_t$ AR(1)

$\rho_k = (-0{,}4)^1 = -0{,}4$, in the other formula with a negative coefficient the correlation is negative $-0{,}4$

$\rho_k = -0{,}4$ between Z_t and Z_{t-1}

If we look at a correlation two steps back (Z_t, Z_{t-2}) we will get the same correlation for these ~~bot~~ models, but with one step back we get different correlations.

2

2.3 $Z_t = 0{,}4 \cdot Z_{t-1} + \varepsilon_t$ AR(1)

$corr(Z_t, Z_{t-2}) \Rightarrow \rho_2 = a^2 = 0{,}4^2 = 0{,}16$

model 1: $corr(Z_t, Z_{t-2}) = 0{,}16$

$Z_t = -0{,}4 Z_{t-1} + \varepsilon_t$

$Corr(Z_t, Z_{t-2}) = (-0{,}4)^2 = 0{,}16$

model 2: $corr(Z_t, Z_{t-2}) = 0{,}16$

2

SU, DEPARTMENT OF STATISTICS

Room: VÄ Anonymous code: 0005-JSK Sheet number: 8

4.

time, t	0	1	2	3	4
Index, X_t	100	101,2	98,3	98,2	–
Change, ε_t	–	1,2	-2,9	-0,1	–
Variance, h_t	0	1	1,652	7,4236	2,435

Garch (1,1) – we dont need to explicitly know the trend because we already have the values

$X_t = a_0 + X_{t-1} + \varepsilon_t$

$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_1$.

$\varepsilon_1 = X_t - X_{t-1}$ ~~$= a_0$~~ $\Rightarrow 101,2 - 100 = 1,2$

$h_2 = 0,2 + 0,8 \cdot 1,2^2 + 0,3 \cdot 1 = \underline{1,652}$

$h_3 = 0,2 + 0,8 \cdot \varepsilon_2^2 + 0,3 \cdot h_2 \Rightarrow 0,2 + 0,8 \cdot (-2,9)^2 + 0,3 \cdot 1,652$

$\varepsilon_2 = X_2 - X_1 \Rightarrow 98,3 - 101,2 = -2,9$

$h_4 = 0,2 + 0,8\ \varepsilon_3^2 + 0,3 \cdot h_3 =$

$\varepsilon_3 = X_3 - X_2 \Rightarrow 98,2 - 98,3 = -0,1$

$h_3 = 7,4236$

$h_4 = 0,2 + 0,8 \cdot (-0,1)^2 + 0,3 \cdot 7,4236 = 2,435$

8