Compositionality in Kaplan style semantics*

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1 Introduction

At first sight, the idea of compositionality doesn't seem to sit well with a semantics taking (extra-linguistic) context seriously. The semantic value (meaning, content, extension, etc.) of the whole is supposed to be determined by the values of the immediate parts and the mode of composition, but what if input from context is required? It would appear that — unless context is *fixed* — compositionality could only apply to rather abstract semantic values, themselves functions from contexts to other values. However, the notion of compositionality *generalizes* to cases where the value is assigned to a syntactic item *and* a contextual item. In fact, it does so in two natural but distinct ways.

The last observation is implicit in some of the literature, and explicit in a few places.[1] Here I present it in a systematic way, within the framework of what I shall call *Kaplan style semantics*. This framework — despite the label — is not strictly tied to Kaplan's particular way of doing semantics (cf. [13]); rather, it is a *format* that covers his as well as many others theorists' preferred accounts of context-dependent meaning. For example, besides the abstract semantics mentioned above, which essentially is what Kaplan called *character*, it applies to intermediate levels of (functional or structured) *content* or *intension* used by many theorists, as well as to model-theoretic *extensions*.

*An early version of the observations here was presented at the Cognitive Science Symposium in Kista, Stockholm, June 2003. Much later did I realize that they might be useful in the current debate on various forms of context-sensitive semantics. I am grateful to the editors of this volume for allowing me to present them here, as well as to those who made comments or suggestions, in Kista and later, on my attempts in this area; in particular, Alexander Almér, Denis Bonnay, Elisabet Engdahl, Jerry Fodor, Ragnar Francén, Ernie Lepore, Larry Moss, François Recanati, Barry C. Smith, Jason Stanley, and two anonymous referees. Particular thanks go to my constant interlocutor and coworker on matters compositional, Peter Pagin. The work was made possible by a grant from the Swedish Research Council.

[1] Notably Pagin [27], Pagin and Pelletier [28], Pagin and Westerståhl [29], and Recanati [34] ([this volume]). See also footnote 27.

The set-up can furthermore be adapted to recent *relativist* modifications of Kaplan's original framework. It also applies, perhaps surprisingly, to the (controversial) phenomenon of *unarticulated constituents*, and to so-called *modulation* or *pragmatic intrusion* (Recanati [this volume]). In addition, *situation semantics* can be made to fit this format, as can several other early or recent ideas about context dependence (in a wide sense) from the literature.[2]

Within this general format there are thus several ways of associating semantic values with expressions (and contexts), preferred by different theorists. *For each of these*, the issue of compositionality can be raised. My aim here is not to suggest any particular way of doing semantics, or to judge which semantics are likely to be compositional, but only to map out the possibilities, and the logical relations between them.[3]

2 Kaplan style semantics

2.1 Context and circumstance

By *Kaplan style semantics* I mean a semantic theory permitting a distinction between two kinds of contexual factors: in Kaplan's terminology, used here, *contexts* (of utterance) and *circumstances* (of evaluation). Formally, I assume that a set

$$CU$$

of utterance contexts and a set

$$CIRC$$

of circumstances are given. Exactly what goes into the two compartments varies. Kaplan's Logic of Demonstratives [13] associates with each context c in CU a quadruple $\langle c_A, c_T, c_P, c_W\rangle$ of the *agent*, *time*, *position*, and *world of* c, and circumstances are pairs of times and worlds: $CIRC = T \times W$. For Lewis in [19], circumstances — which he calls *indices* — are also tuples of times, worlds, locations, etc., whereas contexts are situations in which (normally) someone says something: parts of worlds, rich in structure and not reducible to tuples of independently specifiable parameters.

Kaplan and Lewis agreed that both factors are needed: their basic truth relation is

φ is true at context c in circumstance d

Indeed, most semanticists in the model-theoretic tradition since Montague have found some such distinction necessary, including Montague himself (in [26]; cf.

[2] Recanati [33] mentions Aristotle, the Stoics, Bar-Hillel, Prior, Hintikka, Evans, Dummett, Stalnaker, Barwise, and Perry, among others, as well as participants in the recent debate about contextualism versus relativism; see section 4.5 below.

[3] As noted, I only deal with extra-linguistic context here. For the beginnings of a treatment of (certain aspects of) linguistic context along similar lines, see Pagin and Westerståhl [29].

p. 228). A common criterion for putting a feature in *CIRC* is that it is *shiftable*: some linguistic operator 'shifts' the time or world, say, of the context to another time or world, as the perfect past tense and the modal 'necessary' in English can be taken to do. But some features of context, such as the agent/speaker or position/location denoted by the indexicals 'I' and 'here', respectively, cannot be shifted; to take Kaplan's example, even if I try, I cannot make

(1) In some contexts it is true that I am not tired now.

say that some other person than me is not tired at some other time than the time of my speaking. I come back to this issue in section 6.3.

A related point often made is that features of contexts cannot be arbitrarily replaced by other features of the same kind. If you replace the *time*, say, of a context c with another time but change nothing else, you may not end up with a context at all; e.g. if the new time is one when the *speaker* of c wasn't born. Kaplan's contexts, even though theoretical constructs, are *proper* in this sense: the speaker must *exist* in the world of the context and be *at* the place of the context. Circumstances, on the other hand, are not required to obey such constraints.

Other formats impose no restrictions. *Two-dimensional* accounts, introduced in semantics in Kamp [12] and systematized in Segerberg [37], often identify *CU* and *CIRC*. For example, with both equal to a set T of time points, one can express that 'sentence φ uttered at t_1 is true at t_2', where $t_1, t_2 \in T$. Two- or multi-dimensional accounts are now a well established tool in intensional logic and formal semantics, not only as a means of dealing with context-dependence; cf. Garcia-Carpintero and Macia [6].

In this paper, the point is not how contexts and circumstances are distinguished, but the semantic import of the distinction. The contrast is with what is sometimes called *index semantics*, where all contextual factors are lumped together (say, in one long tuple of features) and play the same role.[4] It will be important, however, that shiftable features (in the above sense) go into the circumstances.[5]

2.2 Content and character

Once the distinction between the two kinds of contextual factors is made, it is possible — though not obligatory — to identify a level of *content*, as that which you obtain by fixing the utterance context, but not the circumstances. I use Kaplan's term here, and likewise I will use *character* for the function taking

[4] This was the advice of Scott [36], though, as noted, not many semanticists have followed it (at least not for very long). But even index semantics could be seen as compatible with the framework here: just think of one of *CU* and *CIRC* as containing exactly one fixed element.

[5] Following Kaplan and Lewis, Stanley [38] (pp. 147–152) argues that *only* shiftable features should go into the circumstances. MacFarlane [24] disagrees, and indeed relativists place judges or standards, which are not thought to be shiftable, in that compartment; cf. section 4.5. My concern here is with the inverse claim: that *all* shiftable contextual factors belong to the circumstances. As we will see in section 6.3, this is not just a terminological matter.

expressions to contents.

Again, this terminology is meant to cover a host of related notions. There are Carnap's *intensions*; originally used for functions from worlds to extensions but often extended to take other contextual factors as arguments.[6] Another terminology adapts Quine's distinction in [32] between standing and occasion sentences to *standing* and *occasion meaning.*[7] Recanati [33] uses the Stoic *lekton* for propositional content, but also Barwise's term *Austinian proposition* (Barwise and Etchemendy [1]). This marks a distinction *within* the notion of content, which requires a comment.

Kaplanian contents are *incomplete* in the sense that you need a circumstance to arrive at the *extension* — in the propositional case, a truth value. This was also true of the Stoic *lekta*: a *time* was needed for a truth value. But it contrasts starkly with Frege's thoughts or Russell's propositions, which are absolutely true or false. So it seems we should simply distinguish *complete* and *incomplete* propositions, and similarly for subsentential contents.

But there appears to be a complication: many regard propositions where only the *world* is not fixed as already complete, whereas if also a time, or a location, or a standard of taste, is required to obtain a truth value, the proposition is incomplete. It is an interesting issue what, if anything, gives worlds such a privileged status, but it is not one we need to resolve here.[8] *By definition*, contents are incomplete, precisely in the sense that you (normally) need a circumstance to get an extension (or, in the structured case, to get something that *has* an extension), even if that circumstance is just a world.

Note that although character and content *can* depend on context and circumstance, respectively, they don't have to. There will usually be expressions, such as proper names or mathematical predicates, whose extension is *independent* of one or both kinds of contextual factors. In the present set-up, these are represented as *constant* functions (or structured objects without any 'holes'; see section 3.2). In this sense we can think of Fregean or Russellian semantics, where *all* expressions are treated in this way, as a limiting case of Kaplan style semantics (see also section 5.2).

To repeat, what I call Kaplan style semantics is *not* a formalization of any particular semantic theory. It is a framework that *fits* many different accounts. The fit may be more or less rough, but (I claim) for the discussion of context-

[6]The subject index of the volume [6] mentioned above lists 18 different kinds of intension!

[7]'Standing meaning', essentially for character, is used, for example, in Heck [8], Pagin [27], King and Stanley [16], and Recanati [this volume]).

[8] Most semanticists in the possible worlds tradition regard functions from possible worlds to truth values as complete, in particular, as entities that can be the objects of propositional attitudes. For example, Recanati's *lekta* are called incomplete, whereas Austinian propositions are complete, but can be true in one world and false in another. One argument is that you get an absolute truth value by plugging in the *actual* world (cf. Evans [4]), the idea being that there is just one actual world but lots of available times. To a *presentist*, on the other hand, the actual time has the same status as the actual world. And some theorists (e.g. MacFarlane [22]) consider speakers' utterances in non-actual worlds. Glanzberg [7] attempts a general argument that worlds aren't really an important part of the semantic analysis. He says (his footnote 2), however, that a similar argument would go through for world-time pairs, whereas the issue here is whether worlds have a different status than e.g. times.

dependent forms of compositionality, the distinctions made in this framework are all you need.

2.3 Extensions and models

I also assume there is a given set

$$M$$

of *extensions*. Usually, formal semantics employs the notion of a *model* which, besides supplying the sets *CU*, *CIRC*, and a domain M_0 of individuals, also *interprets* the non-logical atomic expressions of the language. For example, the name *Saul* could denote a fixed element of M_0, in all contexts and under all circumstances, and the predicate *red* might be interpreted, in all contexts, as a certain function from *CIRC* to the power set of M_0 (i.e. as having a fixed content).

For almost all of this paper, it is not necessary to mention models (except for claim (III) in section 6.2). We may think of a model $\mathcal{M}_0$ as *fixed*, and M as the set of all semantic objects (over M_0) that expressions can refer to in this model. For example, M could be a type-theoretic universe built up from M_0.

3 Content

To repeat, a *content* yields, by definition, for each $d \in \mathit{CIRC}$, an extension in M. There are two main ways in which it is supposed to do this.

3.1 Contents as functions

The simplest notion of content is that of a *function* from *CIRC* to M. I use $[X \longrightarrow Y]$ for the set of functions from X to Y. Thus,

(2) $\mathit{CONT} = [\mathit{CIRC} \longrightarrow M]$

In particular, if extensions of sentences are truth values, the set of *propositions* is

(3) $\mathit{PROP} = [\mathit{CIRC} \longrightarrow \{\mathrm{T}, \mathrm{F}\}] \subseteq \mathit{CONT}$

Contents that are independent of circumstance (e.g. for expressions that, in a given context, *directly refer* to something in M, such as indexicals like *I*, *you*, *here*, *tomorrow*, and deictic third-person pronouns) are treated as constant functions.

3.2 Structured contents

For certain tasks, functional contents seem too coarse-grained. Most dramatically, there are no distinctions among necessarily true propositions: there is just one function taking the value T for all $d \in \mathit{CIRC}$. For example, there would be just one true mathematical theorem on this account. More subtly, as Kaplan points out, there is no principled distinction between a directly referential term, like *I*, and a singular term that just happens to denote one and the same object in every circumstance, like *the smallest natural number*: their content (in a given context) will both be *constant* functions from *CIRC*.

This can be avoided by using instead some kind of *structured contents*. The idea goes back to Frege and Russell; for an overview of its motivation and its most common implementations, see King [15]. For our purposes, it suffices to think of a structured proposition, or in general a structured content, as something like a list or a tree, where (complex) properties, descriptions, etc., but also individuals, may literally *occur*.

We need not choose here between Frege style propositions, belonging wholly to the realm of thought, and Russell style propositions, in which worldly objects can occur, and which fit into Kaplan's theory of direct reference. I will simply use the following picture: incomplete propositions have 'holes' that can be 'filled' by elements of *CIRC*, resulting in Frege or Russell style structured propositions. Similarly for other kinds of structured contents. So there is a set

$$\mathit{SCONT} = \mathit{I\text{-}SCONT} \cup \mathit{C\text{-}SCONT}$$

of *structured contents*, partitioned into two disjoint subsets of *incomplete* and *complete* structured contents, respectively. I indicate that $p \in \mathit{SCONT}$ has a 'hole' by writing $p = p[\,]$, and letting $p[d] \in \mathit{C\text{-}SCONT}$ be the result of 'filling' that hole with $d \in \mathit{CIRC}$. Also, there is a function

$$\mathit{ref}$$

from *C-SCONT* to M, reflecting the fact that complete contents have unique referents (extensions). The result is that for each $p \in \mathit{I\text{-}SCONT}$ we have a corresponding functional content $p^* \in [\mathit{CIRC} \longrightarrow M]$ given by

(4) $$p^*(d) = \mathit{ref}(p[d])$$

That p^* is more coarse-grained than p means that we can have ${p_1}^* = {p_2}^*$ even though $p_1 \neq p_2$, i.e. that the function * need not be one-one (e.g. when p_1 and p_2 are two distinct but true mathematical claims, or two distinct definite descriptions of the same object, etc.).

In fact, we can extend the function * to all of *SCONT*: for the content of circumstance-independent expressions we still write $p = p[\,]$, but nothing happens when one 'fills' with d: $p[d] = p$. Then p^* is a constant function,

$$p^*(d) = \mathit{ref}(p)$$

for all $d \in \mathit{CIRC}$, just as it should be.

4 The general format

4.1 The circumstance of the context

Contexts play a double role in Kaplan style semantics. On the one hand, $c \in CU$ fixes the extension of indexicals and demonstratives. On the other hand, most theorists in this tradition hold that *utterances*, or *occurrences* of sentences in utterance contexts, can be true or false.[9] Since a circumstance is needed besides the context to obtain a truth value, the context must be taken to *determine* a particular circumstance, such as a world or a time. In many accounts, such as those of Kaplan and Lewis mentioned in section 2.1, this is just the world, the time, etc. *of* the utterance.

However, this could be relaxed. Consider Perry's example of his son, who is standing in Perry's house in Palo Alto, and talking on the phone to his brother in Murdock. Perry asks *What's the weather like?*, and his son replies *It's raining*, meaning that it is raining in Murdock (Perry [30]). The location *of* that utterance context is Palo Alto, while the location used to evaluate what is said is Murdock. Similarly, situation semantics, say in the format of Barwise and Etchemendy [1], uses a basic distinction between the situation (circumstance) an utterance *is about*, and the one it is *in*. We capture all of this by just assuming that there is a given function

$$circ\colon CU \longrightarrow CIRC$$

that picks out a circumstance in each context.[10]

4.2 Semantics as assignments of values

Let us say that a *semantics* assigns meanings, or other semantic values, to expressions. In other words, it is a function

$$\mu : E \longrightarrow X$$

where E is a set of *structured expressions* and X is a set of *values*. But we will also consider functions taking contexts or circumstances as additional arguments; such functions too will be called *semantics*.

[9] Israel and Perry [11] argue that utterances are the primary truth bearers, not tokens, because utterances are acts. MacFarlane [22] says just the opposite: if seen as acts, utterances are *not* bearers of truth (he suggests using *accurate* for utterances instead of *true*). Kaplan [13] claims that when doing semantics, not speech act theory, we should take *occurrences* of expressions *in* contexts to have extensions (pp. 524, 546). (Occurrences are a little more specific than tokens: a written token can be 're-used' in another context.) These distinctions are not very important here, but for definiteness, I will use occurrences as Kaplan does.

[10] This may need refinement. In MacFarlane [23], there can be more than one circumstance associated with a context; I disregard that here. Note also that I am not taking a stance in the debate about whether the circumstance determined by the context can *ever* be distinct from the circumstance *of* the context. Recanati [33] calls the claim that it cannot the *Generalized Reflexion Principle* (GRC), and devotes the final part of his book to questions concerning the GRC. *circ* covers both: if you believe in the GRC, *circ* always picks out the circumstance of the context.

In principle, the elements of E are *types* of syntactic objects, *occurrences* of which may be used in acts such as utterances. Most writers follow Kaplan in taking character to apply to types, and content to occurrences. This makes good sense: the character of an expression assigns it a content in every context of utterance. Character reflects linguistic rules, what a speaker of the language must know. However, the character of a type transfers immediately to any occurrence of that type (cf. footnote 9). If we want to compare distinct assignments of semantic values, they had better apply to the same kinds of arguments. So I shall take all of these semantic functions to apply to *occurrences* of expressions.

4.3 Four semantic functions

If character is seen as the basic semantic function, we obtain the following picture.[11] I use e, c, and d (sometimes with subscripts) for elements of E, CU, and $CIRC$, respectively. Recall that $CONT = [CIRC \longrightarrow M]$.

(5) **Kaplan style semantics 1** (functional case)

a. $char\colon E \longrightarrow [CU \longrightarrow CONT]$ is a given semantics.

b. $cont\colon E \times CU \longrightarrow CONT$ is defined by $cont(e, c) = char(e)(c)$; the *content of* e *at* c.

c. $ext\colon E \times CU \longrightarrow M$ is defined by $ext(e, c) = cont(e, c)(circ(c))$; the *extension of* e *at* c.

d. $poe\text{-}sem\colon E \times CU \times CIRC \longrightarrow M$ is defined by $poe\text{-}sem(e, c, d) = cont(e, c)(d)$; the value of e at the *point of evaluation* (c, d).

Thus, with each expression-in-context, *cont* associates a unique content in $CONT$, and *ext* a unique extension in M, via the circumstance picked out by *circ*. Points of evaluation are in a sense theoretical constructs, but when a formal semantics is given for a language fragment, one invariably starts with the point of evaluation semantics (*poe-sem*), and defines the others in terms of it. From this perspective, we begin at the other end, and get the following picture:[12]

(6) **Kaplan style semantics 2** (functional case, alt. formulation)

a. $\mu\colon E \times CU \times CIRC \longrightarrow M$ is a given semantics.

b. $\mu_{cont}\colon E \times CU \longrightarrow CONT$ is defined by $\mu_{cont}(e, c)(d) = \mu(e, c, d)$.

c. $\mu_{char}\colon E \longrightarrow [CU \longrightarrow CONT]$ is given by $\mu_{char}(e)(c)(d) = \mu(e, c, d)$.

d. $\mu_{ext}(e, c) = \mu_{cont}(e, c)(circ(c))$

[11] I omit here the *assignments* of individuals to variables required for handling quantification. The reader familiar with standard Tarskian semantics will easily see how a set of assignments can be added to the picture; cf. also section 6.2.

[12] MacFarlane [20], uses the term *point of evaluation* (taken from Belnap); Montague [25] uses *point of reference*. In [20], MacFarlane calls μ the 'semantics proper', and μ_{ext} the 'post-semantics', since, for sentences, μ ('truth at a point') is the semantics needed in the recursive truth definition, whereas μ_{ext} ('truth at a context of utterance') is what comes closest to our normal use of 'true'.

Here is a version of (5) for structured contents:

(7) **Kaplan style semantics 3** (structured case)

a. $char_s : E \longrightarrow [CU \longrightarrow SCONT]$ is a given semantics.

b. $cont_s : E \times CU \longrightarrow SCONT$ is as before: $cont_s(e, c) = char_s(e)(c)$.

c. Via the mapping * (recall (4) in section 3.2) we get corresponding functional versions of character and content:

$$cont(e, c) = cont_s(e, c)^*$$
$$char(e)(c) = cont(e, c)$$

d. *ext* and *poe-sem* are then defined from *cont* as in (5)c,d.

This time, however, there is no obvious way to go in the other direction, from a semantics for points of evaluation to a content-assigning semantics, since the former has nothing to say about structured contents.

4.4 Propositions and truth

A sentence (occurrence) φ is true at $c \in CU$ if the proposition it expresses at c is true at c:

(8) **Truth of propositions and sentences**

a. A proposition p — in the functional case, a function from *CIRC* to {T,F}, and in the structural case, an element of *SCONT* — is *true at* $d \in CIRC$ iff $p(d) = \mathrm{T}$ (respectively, $p^*(d) = \mathrm{T}$).

b. φ *expresses* p *at* c iff $p = cont(\varphi, c)$ (resp. $p = cont_s(\varphi, c)$).

c. φ is *true at* c iff $ext(\varphi, c) = \mathrm{T}$.

To illustrate, suppose I produce an occurrence φ of

(9) I am sitting.

The context c provides a referent for the indexical *I*, so φ expresses the proposition $cont(\varphi, c)$ = *that* Dag is sitting, which can be true or false at various circumstances, and which happens to be true at the circumstance of my utterance. If circumstances are world-time pairs as in Kaplan [13], and we adopt the functional view, it is that function p_1 from *CIRC* to truth values such that $p_1(w, t) = \mathrm{T}$ iff Dag is sitting in w at t. Fixing (w, t) to the *actual* circumstance, $circ(c) = (w_c, t_c)$, we get $ext(\varphi, c) = \mathrm{T}$. On one (of several) structural account, the proposition expressed could be something like $cont_s(\varphi, c) = p_2[\,] = \langle \mathrm{SITTING}[\,], \mathrm{Dag}\rangle$. Supplying the actual circumstance gives $p_2[w_c, t_c] = \langle \mathrm{SITTING}[w_c, t_c], \mathrm{Dag}\rangle$; again $ext(\varphi, c) = (p_2)^*(w_c, t_c) = \mathrm{T}$.

If you don't accept propositions like p_1 or p_2 your must regard t_c, or both w_c and t_c, as fixed in the proposition. In the latter case, you get directly to T with the functional version, and to something like $p_2[w_c, t_c]$ with the structural version. In the former case, the actual world must be plugged in.[13]

Finally, a model-theoretic semanticist accounts for (9) using *poe-sem*, without resorting to either character or content: $\textit{poe-sem}(\varphi, c, w, t) = \mathrm{T}$ iff the speaker at c is sitting in w at t.

Consider also

(10) I am sitting now.

On one natural view, (10), uttered in the same circumstance c, corresponds to a slightly different proposition, namely, one where the time has been fixed to t_c via the indexical *now*. This is (in the functional case) the function p_3 such that $p_3(w, t) = \mathrm{T}$ iff Dag is sitting in w at t_c, i.e. essentially the same proposition that (9) expresses if only worlds are taken to be circumstances of evaluation (so on that view, the difference between (9) and (10) disappears at the propositional level). Here the time argument plays no role, but the world argument does, since I could be standing at t_c in some $w \neq w_c$. But the utterance fixes w to the actual world, so the truth value of the two utterances is of course the same.[14]

4.5 Variants: dealing with assessment

The truth value of some sentences seems to depend on an irreducibly *subjective* element, for example, claims about taste, beauty, morals, about what is funny, what is likely, etc. A recent debate in the philosophy of language concerns how such claims (and disagreements about them) should be treated within a Kaplan style framework.[15] For simplicity, think of all such claims as needing an *assessor* or a *judge*.[16]

A *contextualist* account fits directly into Kaplan style semantics as defined so far: simply consider the assessor as a feature of the context of utterance, which thus helps determine which proposition is expressed. So if I say *Moby Dick is a funny novel* and you deny this, we are simply making claims about

[13] Note that while t_c might be said to correspond to the present progressive tense in (9), nothing in the sentence corresponds to w_c. In this sense, the world would be an *unarticulated constituent*; see also section 5.2 below.

[14] An alternative is to take *now* to be a temporal operator, setting t to the actual time. The sentence then has the form 'Now(I am sitting)'. On the functional view of content — but not on the structural view — the corresponding proposition would still be the same as when *now* is a simple indexical.

An *eternalist* about time lets the time parameter be fixed by the context of utterance; so that different propositions (still incomplete wrt worlds) are expressed by different utterances. This is compatible with the increasingly popular idea that tenses, in, say,

(i) Dag will be standing.

are not to be handled, as Kaplan did following Prior, by tense operators but rather by quantification over times. See King [14] for arguments, and Recanati [33], chapter 6, for a defense of the Prior-Kaplan treatment. Nothing of what I say here turns on this issue.

[15] For the current state of this debate consult, for example, the collection Garcia-Carpintero and Kölbel [5], the *Synthese* issue [39], or the 2008 issue of *Philosophical Perspectives* [31].

[16] Alternatively, a *standard* of taste, or of what is funny, or a moral or epistemic standard.

ourselves (so there is no real disagreement), or, if the contexts happen to single out other assessors, claims about what these assessors find funny.

According to *relativists* (about truth), this misses the disagreement aspect of such exchanges. Instead, they place the assessor among the circumstances. On one account, the utterance context still determines the assessor, but the proposition expressed needs not only a time, a world, etc. to yield a truth value, but also an assessor.[17] Now you and I can be said to dispute the *same proposition* in the above exchange (i.e. the proposition which is true at an ordinary circumstance d and an assessor a iff Moby Dick is funny for a in d), although both our utterances may be true, since the respective contexts, via the function *circ*, may determine different assessors (so-called *faultless* disagreement).

Other relativists see assessment as *independent* of the original utterance, i.e. not determined by the utterance context.[18] This context still fixes the assessment-relative proposition expressed, but not the truth value of the utterance or of any assessment of it; for that an independently chosen assessor (or a context of assessment) is required.

Yet another variant is relativist about *content* instead of truth.[19] That is, propositions do not take assessors as argument (in this sense, this approach is still contextualist), but in order to determine which proposition is expressed you need an assessor, which can be independent of the utterance context.

What I have said so far is not enough to even begin to discuss the hotly debated pros and cons of these various approaches, but it should be sufficient, I hope, for seeing that they all fit straightforwardly — in the relativist cases with slight adjustments — into Kaplan style semantics.

4.6 Does the choice of semantics matter?

μ_{cont} and μ_{char} in (7) are simply obtained by *currying* μ, and inversely, *cont* and *poe-sem* are simply obtained by *uncurrying* the function *char* (see section 5.2 for definitions). From a mathematical point of view, these functions are often simply *identified*. So how can their differences matter semantically? That depends on what you want them to model or explain. For example, if you think *propositions* are important, say, as the objects of attitudes, you need something like *cont*. If *utterance truth* is what 'truth' in ordinary language stands for, *ext* provides a useful rendering of it. If your ambition is, to the contrary, to get by without propositions in our sense, you may focus on *poe-sem* or on *char* instead.

The differences also surface in notions of logical or necessary truth and consequence tied to the various semantic formats. Famously, Kaplan was able to account for the strong intuition that a sentence like

(11) I am here.

is contextually true (true whenever uttered), but still not necessary (since I

[17]For example, Kölbel [17] and Brogaard [2].
[18]Notably, MacFarlane [21] and Lasersohn [18].
[19]Cappelen [3].

might have been somewhere else).[20]

Regardless of these issues in the philosophy of language, it turns out that for compositionality, the choice of values matters a great deal. This was observed by Lewis in [19]. He argued that for most purposes the choice between (what I have called) *cont* and *poe-sem* is a matter of taste: a 'distinction without a difference', precisely because you can go between the two at will. But he also noted that with the wrong choice of arguments, contents may become non-compositional (and thereby, according to him, disqualified as a semantic values). We will now see how compositionality fares in the presence of contextual factors. In particular, I will explain and generalize (in section 6) Lewis' point about content.

5 Compositionality and context

We introduced four semantic functions, and noted that recursive truth definitions usually appear at the point of evaluation level. So it is natural to ask at which levels compositionality applies, and how compositionality of one of the semantic functions is related to that of the others.

5.1 Standard compositionality

To formulate compositionality, we must make explicit how expressions in E are *structured*. There are various ways to do this. Here, I shall simply assume that there is a set Σ of functions from E^n to E ($n \geq 1$) that *generate* E from some given set $A \subseteq E$ of *atoms*.[21] That is, every expression is either an atom or obtained from atoms by repeated applications of functions in Σ. If μ is a semantics for E, i.e.

$$\mu\colon E \longrightarrow X$$

for some set X of semantic values, standard compositionality of μ means that the value of any complex expression is *determined* by the values of its immediate constituents and the rule applied. In other words:

Funct(μ) For every syntactic rule α there is an operation r_α such that for all $e_1, \ldots, e_n \in E$, $\mu(\alpha(e_1, \ldots, e_n)) = r_\alpha(\mu(e_1), \ldots, \mu(e_n))$.

[20] Just restricting attention to *proper* contexts (section 2.1) isn't enough, according to Kaplan: it makes (11) true in every context, but it also makes

(i) Necessarily, I am here.

true. But if context first fixes the reference of *I* and *here*, and necessity means that the *resulting* proposition is true at all circumstances, then (i) is false. In hindsight, however, the same distinction is available, though less obviously, to one who uses only *poe-sem* and *circ*.

[21] Thus, I ignore (lexical and structural) *ambiguity*, the occurrence of which requires one to assign semantic values to *derivations* of expressions rather than the expressions themselves. This can be done by means of a *term algebra* corresponding to (E, A, Σ); see Hendriks [9] for an account using *many-sorted* term algebras, Hodges [10] for an account with *partial* term algebras, and Westerståhl [40] for some remarks on the relation between the two. All notions and results here can easily be reformulated for this more accurate setting.

This is the *functional* version of compositionality, expressing directly the 'determination' idea. There is an equivalent *substitutional* version, saying that appropriate replacement of synonymous (not necessarily immediate) constituents preserves meaning; see Pagin and Westerståhl [29] for details. Here I use the functional version, although the various principles of 'contextual' compositionality below all have equivalent substitutional versions.

5.2 Compositionality for incomplete meanings

Standard compositionality applies only to character: Funct($char$) makes immediate sense, since character assigns a semantic value directly to expressions. For semantic functions taking contextual arguments, the notion of compositionality must be revised. Abstractly, we have a function

$$F\colon E \longrightarrow [Y \longrightarrow Z]$$

and its *uncurried* version:[22]

$$F_{uc}\colon E \times Y \longrightarrow Z$$

The task is to reformulate the compositionality condition for F_{uc}. In fact, there are *two* natural ways to do this:

C-Funct(F_{uc}) For every syntactic rule α there is an operation r_α such that for all $e_1, \ldots, e_n \in E$ and all $y \in Y$,

$$F_{uc}(\alpha(e_1, \ldots, e_n), y) = r_\alpha(F_{uc}(e_1, y), \ldots, F_{uc}(e_n, y)).$$

C-Funct(F_{uc})$_w$ For every syntactic rule α there is an operation r_α such that for all $e_1, \ldots, e_n \in E$ and all $y \in Y$,

$$F_{uc}(\alpha(e_1, \ldots, e_n), y) = r_\alpha(F_{uc}(e_1, y), \ldots, F_{uc}(e_n, y), y)$$

When C-Funct(F_{uc}) (C-Funct(F_{uc})$_w$) holds, I will say that F is (*weakly*) *contextually compositional*. Also, I will say that F is (weakly, contextually) α-*compositional* when the corresponding condition above holds for a particular $\alpha \in \Sigma$.

The seemingly small difference between the two variants — that the contextual element is an argument of the semantic operation in the weak but not the

[22] Or, start from

$$\mu\colon E \times Y \longrightarrow Z$$

and apply *currying*:

$$\mu_{curr}\colon E \longrightarrow [Y \longrightarrow Z]$$

where $\mu_{curr}(e)(y) = \mu(e, y)$. The two operations are *inverses* to each other:

(i) $(F_{uc})_{curr} = F$ and $(\mu_{curr})_{uc} = \mu$.

strong version — is in fact significant, and corresponds to different accounts of how contextual elements interact with meaning. Here is one illustration:

(12) John's books are are thick.

requires a 'possessor relation' fixed by the utterance context: it might be the books he owns, or has borrowed, or authored, or sold, or the ones he is standing on to reach the upper shelf. But *John* doesn't single out such a relation, and neither does *books* or the possessive *'s*. The possessive *'s* indicates the presence of a 'possessor relation' R, but doesn't by itself specify which one. Consider a context c_1 where R is 'authored by' and another context c_2 where it is 'standing on to reach the upper shelf', but otherwise similar to c_1. One natural analysis (others are possible) has $cont(John, c_1) = cont(John, c_2)$, $cont('s, c_1) = cont('s, c_2)$ (using a free parameter for R), and also $cont(John's, c_1) = cont(John's, c_2)$. Furthermore, $cont(books, c_1) = cont(books, c_2)$, but on the next level the value of R could be specified, and so $cont(John's\ books, c_1) \neq cont(John's\ books, c_2)$.[23] If so, C-Funct(*cont*) fails, but C-Funct$(cont)_w$ may still hold.

This example illustrates what Pagin [27] calls *context-shift failure*: the failure of compositionality does not arise from substituting synonymous expressions, but merely from changing the context: all immediate subexpressions have the same value in both contexts, but the complex expression in question doesn't.

The same situation may apply if *unarticulated constituents* occur. Consider

(13) It is raining.

If one argues that *it* and *rain* have no location argument but (13) does, we have a case of unarticulated constituents that yields the same kind of counter-example to C-Funct(*cont*) (but not to C-Funct$(cont)_w$) as above. Thus, unarticulated constituents are allowed by C-Funct$(cont)_w$ (a point noted and discussed in detail in Pagin [27]).

This can be generalized. When Recanati [this volume] argues that phenomena like *modulation* falsify a principle of compositionality that only allows 'indexical-like' forms of context dependence, he is in effect claiming that C-Funct(*cont*) fails.[24] He discusses a proposal from Pagin and Pelletier [28] (recursively using contextually modulated meanings as building blocks of meaning)

[23] I am assuming that *John's books* contains *John's* and *books* as immediate parts, and *John's* is formed from *John* and the genitive *'s*. The reason why one shouldn't fix R from the start in the context comes from cases with relational nouns like *John's bride*: this could be his spouse but also the bride he has been assigned to escort on some occasion. Until the level where *John's* and *bride* are combined, the former relation is not 'available'.

[24] "... [W]e cannot maintain that the meaning of a complex phrase is (wholly) determined by the meanings of its parts and their mode of combination", where the latter meanings are taken to be "standing meanings (*and* Kaplanian contents)". (p. XXX) A typical example of modulation appears in

(i) The policeman stopped the car.

where we get different meanings depending on whether the policeman was outside the car or driving it, and where that variation is not attributable — so the story goes — to an implicit parameter for 'ways of stopping cars' or indeed to anything linguistic.

that in a weak sense salvages compositionality. But he also observes that if the context itself is taken to be an argument of the composition function, modulation is straightforwardly covered. After all, the idea is precisely that *context* influences the build-up of the meaning of an expression, sometimes in ways not predictable solely from the (possibly context-dependent) meanings of its parts and the mode of composition.

The relations between our three notions of compositionality are as follows:[25]

Lemma 1
C-Funct$(F_{uc}) \Longrightarrow$ *C-Funct*$(F_{uc})_w \Longrightarrow$ *Funct*(F), *but none of these arrows can be reversed.*

Proof. Clearly, C-Funct(F_{uc}) entails C-Funct$(F_{uc})_w$. Suppose C-Funct$(F_{uc})_w$ holds. We have:

$$\begin{aligned} F(\alpha(e_1,\ldots,e_n))(y) &= F_{uc}(\alpha(e_1,\ldots,e_n),y) \\ &= r_\alpha(F_{uc}(e_1,y),\ldots,F_{uc}(e_n,y),y) \\ &= s_\alpha(F(e_1),\ldots,F(e_n))(y) \end{aligned}$$

where $s_\alpha : [Y \longrightarrow Z]^n \longrightarrow [Y \longrightarrow Z]$ is defined by

$$s_\alpha(f_1,\ldots,f_n)(y) = r_\alpha(f_1(y),\ldots,f_n(y),y)$$

for $f_1,\ldots,f_n \in [X \longrightarrow Z]$ and $y \in Y$. The third equality above then follows, and since the argument holds for any $y \in Y$, we have

$$F(\alpha(e_1,\ldots,e_n)) = s_\alpha(F(e_1),\ldots,F(e_n))$$

Thus, Funct(F) holds.

Counterexamples to the converse of the first implication were just given. The following counter-example to the converse of the second implication is instructive, and will be generalized in section 6.2. Consider a language for propositional modal logic, with atomic formulas $p_1, p_2, \ldots$ and complex formulas of the forms $\neg\varphi$, $\varphi\wedge\psi$, and $\Box\varphi$. A given model assigns, for every world $w \in W$, a truth value $F(p_i)(w)$ to each p_i ($F(\varphi)(w) = \mathrm{T}$ is usually written $w \models \varphi$). Truth values to complex formulas in worlds as assigned in the usual (S5) way, in particular,

$$F(\Box\varphi)(w) = T \text{ iff for all } w' \in W,\ F(\varphi)(w') = T.$$

We may think of the meaning $F(\varphi)$ of a formula φ as the set of worlds where φ is true. Clearly, Funct(F) holds. But C-Funct$(F_{uc})_w$ normally fails. Specifically, suppose p_1 is true in all worlds, but there are worlds w' and w'' such that p_2 is true in w' but false in w''. Then $F_{uc}(p_1, w') = F_{uc}(p_2, w') = \mathrm{T}$, but $F_{uc}(\Box p_1, w')$ $= \mathrm{T}$ and $F_{uc}(\Box p_2, w') = \mathrm{F}$. So there can be no operation r that computes the truth value of $\Box\varphi$ in w from just the truth value of φ in w, and possibly w itself. □

[25] This result, mentioned in my Kista talk (see the introductory footnote), is also proved in Pagin [27], Appendix 1.

It will be convenient for applications to formulate a slightly more general version of this lemma. Suppose

$$G\colon E \times X \longrightarrow [Y \longrightarrow Z]$$

is given, with its uncurried version

$$G_{uc}\colon E \times X \times Y \longrightarrow Z$$

as usual: $G_{uc}(e,x,y) = G(e,x)(y)$. Both G_{uc} and G take contextual arguments. The proof of the following lemma is a minor variation of the proof of the first implication in Lemma 1 (which can be obtained as a special case of Lemma 2, taking X as a singleton).

Lemma 2
C-Funct(G_{uc}) $\Longrightarrow$ C-Funct(G), and similarly for the weak case.

Sometimes it is natural to *fix* the context, thus transforming a semantics F taking contextual arguments to one taking only expressions. That is, for some fixed $y_0 \in Y$, one considers $F_{y_0}\colon E \longrightarrow Z$ given by

$$F_{y_0}(e) = F(e)(y_0)$$

Much of classical formal semantics is done under such a *fixed context assumption.* We have the following result:[26]

Fact 3
C-Funct(F_{uc})$_w$ is equivalent to the requirement that for all $y \in Y$, Funct(F_y) holds.

Proof. Suppose that for all y in Y, Funct(F_y). We then have, for each y,

$$\begin{aligned} F_{uc}(\alpha(e_1,\ldots,e_n),y) &= F_y(\alpha(e_1,\ldots,e_n)) \\ &= r_{\alpha,y}(F_y(e_1),\ldots,F_y(e_n)) \text{ (for some operation } r_{\alpha,y}) \\ &= s_\alpha(F_{uc}(e_1,y),\ldots,F_{uc}(e_n,y),y) \end{aligned}$$

where

$$s_\alpha(p_1,\ldots,p_n,y) = r_{\alpha,y}(p_1,\ldots,p_n)$$

The converse direction is even simpler. □

So weak contextual compositionality is equivalent to the condition that ordinary compositionality holds *however* the contextual argument is fixed (for an application, see section 6.6).

Without a fixed context assumption it will still be the case that *some* expressions e are *context-independent* in the sense that $F(e)$ is a constant function. One may think of Frege style semantics as claiming that *all* expressions are

[26] See also Pagin [27], footnote 11.

context-independent. Then, unsurprisingly, the three notions of compositionality collapse into one. I leave the straightforward verification of the following fact as an exercise.

Fact 4
If $Y = \{y_0\}$ or, more generally, if all expressions are context-independent, then $\mathit{Funct}(F) \Longrightarrow \mathit{C\text{-}Funct}(F_{uc})$.

6 Compositionality in Kaplan style semantics

6.1 The general picture

Notions of contextual compositionality apply directly to the functional version of the set-up in section 4, where we started with the function *char*, and defined the other semantics from it.[27] Let char_0 be the function from E to $[\mathit{CU} \times \mathit{CIRC} \longrightarrow M]$ obtained by currying both contextual arguments of *poe-sem*:

(14) $\quad \mathit{char}_0(e)(c,d) = \mathit{poe\text{-}sem}(e,c,d)$

It ought to make little or no difference whether one thinks of character in terms of *char* or char_0; the result below confirms this. The following picture results.

Proposition 5

$$\begin{array}{ccccc}
\mathit{C\text{-}Funct}(\mathit{poe\text{-}sem}) & \Longrightarrow & \mathit{C\text{-}Funct}(\mathit{poe\text{-}sem})_w & \Longrightarrow & \mathit{Funct}(\mathit{char}_0) \\
\Downarrow & & \Downarrow & & \Updownarrow \\
\mathit{C\text{-}Funct}(\mathit{cont}) & \Longrightarrow & \mathit{C\text{-}Funct}(\mathit{cont})_w & \Longrightarrow & \mathit{Funct}(\mathit{char})
\end{array}$$

Proof. The rightmost horizontal arrow in the lower row is an instance of Lemma 1, with $Z = \mathit{CONT} = [\mathit{CIRC} \longrightarrow M]$. The one in the upper row follows from the same lemma with $Y = \mathit{CU} \times \mathit{CIRC}$. The two leftmost downward implications follow from Lemma 2. Suppose Funct(char_0) holds. To prove Funct(*char*) we calculate:

(15)

$$\begin{aligned}
\mathit{char}(\alpha(e_1,\dots,e_n))(c)(d) &= \mathit{char}_0(\alpha(e_1,\dots,e_n))(c,d) \\
&= r_\alpha(\mathit{char}_0(e_1),\dots,\mathit{char}_0(e_n))(c,d) \\
&= s_\alpha(\mathit{char}(e_1),\dots,\mathit{char}(e_n))(c)(d)
\end{aligned}$$

[27] Kaplan's principle (F1) in [13], p. 507, is the substitutional equivalent of Funct(*char*). His (F2) appears to be intended as a substitutional equivalent of C-Funct(*cont*), but it isn't; in fact C-Funct(*cont*) has no simple such equivalent. Indeed (F2) seems mistaken, but if it is reformulated for one context instead of two, it becomes equivalent to C-Funct$(\mathit{cont})_w$. Kaplan does not discuss these matters, however. Lewis [19] implicitly uses context-dependent compositionality when discussing requirements on semantic values, but without explicit formulations.

where, for any $f_1, \ldots, f_n \colon CU \longrightarrow CONT$, $c \in CU$, and $d \in CIRC$,

$$s_\alpha(f_1, \ldots, f_n)(c)(d) = r_\alpha((f_1)_{uc}, \ldots, (f_n)_{uc})(c, d)$$

Here, as before, $(f_i)_{uc}(x, y) = f_i(x)(y)$. Since we have

$$(char(e_i))_{uc} = char_0(e_i)$$

it follows that the last equality of (15) holds. This shows

$$char(\alpha(e_1, \ldots, e_n)) = s_\alpha(char(e_1), \ldots, char(e_n))$$

The proof that Funct($char$) entails Funct($char_0$) is similar (using the curryings of the functions $m_1, \ldots, m_n \colon CU \times CIRC \longrightarrow M$). □

Thus, the point of evaluation semantics is indeed basic: if it is (weakly) compositional, so is the content semantics, and the character semantics. None of the unidirectional arrows in Proposition 5 can be reversed in general. We show in 6.4 that under a certain natural condition, some of the arrows reverse. First, however, an important comment to Proposition 5 is required.

6.2 Compositional vs. recursively defined semantics

The pleasant symmetric picture of Proposition 5 is in many cases somewhat illusory, since it often happens that the point of evaluation semantics has a *recursive truth definition* but is *not even weakly contextually compositional.* Tarski's truth definition for first-order logic is a familiar example: it recursively specifies when an *assignment* (of individuals to variables) satisfies a *formula* in a *model.* Regarding the model as fixed, and thinking of assignments as points of evaluation, the clause for the existential quantifier is:

(16) $\quad \mu(\exists x\varphi, f) = \mathrm{T}$ iff for some a, $\mu(\varphi, f(a/x)) = \mathrm{T}$

where $f(a/x)$ is like f except that it assigns a to x. The clause is recursive in that the right-hand side of (16) uses the value of μ for an expression of lower complexity. But the assignment argument is not the same as on the left-hand side, and precisely for that reason, μ is *not* (weakly) contextually compositional. Suppose, for example, that (in the given model) P denotes the empty set but R doesn't, and that f assigns to x an individual which is not in the denotation of R. Then $\mu(Px, f) = \mu(Rx, f) = \mathrm{F}$, but $\mu(\exists xPx, f) \neq \mu(\exists xRx, f)$. So $\mu(\exists xPx, f)$ cannot be calculated from $\mu(Px, f)$ and f.

Should we say that Tarski's definition is not compositional? But it is a familiar fact that if we take semantic values of formulas to be *sets of assignments* instead, we regain compositionality: the set of assignments satisfying $\exists x\varphi$ *can* be calculated (indeed using the clause (16)) from the set of assignments satisfying φ. In fact, this is just another way of saying that the *currying* of μ is compositional.

I will state some facts about the general situation, without attempting full detail or formal precision. In this setting, a recursive definition is best thought of as defining a *relation* S — so that in the case of a function we define its *graph* — by means of a sentence Φ in some suitable (usually first-order) interpreted meta-language, in which one can also talk about the syntactic objects in E. When (the graph of) a semantics F is being defined, Φ is usually a disjunction of *base clauses* (for atomic expressions) and *inductive clauses* for the operators (elements of Σ in our case). I restrict attention here to the case of sentence operators Δ. With Φ_Δ as the corresponding disjunct, we may then have

(17) $\quad F(\Delta\varphi, c, d, w) = \mathrm{T}$ iff $\Phi_\Delta(F(\varphi, c, t[d], w), c, d, w)$

Here c, d, w are parameters, varying over C, D, W, respectively. Think of c as a *context parameter* and w as a *circumstance parameter*, e.g. a world. I will call d a *shifted* parameter, since it is changed on the right-hand side: in its place there is a term constructed from d and possibly other terms, such as bound variables.[28] So F is a point of evaluation semantics. In particular cases some of the parameters can be missing; for example, in (16) above both c and w are absent, and $\Phi_\Delta(F(\varphi, t[f]), f)$ is, roughly, $\exists v F(\varphi, f(v/x)) = \mathrm{T}$.[29]

Various more abstract semantics, or notions of *content*, are obtainable by currying. Let F_{w} be the semantics obtained by currying w, $F_{\mathrm{d,w}}$ by also currying the shifted parameter d, and F_{d} by currying only d:

$$F_{\mathrm{w}}(\psi, c, d)(w) = F(\psi, c, d, w) = F_{\mathrm{d,w}}(\psi, c)(d)(w) = F_{\mathrm{d}}(\psi, c, w)(d)$$

We can now state various facts about the compositionality of F and its curried versions. To begin,

(I) $\quad$ *F_{d} is at least weakly contextually compositional. Hence, so is $F_{\mathrm{d,w}}$.*

The second claim follows from the first by Lemma 2. The following argument is not a strict proof of the first claim, but gives the idea. Consider an operator Δ with defining clause as in (17), and let r_Δ be an operation taking functions g in $Z = [D \longrightarrow \{\mathrm{T}, \mathrm{F}\}]$ and $c \in C$ and $w \in W$ to functions in Z, defined by

$$r_\Delta(g, c, w)(d) = \mathrm{T} \text{ iff } \Phi_\Delta(g(t[d]), c, d, w)$$

Then

$$\begin{aligned} r_\Delta(F_{\mathrm{d}}(\varphi, c, w), c, w)(d) = \mathrm{T} \quad &\Leftrightarrow \quad \Phi_\Delta(F_{\mathrm{d}}(\varphi, c, w)(t[d]), c, d, w) \\ &\Leftrightarrow \quad \Phi_\Delta(F(\varphi, c, t[d], w), c, d, w) \\ &\Leftrightarrow \quad F(\Delta\varphi, c, d, w) = \mathrm{T} \\ &\Leftrightarrow \quad F_{\mathrm{d}}(\Delta\varphi, c, w)(d) = \mathrm{T} \end{aligned}$$

Since this holds for all d,

[28] This conforms to the usual notion of 'shiftiness' in the literature, e.g. in Lewis [19], but not to Recanati [33], who instead uses the term for cases when (in our terms) *circ*(c) is not the circumstance *of* c; see section 4.1.

[29] I.e. $t[f]$ describes how the assignment $f(v/x)$ is formed from f by assigning v to the object language variable x.

$$F_{\mathrm{d}}(\Delta\varphi, c, w) = r_\Delta(F_{\mathrm{d}}(\varphi, c, w), c, w)$$

and we have weak (if c or w occurs in Φ_Δ; otherwise strong) contextual Δ-compositionality for F_{d}. □

Recall that in Proposition 5, *cont* = *poe-sem*$_{curr}$. Here w is absent, or treated as part of the shifted d parameter. Thus, when *poe-sem* is recursively definable but not (contextually) compositional, so that Proposition 5 doesn't apply, (I) claims that *cont* is nevertheless (contextually) compositional. In this sense, the general picture given by Proposition 5 is still valid.

Next, I will call the shift in (17) *trivial* if

(18) for all c, d, w, if $t[d] \neq d$, then $\Phi_\Delta(\mathrm{T}, c, d, w) \Leftrightarrow \Phi_\Delta(\mathrm{F}, c, d, w)$

For example, Kaplan defines in [13] a sentential operator *yesterday* (Y):

$$F(Y\varphi, d) = \mathrm{T} \;\text{ iff }\; F(\varphi, d-1) = \mathrm{T}$$

Here the contextual parameter varies over *days*, representable as integers. So the day parameter is non-trivially shifted with $t[d] = d-1$ (since today $\neq$ yesterday and $\mathrm{T} = \mathrm{T}$ is not equivalent to $\mathrm{T} = \mathrm{F}$). Likewise, the shifts occurring in (16) above, and in the modal example from the proof of Lemma 1, are non-trivial.

(II) *If the contextual parameter is only trivially shifted in the inductive clause for the operator Δ, then F is weakly contextually Δ-compositional. Hence, so is F_{w}.*

The second statement uses (the proof of) Lemma 2. For the first statement, suppose (18) holds. We claim that

(19) $\Phi_\Delta(F(\varphi, c, t[d], w), c, d, w) \iff \Phi_\Delta(F(\varphi, c, d, w), c, d, w))$

For if $t[d] = d$ this is obvious, and if $t[d] \neq d$, the claim follows from triviality. (19) means that

$$F(\Delta\varphi, c, d, w) = \mathrm{T} \iff \Phi_\Delta(F(\varphi, c, d, w), c, d, w)$$

But then it is clear that F is weakly contextually Δ-compositional. □

(III) *If the d-parameter is non-trivially shifted in a recursive definition of F, then F_{w} is not weakly contextually compositional. As before, it follows that neither is F.*

This time we need to assume something — the only occasion in this paper — about how F relates to models; roughly, that one can always choose interpretations of primitive symbols to attain relevant semantic values. Suppose, then, that there is an operator Δ satisfying (17) for which (18) fails, so that there are c_0, d_0, w_0 such that $t[d_0] \neq d_0$ and, say, $\Phi_\Delta(\mathrm{T}, c_0, d_0, w_0)$ but not

$\Phi_\Delta(\mathrm{F}, c_0, d_0, w_0)$. Our assumption is that we can find a model and sentences φ and φ' such that

$$F(\varphi, c_0, d_0) = F(\varphi', c_0, d_0)$$

and

$$F(\varphi, c_0, t[d_0], w_0) = \mathrm{T}$$
$$F(\varphi', c_0, t[d_0], w_0) = \mathrm{F}$$

Since $\Phi_\Delta(\mathrm{T}, c_0, d_0, w_0) \not\Leftrightarrow \Phi_\Delta(\mathrm{F}, c_0, d_0, w_0)$, $F(\Delta\varphi, c_0, d_0, w_0) \neq F(\Delta\varphi', c_0, d_0, w_0)$, and hence $F_{\mathrm{w}}(\Delta\varphi, c_0, d_0) \neq F_{\mathrm{w}}(\Delta\varphi', c_0, d_0)$. This contradicts weak contextual Δ-compositionality for F_{w}. □

These observations generalize the remark by Lewis in [19] mentioned in section 4.6. He noted that if *location* is shifted by some operator, but content doesn't take location as an argument, then content will not be compositional: the content of *Somewhere the sun is shining* at the location of c will depend on the content of *The sun is shining* at some other location. This is an instance of (III), with d as the location parameter and w as a world parameter (assuming the point of evaluation semantics can be recursively defined). Indeed, neither the point of evaluation semantics nor the one treating contents as functions from worlds to truth values is (weakly contextually) compositional. Lewis' point was precisely that the latter kind of content would not be compositional if the location parameter is shifted.

(II) generalizes the inverse claim, that if you *don't* shift the location parameter, (weak contextual) compositionality of content obtains. And (I) observes that if you curry shifted parameters, compositionality of the corresponding notion of content is guaranteed.

6.3 Compositionality and monsters

Semanticists in the tradition focused on here agree that the point of evaluation semantics provides the truth value of φ at context c in circumstance d, but differ as to what separates contexts from circumstances (section 2.1). But since a context always *determines* a circumstance (via the function *circ*), an abstract picture could be:

context c: $\langle \mathit{speaker}_c, \mathit{time}_c, \mathit{location}_c, \mathit{world}_c, \ldots\rangle$
circumstance d: $\langle \mathit{judge}_d, \mathit{time}_d, \mathit{world}_d, \ldots\rangle$

(I have included an optional judge for the relativist's sake; cf. section 4.5). A further point of agreement is that *shiftable* parameters belong to the circumstances. In view of (I) above, if *poe-sem* can be recursively defined, we have an *argument* for this:

(IV) *Circumstance parameters allow content to be compositional.*

Note that (IV) doesn't prevent non-shiftable parameters like judges from being placed among the circumstances. But shiftable parameters must *not* go in the context. In Kaplan's terminology, that would create *monsters*. He claimed that monsters don't exist in English, in fact, that 'none could be added' ([13], p. 510). The claim has been debated,[30] but we can use (III) to find a rationale for the quoted phrase in terms of compositionality:

(V) *Monsters destroy the compositionality of content.*

Given that compositionality of content is desirable, monsters are not. If you discover a monstrous contextual feature, relegate it to the circumstances.

6.4 Extensional composition

We have talked about *poe-sem*, *cont*, and *char*, but what about the semantics *ext*, i.e. in the case of sentences, the notion of truth at a context? MacFarlane [20] (p. 328–9) adapts a counter-example by Kaplan:

(20) a. It is always the case that I am here.
b. It is always the case that 2 + 2 = 4.

Clearly, *ext*('I am here',c) = *ext*('2 + 2 = 4',c) = T for all c, but in most contexts, (20-a) is false and (20-b) is true. So *ext* need not be even weakly contextually compositional, even when *poe-sem* (and hence *cont* and *char*) is.

Note that the counter-example involves an intensional (in this case temporal) operator. Is this essential? To appreciate the situation, let us lay down the following terminology.

(21) **Extensional semantic operations**

a. An operation $r : [X \longrightarrow Y]^n \longrightarrow [X \longrightarrow Y]$ is *extensional* iff $f_i(x) = f'_i(x'), i = 1, \ldots, n, \Longrightarrow r(f_1, \ldots, f_n)(x) = r(f'_1, \ldots, f'_n)(x')$.

b. $r : [X \longrightarrow Y]^n \times C \longrightarrow [X \longrightarrow Y]$ is *extensional* iff for every $c \in C$, r_c defined by $r_c(f_1, \ldots, f_n) = r(f_1, \ldots, f_n, c)$ is extensional.

Consider

(22) The president is sitting.

Calculating the content of (22) at a context c only requires extensional operators: $cont(\textit{the president}, c)$ is a function f which for each circumstance d selects an

[30] Israel and Perry [11] argue that even if there are no English monsters of the kind Kaplan considers ('metaphysical monsters'), this is at most an empirical fact, not a principled one. Moreover, they claim that a proper treatment of propositional attitudes requires context-shifting operators. Schlenker [35] argues that various languages, including English, do have monsters (*two days ago* is said to be an example). Other putative counter-examples, noted by Kaplan too, involve quotation contexts. Finally, consider the following title of an art exhibition in Göteborg (Konsthallen, fall 2008): "Tomorrow always belongs to us." This sentence is not about the day after any particular day.

individual $f(d)$ in M_0, $cont(sitting, c)$ is a function P such that each $P(d)$ is a subset of M_0,[31] and $cont((22), c) = r(f, P)$, where

$$r(f, P)(d) = \mathrm{T} \text{ iff } f(d) \in P(d)$$

Clearly, the operation r is extensional. But as we just saw, the presence of intensional operators in the language changes the situation. In such a language, we may expect *content* to be contextually compositional with extensional composition operations only for restricted fragments. Proposition 7 below confirms this expectation.

What about *character*? By Proposition 5, this is the weakest form of compositionality, and one certainly expects it to hold, but with what kind of composition operations? Suppose the speaker in c is the addressee in c', and the character of *sitting* is constant. Then *I am sitting* expresses the same (functional or structured) proposition in c as *You are sitting* does in c'. Again the simple example is perfectly extensional. But now recall Kaplan's injunction against monsters: no operators shift context. If so, there is no way to produce a counter-example like the previous one. So could we always require character compositionality to use extensional composition operators?

Not if there are context-shift failures as in the case of *John's books*, described in section 5.2. This was a counter-example to C-Funct(*cont*), but in fact shows that Funct(*char*) (which holds by Proposition 5, provided C-Funct$(cont)_w$ holds) cannot use extensional operations. The following result explains the situation.

Proposition 6
C-Funct(cont) is equivalent to Funct(char) holding with extensional composition operations. So in that case, the four conditions Funct($char_0$), Funct(char), C-Funct(cont), and C-Funct$(cont)_w$ are all equivalent.

Proof. One direction follows from the proof of Lemma 1. Assuming C-Funct(*cont*) with composition operations r_α, we in effect showed that Funct(*char*) holds with operations s_α defined by

$$s_\alpha(f_1, \ldots, f_n)(y) = r_\alpha(f_1(y), \ldots, f_n(y))$$

Clearly, the s_α are extensional. In the other direction, suppose Funct(*char*) holds with extensional composition operations r_α. We have

$$\begin{aligned} cont(\alpha(e_1, \ldots, e_n), c) &= char(\alpha(e_1, \ldots, e_n))(c) \\ &= r_\alpha(char(e_1), \ldots, char(e_n))(c) \\ &= r'_\alpha(char(e_1)(c), \ldots, char(e_n)(c)) \quad \text{(for some } r'_\alpha) \\ &= r'_\alpha(cont(e_1, c), \ldots, cont(e_n, c)) \end{aligned}$$

The existence of r'_α in the third equality is guaranteed by the extensionality of r_α: just define, for $p_1, \ldots, p_n \in [CIRC \longrightarrow M]$, $r'_\alpha(p_1, \ldots, p_n)$ to be equal to

[31] Assuming for simplicity that (22) expresses a temporal proposition (so c is irrelevant) and that the domain of individuals is the same in all circumstances.

$r_\alpha(f_1, \ldots, f_n)(c)$ if there exist $f_1, \ldots, f_n \in [CU \longrightarrow [CIRC \longrightarrow M]]$ and $c \in CU$ such that $p_i = f_i(c)$, for $1 \leq i \leq n$ (undefined or arbitrary otherwise). This definition works (is independent of which f_i and c are chosen) precisely because r_α is extensional. Thus, C-Funct(*cont*) holds. □

Note that the first half of the proof doesn't go through if only C-Funct$(\mathit{cont})_w$ is assumed. In that case, s_α was defined by

$$s_\alpha(f_1, \ldots, f_n)(y) = r_\alpha(f_1(y), \ldots, f_n(y), y)$$

and hence need *not* be extensional.

Here are the consequences of (contextual) *content* compositionality with extensional composition operators:

Proposition 7
If C-Funct(cont) holds with extensional composition operators, then C-Funct(poe-sem) and C-Funct(ext) both follow. Similarly for the weak variants.

Proof. Assume C-Funct(*cont*) with extensional operations r_α. We have

$$\begin{aligned} \mathit{poe\text{-}sem}(\alpha(e_1, \ldots, e_n), c, d) &= \mathit{cont}(\alpha(e_1, \ldots, e_n), c)(d) \\ &= r_\alpha(\mathit{cont}(e_1, c), \ldots, \mathit{cont}(e_n, c))(d) \\ &= s_\alpha(\mathit{cont}(e_1, c)(d), \ldots, \mathit{cont}(e_n, c)(d)) \text{ (for some } s_\alpha) \\ &= s_\alpha(\mathit{poe\text{-}sem}(e_1, c, d), \ldots, \mathit{poe\text{-}sem}(e_n, c, d)) \end{aligned}$$

The existence of s_α in the third equality is again guaranteed by the extensionality of r_α. So C-Funct(*poe-sem*) holds. Also, since $\mathit{ext}(e, c) = \mathit{poe\text{-}sem}(e, c, \mathit{circ}(c))$,

(23) C-Funct(*poe-sem*) implies C-Funct(*ext*).

The proof in the weak case is analogous, using (21-b). □

We saw in section 6.2 that, even if *poe-sem* can be recursively defined, it is (given a few assumptions) weakly contextually compositional exactly when no operators in the language shift circumstances. So as soon as there are such operators, C-Funct$(\mathit{cont})_w$ cannot hold with extensional composition operations, although, by (I), it does hold with non-extensional operations. Also, although the implication (23) cannot in general be reversed, in practice C-Funct(*ext*) will fail when C-Funct(*poe-sem*) does, and similarly for the weak version.

6.5 Compositionality with structured contents

Recall, from section 3.2 and (7) in section 4.3, that in this case we started with a character semantics char_s — or cont_s in its uncurried version — and then defined corresponding versions *char* and *cont* for functional contents via the mapping * (where $p^*(d) = \mathit{ref}(p[d])$, for $p \in \mathit{SCONT}$), from which we got the semantics *ext* and *poe-sem* in the usual way.

This already says a lot about how the various notions of compositionality involved are related. A small extra assumption gives us a little more. When $cont_s$ is contextually compositional, think of $SCONT$ as structured by the corresponding semantic operations r_α. Then we can require that the operation * is compositional too, i.e. that there exist operations s_α such that

(24) a. $r_\alpha(p_1, \ldots, p_n)^* = s_\alpha(p_1^*, \ldots, p_n^*)$,
b. and in the weak case, $r_\alpha(p_1, \ldots, p_n, c)^* = s_\alpha(p_1^*, \ldots, p_n^*, c)$

This gives us the following picture.

Proposition 8

$$\begin{array}{ccccc}
\textit{C-Funct}(\textit{poe-sem}) & \Longrightarrow & \textit{C-Funct}(\textit{poe-sem})_w & \Longrightarrow & \textit{Funct}(\textit{char}_0) \\
\Downarrow & & \Downarrow & & \Updownarrow \\
\textit{C-Funct}(\textit{cont}) & \Longrightarrow & \textit{C-Funct}(\textit{cont})_w & \Longrightarrow & \textit{Funct}(\textit{char}) \\
\Uparrow & & \Uparrow & & \\
\textit{C-Funct}(\textit{cont}_s) & \Longrightarrow & \textit{C-Funct}(\textit{cont}_s)_w & \Longrightarrow & \textit{Funct}(\textit{char}_s)
\end{array}$$

Proof. Note that the upper part of the diagram is exactly as in Proposition 5, and it is proved in exactly the same way, since the relations between the semantic functions involved are the same, even when these functions are derived from the given $char_s$. Moreover, the lower horizontal implications are again instances of Lemma 1. Next, let us show that C-Funct($cont_s$) implies C-Funct($cont$):

$$\begin{aligned}
\textit{cont}(\alpha(e_1, \ldots, e_n), c) &= \textit{cont}_s(\alpha(e_1, \ldots, e_n), c)^* \quad \text{(by definition)} \\
&= r_\alpha(\textit{cont}_s(e_1, c), \ldots, \textit{cont}_s(e_n, c))^* \\
&= s_\alpha(\textit{cont}(e_1, c)^*, \ldots, \textit{cont}(e_n, c)^*) \quad \text{(by (24-a))} \\
&= s_\alpha(\textit{cont}(e_1, c), \ldots, \textit{cont}(e_n, c))
\end{aligned}$$

Finally, the weak case is analogous, using (24-b). □

Relevant parts of the results in section 6.2 – 6.4 carry over more or less directly to structured contents; I will not pursue this further here.[32] Likewise, I will not go into the straightforward adjustments that various forms of relativist semantics (section 4.5) require for the results of this section to carry over.

[32] Except to note the following. Recanati [33] gives a functional account of content (the *lekton* for sentential content), but interposes 'Austinian content' between this and the extensions in M, in the form of *pairs* of contents and circumstances. In other words, he uses a function *acont* from $E \times CU \times CIRC$ to $ACONT = CONT \times CIRC$ defined by

(i) $\textit{acont}(e, c, d) = \langle \textit{cont}(e, c), d\rangle$

This is intermediate between a functional and a structured account. But since *acont* doesn't apply the content to the circumstance but only lists the two, one expects it to be (contextually) compositional just when *cont* is. The proof of the following fact is left as an exercise.

Fact 9
C-Funct(acont) is equivalent to C-Funct(cont), and similarly for the weak versions.

6.6 An application: Do characters compose?

King and Stanley [16] argue that speakers don't seem to use the character of *complex* expressions. Rather, they say, speakers use the character of the *simple* subexpressions, then plug in the contextual elements, and *then* compose to get the *content* of the complex expression. This part of the argument concerns the processing of meaning in speakers' minds. Another of their arguments is that it makes no difference for the end result whether characters are composed or not.

It is natural to take this is as a claim that semantic composition is done under the assumption of a fixed context, say c_0, so that, as in section 5.2, *cont* turns into a function ($char_{c_0}$) taking only expressions as arguments. However, if King and Stanley's reasoning is valid, it must surely be independent of any particular context chosen. By Fact 3, we then see that the claim that content is compositional however the context is fixed amounts exactly to weak contextual compositionality for *cont*. That is, although King and Stanley only talk about standard compositionality, their claim in effect expresses one of the versions of contextual compositionality. Interestingly, it is the weak version.

Furthermore, it follows, as we have seen, that King and Stanley's claim *entails* standard compositionality for character. So from the perspective of descriptive semantics, there is no opposition between their proposal and the claim that characters compose. This is of course compatible with the claim that in terms of psychological processing, speakers in fact do not compose character, a claim I will not try to assess here.[33]

7 Conclusions

The pervasive context-dependence of natural languages, in all its forms, may seem to conflict with compositionality, or systematicity. Hopefully, the observations in this paper can alleviate such worries, or at least clarify the issues at stake. We have seen that compositionality and context-dependence are not incompatible, indeed that contextual compositionality of content, far from being opposed to traditional compositionality of character or standing meaning, in fact *entails* it, and that the strong and weak versions of contextual compositionality relate to how semantic theories deal with things like context shift failure, unarticulated constituents, modulation, etc.

Furthermore, I have stated, albeit in a rough way, how the presence of a recursive truth definition for the model theorist's basic form of semantics relates to the compositionality of that same semantics, and to more abstract semantics (for content or character) obtainable from it. I also analyzed the effect of

[33]But note that King and Stanley will have to tell a plausible story about what speakers do when context doesn't allow them to fix the reference of indexicals. Suppose I hear through my hotel room wall someone shouting "You don't love me anymore!", or I find a note on the sidewalk saying "Please help, I am locked in the basement since yesterday!". If the answer is that discourse referents or something similar are introduced as referents of *me*, *you*, etc. so that composition of content can be performed, that doesn't seem very different from forming the complex character of those sentences.

intensional operators on compositionality, with applications to the distinction between context and circumstance, to so-called monsters, and to the conditions under which extension (*Bedeutung*) can be compositional. Finally, I indicated how these facts extend to semantic accounts that posit structured contents.

In sum, the results here, though not surprising or mathematically deep, show in what forms and for which kinds of semantic values one may reasonably raise the issue of compositionality, when extra-linguistic context is taken seriously.

References

[1] John Barwise and John Etchemendy. *The Liar: An Essay on Truth and Circularity*. Oxford University Press, New York, 1987.

[2] Berit Brogaard. Introduction to *Relative Truth*. *Synthese*, 166:2:215–229, 2009 (online 2007).

[3] Herman Cappelen. The creative interpreter: content relativism and assertion. *Philosophical Perspectives*, 22:23–46, 2008 (online version).

[4] Gareth Evans. Does tense logic rest on a mistake? In *Collected Papers*, pages 343–363. Clarendon Press, Oxford, 1985.

[5] Manuel García-Carpintero and Max Kölbel, editors. *Relative Truth*. Oxford University Press, Oxford, 2008.

[6] Manuel García-Carpintero and Josep Macia, editors. *Two-dimensional Semantics: Foundations and Applications*. Oxford University Press, Oxford, 2006.

[7] Michael Glanzberg. Semantics and truth relative to a world. *Synthese*, 166:2:281–307, 2009 (online 2007).

[8] Richard Heck. Do demonstratives have senses? *Philosophers' Imprint*, 2:2:1–33, 2001.

[9] Herman Hendriks. Compositionality and model-theoretic interpretation. *Journal of Logic, Language and Information*, 10:29–48, 2001.

[10] Wilfrid Hodges. Formal features of compositionality. *Journal of Logic, Language and Information*, 10:7–28, 2001.

[11] David Israel and John Perry. Where monsters dwell. In Jerry Seligman and Dag Westerståhl, editors, *Logic, Language, Computation*, pages 303–316. CSLI Publications, Stanford, 1996.

[12] Hans Kamp. Formal properties of 'now'. *Theoria*, 37:227–274, 1971.

[13] David Kaplan. Demonstratives: An essay on the semantics, logic, metaphysics, and epistemology of demonstratives and other indexicals. In Joseph Almog, John Perry, and Howard Wettstein, editors, *Themes from Kaplan*, pages 481–566. Oxford University Press, Oxford, 1989.

[14] Jeffrey C. King. Tense, modality, and semantic values. In John Hawthorne and Dean Zimmerman, editors, *Philosophical Perspectives 17: Language and Philosophical Linguistics*, pages 195–246. 2003.

[15] Jeffrey C. King. Structured propositions. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy.* URL = http://plato.stanford.edu/archives/fall2006/entries/propositions-structured/, Fall 2006.

[16] Jeffrey C. King and Jason Stanley. Semantics, pragmatics, and the role of semantic content. In Zoltan Szabo, editor, *Semantics versus Pragmatics*, pages 111–164. Oxford University Press, Oxford, 2005.

[17] Max Kölbel. Faultless disagreement. *Proceedings of the Aristotelian Society*, 104:53–73, 2004.

[18] Peter Lasersohn. Context dependence, disagreement, and predicates of personal taste. *Linguistics and Philosophy*, 28:643–686, 2005.

[19] David Lewis. Index, context, and content. In Stig Kanger and Sven Öhman, editors, *Philosophy and Grammar*. D. Reidel, Dordrecht, 1980.

[20] John MacFarlane. Future contingents and relative truth. *The Philosophical Quarterly*, 53:212:321–336, 2003.

[21] John MacFarlane. Making sense of relative truth. *Proceedings of the Aristotelian Society*, 105:321–339, 2005.

[22] John MacFarlane. Relativism and disagreement. *Philosophical Studies*, 132:1:17–31, 2007.

[23] John MacFarlane. Truth in the garden of forking paths. In Manuel García-Carpintero and Max Kölbel, editors, *Relative Truth*, pages 81–102. Oxford University Press, Oxford, 2008.

[24] John MacFarlane. Nonindexical contextualism. *Synthese*, 166:2:231–250, 2009.

[25] Richard Montague. Pragmatics. In Richmond Thomason, editor, *Formal Philosophy*, pages 95–118. Yale University Press, New Haven, 1974 (1968).

[26] Richard Montague. Universal grammar. In Richmond Thomason, editor, *Formal Philosophy*, pages 222–246. Yale University Press, New Haven, 1974 (1970).

[27] Peter Pagin. Compositionality and context. In G. Preyer and G. Peter, editors, *Contextualism in Philosophy*, pages 303–348. Oxford University Press, Oxford, 2005.

[28] Peter Pagin and Francis Jeffry Pelletier. Content, context and communication. In G. Preyer and G. Peter, editors, *Content and Context. Essays on Semantics and Pragmatics.* Oxford University Press, Oxford, 2007.

[29] Peter Pagin and Dag Westerståhl. Compositionality I (definitions and variants) and II (arguments and problems). *Philosophy Compass*, 5(3):250–282, 2010.

[30] John Perry. Thought without representation. *Supplementary Proceedings of the Aristotelian Society*, 60:137–152, 1986. Reprinted in Perry, *The problem of the Essential Indexical and Other Essays*, Oxford University Press, Oxford, 1993.

[31] Philosophical Perspectives. *Special issue: Philosophy of Language*, volume 22. 2008 (online version).

[32] Willard Van Orman Quine. *Word and Object*. MIT Press, Cambridge, 1960.

[33] François Recanati. *Perspectival Thought. A Plea for (Moderate) Relativism*. Oxford University Press, Oxford, 2007.

[34] François Recanati. Compositionality, flexibility, and context-dependence. In *this volume*. 2008.

[35] Philippe Schlenker. A plea for monsters. *Linguistics and Philosophy*, 26:29–120, 2003.

[36] Dana Scott. Advice on modal logic. In Karel Lambert, editor, *Philosophical Problems in Logic*, pages 143–173. D. Reidel, Dordrecht, 1970.

[37] Krister Segerberg. Two-dimensional modal logic. *Journal of Philosophical Logic*, 2:77–96, 1973.

[38] Jason Stanley. *Knowledge and Practical Interests*. Oxford University Press, Oxford, 2005.

[39] Synthese. *Special issue: Relative Truth*, volume 166:2. 2009.

[40] Dag Westerståhl. On the compositional extension problem. *Journal of Philosophical Logic*, 33:549–582, 2004.