Explaining Quantifier Restriction: Reply to Ben-Yami

Dag Westerståhl
Stockholm University and University of Gothenburg

Natural language quantification is *restricted*, in the sense that the truth or falsity of a sentence like

(1) Most students came to the party

is independent of (a) how many non-students there were at the party, and (b) how many individuals in the universe of discourse are neither students nor party guest. That is, quantification is restricted to the set of students. Ben-Yami (2009) argues that the usual account of quantification in terms of generalized quantifier theory[1] has a "serious flaw" in that it does not *explain* this phenomenon, but merely treats it as a property that quantifiers interpreting natural language determiners happen to have. Instead, he proposes an alternative account, inspired by Geach, designed to provide the desired explanation. He also suggests that "the ability of competing theories to supply an explanation [of restriction] should be a criterion for deciding between them." (p. 309)

While I agree that restriction in this sense is a phenomenon we should explain, I don't see that Ben-Yami's suggestion improves the situation, nor that a choice between formal semantic accounts of quantification should be based solely on this question. But the issue raises points that may be worth getting clear about. In what follows I will briefly contrast the standard account with Ben-Yami's alternative and explain why I still think the former is preferable.

1 The standard account

In model theory, an *n-ary quantifier* can be identified with a class of structures of an n-ary monadic signature, usually taken to be closed under isomorphism (satisfying ISOM), or, equivalently, as a functional Q assigning to each universe M an n-ary relation Q_M between subsets of M. The standard account of how

[1] See Peters and Westerståhl (2006) for the most recent survey, and for explanation of all unexplained terminology here.

this notion is used in natural language semantics essentially boils down to the following:[2]

(DQ) a. Determiners are interpreted as binary quantifiers.

b. The truth conditions of a sentence of the form 'Det S are P' are $Q_M(A, B)$, where M is a universe, A is the extension of S in M, B the extension of P in M, and Q interprets Det.

Thus, for example, *all* is interpreted, on each M, as the subset relation on M, *some* as the relation of having non-empty intersection, (one reading of) *most* as, for $A, B \subseteq M$,

$$most_M(A, B) \text{ iff } |A \cap B| > |A - B|,$$

where $|X|$ is the number of elements in X, and (one reading of) *Mary's* as

$$Mary's_M(A, B) \text{ iff } \emptyset \neq A \cap \{b \in M : R(m, b)\} \subseteq B,$$

where m is Mary and R is a 'possessor relation'.[3]

Model theory conveniently extends the syntax of first-order logic (FO) to other quantifiers than $\forall$ and $\exists$, so that, for example, the claim $most_M(A, B)$ corresponds to the sentence

(2) $most\, x(Ax, Bx)$

being true in the model $\mathcal{M} = (M, A, B)$.[4] This gives us a language $FO(most)$ which extends the expressive power of FO by allowing quantification with *most* as well.

Now, we can define the property of being restricted. If Q is binary, define another binary quantifier Q^r as follows:

(3) $Q^r_M(A, B)$ iff $Q_A(A, A \cap B)$

Then we say that Q is *restricted* iff $Q = Q^r$. Clearly, *all*, *some*, *most*, and *Mary's*, as defined above are restricted. Indeed it seems to be the case that:

(RU) All interpretations of natural language determiners are restricted.

Here are two non-restricted quantifiers:

(4) a. $I_M(A, B)$ iff $|A| = |B|$ (the Härtig quantifier)

[2] I disregard in this note determiners taking more than one noun argument as well as uses of polyadic quantification in natural language semantics.

[3] I use the convention that the English word also names the quantifier. In the case of *most* one usually assumes that M is finite, so that it means *more than half*. The last example illustrates that the Isom requirement is sometimes dropped. One can discuss to what extent these interpretations give the right truth conditions, e.g. whether or not in English *all* and *most* lack but *Mary's* has existential import (as in the interpretations above), but these are not the issues under debate here.

[4] Using for convenience 'A','B' both as symbols for subsets of M and as unary predicate symbols in the formal language; similarly for '*most*'.

b. $Q^C_M(A,B)$ iff $|A \cap B| = |M - (A \cap B)|$

Historically, restrictedness was approached via the two independent properties of *conservativity* (Conserv), $Q_M(A,B) \Leftrightarrow Q_M(A, A \cap B)$, and *extension* (Ext), which applies to any quantifier and says that the part of the universe outside the union of the argument sets is irrelevant. The Härtig quantifier above satisfies Ext but not Conserv, whereas the inverse holds for Q^C. If (RU) is correct, no natural language determiner could denote either of these. Clearly, a binary quantifier is restricted iff it is Conserv and Ext.

So far we have, as Ben-Yami correctly notes, only identified the restricted quantifiers among the larger class of all binary quantifiers, but we have not explained *why* (RU) holds. It would be an exaggeration, however, to say that that the standard account has *nothing* to offer here. A starting-point for this account was that basic quantified sentences in (many) natural languages have the constituent structure $[[\text{Det N}]_{\text{NP}}\ \text{VP}]_{\text{S}}$. Thus, the *noun argument* (coming from the N), i.e. the first argument of the binary quantifier interpreting the Det, has a different syntactic role than the second, *verb argument* (coming from the VP). It is quite natural that this syntactic difference has semantic effect, and the standard account takes this effect to be precisely (RU).

It is true that the logical syntax (2) or, in general,

(5) $Qx(\varphi, \psi)$

does not mark this effect. But the standard GQ treatment is not wedded to that syntax. In fact, other formats are often used that distinguish the role of the two arguments, e.g.

(6) a. $(Q\hat{x}[\varphi])\eta$

b. $[Qx\colon \varphi]\psi$

The format (6a) is from Barwise and Cooper (1981), where $\hat{x}[\varphi]$ (together with unary predicate symbols) is a special kind of term in the formal language, a *set term*, which together with a *determiner* Q forms a *noun phrase*,[5] which in turn together with another set term η (that in the case of (5) would have the form $\hat{x}[\psi]$) forms a sentence (formula). (6b) is also a fairly common format. Furthermore, Barwise and Cooper as well as Keenan (e.g. Keenan and Stavi (1986)) treat (local) binary quantifiers not as binary relations between sets but as *functions* from sets (the verb argument) to unary (local) quantifiers (denoted by the NP), thus directly reflecting the syntactic form.

However, Ben-Yami points out that saying that the noun argument has a special role is a far cry from explaining why it is that *particular* role and not another role, or several roles. I think this is essentially correct. The question is, what should we expect from such an explanation?

[5] Barwise and Cooper call these *quantifiers*.

2 Ben-Yami's alternative

The key to Ben-Yami's proposal to explain why quantification is restricted is a different view of *nouns*, as they occur in the simple quantified sentences we are here concerned with: (a) they are *logical subject terms*, in contrast with the *predicate* (coming from the VP), and (b) they have *plural reference*. Beginning with the latter, the term 'students', in a use of 'Some students failed the exam' may refer to the students in a specific course. "Notice that it *does not* refer to *some* of these students, but to *all* of them." (p. 321)

Despite a venerable medieval tradition of speaking of reference in this way, I confess that I fail to see its deep significance. Suppose the relevant students are Tom, Bill, and Sue. The use of 'students' is said to refer to all of them, which presumably means to each of Tom, Bill, and Sue. This cannot be ordinary reference, since one cannot use 'students' to talk about only Bill, for example, even though the term is said to refer to him. Indeed, it is *plural* reference. However, these difficulties (of mine?) do not really matter here. Let the *extension* of this sort of use of a noun be the set of individuals it refers to. Then it is clear that only the extension matters for the truth conditions that Ben-Yami gives for sentences of this form. Similarly for the predicate: only its extension (the set of things falling under it) matters.

Now, Ben-Yami's truth conditions for a sentence 'Det S are P' are as follows. The logical subject noun S introduces the set of objects to be quantified over, i.e. its extension, say A. This is precisely the role of the subject. The extension of the predicate, say B, splits A into two pieces, $A \cap B$ and $A - B$. The determiner tells us how to quantify over A: in general it provides a relation between $A \cap B$ and $A - B$.

But this doesn't seem very different from what we already had. Let us be precise. What are the semantic objects that interpret determiners? In the standard account, they are binary quantifiers. Ben-Yami isn't explicit on this point, but a reasonable reconstruction is that on his account they are functionals q taking an arbitrary set A to a binary relation, say

$$q[A],$$

between subsets of A.[6] Let us call such functionals *binary quantifiers**. In effect, Ben-Yami proposes to replace (DQ) by

(DQ*)a. Determiners are interpreted as binary quantifiers*.

b. The truth conditions of a sentence of the form 'Det S are P' are $q[A](A \cap B, A - B)$, where A is the extension of S, B the extension of P, and q interprets Det.[7]

[6] Thus, $Q_M \subseteq P(M)^2$, and $q[A] \subseteq P(A)^2$.

[7] Ben-Yami (2009), p. 322:

> The quantified sentence can now make a claim either about the quantity of the items in the first sub-plurality (the S's that are P), the *intersective* quantifiers), or about their quantity in the second (S's that are not P, the *co-intersective*), or about the relation between these quantities (*proportionality*).

Cause (a) in (DQ*) is vitally important. It concerns the meaning of determiners, and this meaning is independent of the domain of quantification. The word 'some' doesn't mean one thing in 'some cats' and another thing in 'some students'. It means just one thing, and whether we take a standard approach to what that thing is or some alternate approach, we need to identify the semantic objects in question.

Now, apart from the use of quantifiers*, an obvious difference from (DQ) is that there is no mention of *universes* in (DQ*). This is no accident, but an essential part of Ben-Yami's proposal. Another difference is that restriction to the noun argument is built into the truth conditions. I will comment on the second point first, and then get back to universes.

3 Relation between the two accounts

First, note that although the definition of a quantifier* q allows arbitrary binary relations $q[A]$, in the proposed semantics $q[A]$ will only be used for *disjoint* arguments (in fact, for arguments that *partition* A). Next, we observe the following:

(7) Given a binary quantifier* q, define a binary quantifier $q^\dagger$, for any M and any $A, B \subseteq M$, by

$$q^\dagger_M(A, B) \Longleftrightarrow q[A](A \cap B, A - B)$$

Then $q^\dagger$ is restricted, and the truth conditions for 'Det S are P' are as in (DQ): $q^\dagger_M(A, B)$.

(8) Given any binary restricted quantifier Q, define a binary quantifier* Q^*, for any A and any $X, Y \subseteq A$, by

$$Q^*[A](X, Y) \Longleftrightarrow Q_A(X \cup Y, X)$$

Then the truth conditions for 'Det S are P' are as in (DQ*): $Q^*[A](A \cap B, A - B)$. Furthermore:

a. If Q is restricted, $(Q^*)^\dagger = Q$.

b. On disjoint arguments, $(q^\dagger)^*[A] = q[A]$.

These facts follow easily from the definitions. To give some examples of (8):

(9) If $X, Y \subseteq A$ and $X \cap Y = \emptyset$, then

a. $all^*[A](X, Y) \Longleftrightarrow Y = \emptyset$

b. $some^*[A](X, Y) \Longleftrightarrow X \neq \emptyset$

Apart from the mention of *quantities* here, which amounts to a restriction to the ISOM case (a restriction which Ben-Yami should avoid since he wants to treat noun phrases like 'my children' as well), and from the fact that there are other relations between two quantities than proportionality, this seems to be essentially (DQ*).

c. $most^*[A](X, Y) \iff |X| > |Y|$

d. $Mary's^*[A](X, Y) \iff \emptyset \neq (X \cup Y) \cap \{b \colon R(m, b)\} \subseteq X$

(7) and (8) show how we can go back and forth between quantifier semantics and quantifier* semantics, at least for providing correct truth conditions for the simplest quantified sentences. Before discussing possible reasons for choosing between these approaches, we need to say something about universes.

4 Universes

Standard model-theoretic semantics avails itself of the notion of a *universe of discourse* (sometimes but not always identified with the universe of a *model*). Ben-Yami thinks that, for natural language, this is simply wrong: "*Natural language has no universe of discourse.*" (p. 321). The universe of discourse in predicate logic is "a semantic constituent that has no parallel in natural language." (ibid.)

I am not sure the question whether natural languages employ universes of discourse has a simple yes/no answer obtainable by just looking at the linguistic facts. After all, discourse universes are theoretical tools used by the semanticist. Almost by definition, there is normally no word or part of the sentence or discourse that explicitly denotes such a universe. But if the second quote above means that Ben-Yami takes this fact to show that such universes do not exist, or are not needed, I think he is mistaken.

Suppose I describe a tram ride to my friend, and how at some point the tram stopped due to a power failure, and all passengers had to get out. I might end my discourse with:

(10) Everyone left.

It can be quite natural to assume that my discourse has built a temporary universe consisting of people in the tram (even if I used no expression denoting that set), and so (10) automatically means what I intended it to mean, i.e. that everyone *in the tram* left. Of course, it may not be strictly necessary to do so: we *could* also assume that this particular use of "-one" (plurally) refers to the people in the tram. (But how natural is that?)

However, restricted quantification in natural language is often more sophisticated than this example indicates. This is due to the use of *context sets*. Consider:

(11) Wherever John shows up, most people tend to leave.

Here we are not talking about a particular group of people, we are *quantifying* over (locations and) context sets: at each place it is the set (or plurality) of people in the vicinity of John. This is irrespective of whether a discourse universe is used or not. My point is that there is *no constituent* of (11), and *no token or use* of "people", that refers to these context sets. But this doesn't show that they are not needed; on the contrary, we cannot give a correct interpretation of (11) without them.

Therefore, the fact that no constituent refers to a discourse universe doesn't show that they don't exist. No constituent refers to the context sets in (11), yet there it is obvious that they do 'exist'.

A very different question is this: Would it be *possible* to do natural language semantics in the model-theoretic style that Ben-Yami also employs, but without using discourse universes? I don't know the answer to this question. But even if it were possible to eliminate discourse universes altogether, it seems to me that a more important question is: Would it be *practical*?[8]

5 Expressive power

At this point let me come back to Ben-Yami's suggestion that if it turns out that his account of quantifier restriction — without universes, and using (in my formulation) (DQ*) — fits the linguistic facts better, we should abandon the standard account. A natural objection is that we should also look at *other* things such an account is supposed to do. Consider all the facts and insights that (many people believe) the standard account has provided concerning monotonicity, polarity items, possessive quantification, exceptives, definites, polyadic quantification, reciprocals, etc. etc. One would need to reformulate all of this in Ben-Yami's format. Even if this could be done, would things become *simpler*?

My guess is that the answer is negative, but I will only illustrate with one type of issue: questions about expressive power. Such facts play a role in Ben-Yami's paper too; for example, he relies on the fact, proved in Kolaitis and Väänänen (1995), that the binary quantifier *most* is not definable from any unary quantifiers. More exactly, what they showed was the following: Take any finite number of unary quantifiers $Q_1, \ldots, Q_n$ and form the language $FO(Q_1, \ldots, Q_n)$ by adding these quantifiers to first-order logic as indicated in Section 1. Then there is no sentence in this language, with A, B as its only non-logical symbols, which is logically equivalent to (true in the same models as) sentence (2), i.e. $most\, x(Ax, Bx)$.

Now, the proof of this rather non-trivial fact uses established methods from model theory. And this, I would claim, is an immense advantage of the standard account of quantification: it can rely on a wealth of techniques and results already available from logic.

But can't Ben-Yami do the same? The problem is that standard model theory essentially uses universes. It isn't even clear what notion of logical equivalence should be used to state the result, in the absence of universes.

On the other hand, we could try to *reinterpret* Ben-Yami's proposal as a

[8]See Peters and Westerståhl (2006), Ch. 1.3.3–5 for a discussion of universes and context sets, and Stanley and Szabo (2000) for a detailed treatment of quantifier domain restriction. I take examples like (11) to show that one cannot do without something like context sets in natural language semantics, even though many accounts, including Ben-Yami's, leave them out for simplicity.

suggestion for a special kind of quantifier to be used in the semantics for natural language. Most naturally, quantifiers* would then be *ternary* quantifiers (taking A, X, Y as arguments, subsets of a universe M). Their relation to restricted binary quantifiers is explained à la (7) and (8), and it is then easy to see that if Q is restricted, Q^* is definable from Q and *vice versa*. Thus, Kolaitis and Väänänen's result about *most* extends to *most**.

But with such a reinterpretation, we are already halfway to the standard model-theoretic story. Would we have gained anything?

6 Conclusions

I end by stating my conclusions, some of which have already been indicated, concerning the best way to formulate, and to explain, quantifier restriction.

1.
Starting with practical advantages and disadvantages, I already said that I believe issues of expressive power are best dealt with within the standard format, and that discourse universes are quite useful in semantics, also for natural languages. Moreover, it seems to me that as regards the intuitive meaning of determiners, (DQ*) is more cumbersome than (DQ). It is rather natural to let determiners stand (on each universe) for relations between the noun argument A (the plurality which is the extension of the subject) and the verb argument B (the extension of the predicate), rather than the corresponding relation between $A \cap B$ and $A - B$. For words like 'all', 'some', 'most', the difference may seem minimal; cf. (9). But for possessives like 'Mary's', the truth conditions for 'Mary's S are P' become distinctly less transparent, as (9d) shows.

2.
(DQ) and (DQ*) are equivalent in the sense that one can use one or the other with the same resulting truth conditions. But even if the standard account is easier to use, one could still prefer (DQ*) if it really did what Ben-Yami wants it to do: *explain* domain restriction. But I don't think it does.

The standard account says: Natural language determiners are interpreted as binary quantifiers. Now look! It turns out that all of these are restricted (satisfy Conserv and Ext)! That must mean that the special semantic role of the noun in these sentences is to restrict quantification to its extension.

Ben-Yami's account says: Natural language determiners are interpreted as binary quantifiers*, whose domain of quantification is (by definition) the plurality referred to by the noun. The noun is the logical subject in these sentences, and its role is to restrict quantification to that plurality.

Modulo the talk about extensions on the one hand, and plural reference and logical subjects on the other, the explanatory power of these two accounts seems pretty similar. Maybe one could say that in the standard treatment, restriction first came as a surprise, which then made us realize the special role of the noun. In Ben-Yami's case, on the other hand, we somehow already knew from the start that logical subjects have this role, which is why we used quantifiers* instead of quantifiers. But surely this is not a very significant difference.

3.
One could also put the difference as follows: Ben-Yami builds restriction into the semantics, whereas it is an additional property in the standard account. And one could argue, with Ben-Yami, that since restriction is a *universal* property, it *should* be built in. The reason I think this argument fails was indicated above: restriction is built in *by stipulation*, not for independent principled reasons.

This also has to do with one's view of linguistic universals. Ben-Yami thinks that since restriction is universal, the correct semantic theory must make it *analytic*, even *a priori*, that restriction holds (pp. 322, 324). I suppose I have a more empirical view. That restriction holds in all known languages is surely an interesting discovery. It may point to important features of the language faculty in the human brain. Maybe a language with unrestricted quantification could never evolve with beings like us. But we can *think* of such languages. At least I believe I can easily think of such a language, so someone would have to *show* that what I am thinking of is in fact not a language at all. A mere stipulation would not be enough.

4.
I should also mention another of Ben-Yami's arguments: He claims that the contrast between (in my terminology) binary quantifiers and binary quantifiers* is analogous to the contrast from earlier days between the use of unary vs. binary quantifiers for the analysis of simple quantified sentences (pp. 318–19). Therefore, he says, just as unary quantifiers were abandoned in favor of binary ones, so should binary quantifiers be abandoned in favor of quantifiers*.

But this analogy limps. The first contrast concerns syntactic form and, most importantly, *expressive poverty.* As the result by Kolaitis and Väänänen mentioned above shows, if we use only unary quantifiers, there is *no way* to get the correct truth conditions for simple quantified sentences. The use of binary quantifiers not only eliminates this problem, but gives a more adequate analysis of all of these sentences, even those that could be handled with unary quantifiers (at the expense of introducing propositional connectives not present in originals). The second contrast, on the other hand, is one of *expressive richness.* We don't *need* non-restricted binary quantifiers for the analysis, but nothing is lost by their presence. Furthermore, the syntactic argument in the first case is much more compelling: it is completely obvious that natural language determiners do not stand for unary quantifiers. It is much less obvious — indeed I have argued that the difference is minimal — that they stand for binary quantifiers* rather than binary quantifiers.

5.
In conclusion, my guess is that with the semantic tools we have so far been using, no further explanation of quantifier restriction should be expected. To go deeper, we would need a richer framework. I am aware of only one such at-

tempt: Fernando (2001) combines the standard model-theoretic approach with dependent type theory and the idea of propositions-as-types. Here too the noun and the predicate in simple quantified sentences have distinct roles, but now it is rather the predicate which is reanalyzed (as a dependent type). The paper, which is technically quite demanding, leads up to an intricate *explanation of conservativity*, using also facts about presupposition and anaphora. It would be interesting to go deeper into the sort of explanation offered there, but that must be left for another occasion.

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