

# Does English Really Have Resumptive Quantification And Do ‘Donkey’ Sentences Really Express It?

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(1) divide quantification in natural languages into D Quantification and A Quantification. We note that if A Quantification is taken to be resumptive, English ‘donkey’ sentences do not fit into that category. Indeed, ‘donkey’ sentences with determiners are not counterexamples to the claim, which appears to be correct, that determiners monoselectively bind just one variable. We also observe that while the ‘proportion problem’ applies to a resumptive analysis of ‘donkey’ sentences, it does not apply to proper instances of resumptive quantification in English.

## 1 Why would it matter if English has resumptive quantification?

The University of Massachusetts project on Quantification in Natural Languages (Bach et al. (1)) concluded that language employs two different kinds of quantification: A Quantification and D Quantification.

- *A Quantifiers* bind multiple variables unselectively, express resumptive quantification, and often surface as adverbs.
- *D Quantifiers* bind one specified variable and usually surface as determiners.

We show that while English adverbs can bind multiple variables simultaneously (though perhaps selectively) to express resumptive quantification, English ‘donkey’ sentences are not instances of this phe-

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nomenon, nor are they as some have proposed counterexamples to the claim that determiners bind only one variable.<sup>1</sup>

## 2 What is resumptive quantification?

Resumptive quantification is the meaning expressed by employing a monadic quantifier to bind multiple variables simultaneously in order to quantify over tuples of the entities the variables range over.

The monadic quantifier *EVERY* binds one variable to form the proposition that each entity over which the variable ranges and which satisfies the condition restricting the quantifier's domain also satisfies the condition expressed by the quantifier's scope. Thus

- (1) Every donkey limps.

uses the monadic quantifier *EVERY* to bind, say, the variable  $x$  in both the domain restriction ' $x$  is a donkey' and the scope ' $x$  limps'. In

- (2) An environmentalist always despises a developer.

the resumption of *EVERY* binds, say, the two variables  $x$  and  $y$ , in the domain restriction ' $x$  is an environmentalist and  $y$  is a developer' as well as in the scope ' $x$  despises  $y$ '.<sup>2</sup> Thus (2) expresses the claim that all pairs of an environmentalist and a developer satisfy the condition that the former despises the latter.

*Monadic* quantification is quantification over individual entities. *Polyadic* quantification is quantification over ordered pairs, triples, or, in general,  $n$ -tuples of entities. Resumptive quantification is thus a (very) special kind of polyadic quantification. While a polyadic quantifier over pairs can be any of a wide range of properties of, or relations between, sets of pairs of individuals, a resumptive pair quantifier is a property (or relation) given by a monadic quantifier applied to pairs.

Thus, in (2), *always* can be taken to stand for the relation of inclusion between the set  $E \times D$  of pairs of environmentalists and developers and the set  $R$  of pairs  $(x, y)$  such that  $x$  despises  $y$ , a relation given by the monadic quantifier *EVERY*. Contrast this with

- (3) Every environmentalist despises at least two developers.

<sup>1</sup>Chierchia (2) came to similar conclusions to our Claims I and II (v. Section 7 below). We extend the data and reasoning with aid of concepts and results from Kanazawa (6) and Westerståhl (14).

<sup>2</sup>We use small caps for the abstract variable-binding operators — quantifiers — corresponding to certain English determiners and adverbs. We furthermore extend this notation to the second-order relations — often called 'quantifiers' as well — corresponding to such operators.

This combines two monadic quantifiers, EVERY and AT LEAST TWO, but from a logical point of view it can also be seen as polyadic quantification, expressing another relation between  $E \times D$  and  $R$ , namely, (on the default scope reading) that each element in  $E$  stands in the relation  $R$  to at least two elements of  $D$ . But this pair quantifier is *not* a resumption; there is no single corresponding monadic quantifier in this case.<sup>3</sup>

Therefore, while a claim that English A Quantification, in the case where two variables are bound, always expresses quantification over pairs would be trivially true, the claim that such quantification always expresses resumptive quantification would be much stronger — and in fact false (cf. section 7).

### 3 Does English have resumptive quantification?

Dorothy Parker quipped:

- (4) Men seldom make passes at girls who wear glasses.

Following (9), the adverb *seldom* is commonly analyzed as expressing a polyadic quantifier SELDOM which is the resumption of the monadic quantifier FEW. Accordingly (4) is taken to mean

- (5) Few pairs of a man and a girl who wears glasses are such that the former makes passes at the latter.

Similarly,

- (6) New cars are seldom incapable of climbing steep hills.

means

- (7) Few pairs of a new car and a steep hill are such that the former is incapable of climbing the latter.

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<sup>3</sup>This can be shown as follows: Suppose (for reductio) that (3) expresses the resumption of a monadic quantifier  $Q$ . Then, presumably, the following sentence also involves the resumption of  $Q$ :

Every developer is despised by at least two environmentalists.

Now, assuming that  $Q$ , like monadic natural language quantifiers in general, is invariant for permutations of objects in the domain, it follows that the resumption of  $Q$  is invariant for permutations of *pairs* of such objects. But the function mapping  $\langle a, b \rangle$  to  $\langle b, a \rangle$  is a permutation of pairs, which maps the relevant sets of pairs in these two sentences to each other. For example, it maps the set of environmentalist-developer pairs to the set of developer-environmentalist pairs. From this it follows that the two sentences are logically equivalent. But clearly they are not equivalent. Hence they are not resumptions (of the same  $Q$ ).

Lewis's example was

- (8) Politicians are usually willing to help constituents.

Under the hypothesis that the adverb *usually* here expresses a polyadic quantifier USUALLY which is the resumption of the monadic quantifier MOST, (8) means

- (9) Most pairs of a politician and a constituent are such that the former is willing to help the latter.

The hypotheses about the interpretation of the adverbs in sentences such as these are widely though not universally accepted as being reasonably well supported, and the predicted truth conditions for sentences (2), (4), (6), and (8) are apparently correct.

#### 4 Why would it matter whether 'donkey' sentences really express resumptive quantification?

If English D Quantifiers show up as determiners and really bind one specified variable, then quantificational determiners should always be monadic. In point of fact, not only quantificational determiners but also quantificational agreement affixes of verbs and quantificational focus particles may well monoselectively bind just one variable.

Recall, however, that Kamp's Discourse Representation Theory (Kamp (5)) and Heim's File Change Semantics (Heim (4)) were originally motivated in part by the fact that the meaning of sentences like

- (10) Every farmer who owns a donkey beats it.

apparently involves universal quantification over pairs of a farmer and a donkey he or she owns, and by the similarity of this sort of quantification to the adverbial quantification studied in (9). This similarity is re-emphasized in (1). If their analysis of this fact is correct, determiner quantified 'donkey' sentences do express resumptive quantification and thus are counterexamples to the claim that quantificational determiners monoselectively bind just one variable and express monadic quantification.

#### 5 Do 'donkey' sentences really express resumptive quantification?

Kamp's and Heim's rules for giving sentence (10) its meaning assign no quantifier to the 'donkey' determiner *a*, thus keeping the 'donkey'



variable free to recur as the interpretation of the ‘donkey’ pronoun *it*. Then they have the ‘farmer’ determiner *every* bind two variables, expressing resumptive quantification.

Using these rules on sentences like (11) yields the wrong meaning altogether, as Partee pointed out.

(11) At least two farmers who own a donkey are happy.

Statement (11) is false if Jones and Smith are the only existing farmers, farmer Jones is unhappy, and farmer Smith is happy. However, the resumptive quantification analysis Kamp’s and Heim’s rules give (11) would make it true if farmer Smith owns at least two donkeys, despite the obvious irrelevance of this fact to the truth value of statement (11).

Likewise, (6) pointed out that

(12) At least two farmers who own a donkey beat it.

is false if farmer Jones doesn’t beat the one donkey he owns while farmer Smith beats both of his donkeys (again assuming no other farmers exist).

(12) drew attention to the corresponding failure of Kamp’s and Heim’s resumptive quantification analysis for

(13) Most farmers who own a donkey beat it.

These examples illustrate what has become known as the ‘proportion problem’; cf. also section 7 below. Far from being an exception, this problem arises for almost all quantifiers in the ‘farmer’ position of ‘donkey’ sentences.<sup>4</sup>

## 6 What do ‘donkey’ sentences really express?

(6) showed that regardless of what quantifier the ‘farmer’ determiner expresses, a ‘donkey’ sentence such as

(14) Q farmer(s) who own(s) a donkey beat it.

expresses one (or sometimes, ambiguously, more) of the following three propositions.<sup>5</sup>

<sup>4</sup>(3), who deals with several of the issues discussed in this note, gives a characterization of the quantifiers for which the ‘proportion problem’ for the corresponding ‘donkey’ sentences does *not* arise: essentially these are just the quantifiers ALL, NO, SOME, and NOT ALL.

<sup>5</sup>Actually, Kanazawa did not include (17) as an alternative, claiming that in all *clear* cases, either (15) or (16) is meant. (17) was suggested in (12) as a possible reading of (13). Intuitions about the precise meaning of (13) are notoriously unclear. We include (17) here as a candidate reading of ‘donkey’ sentences, without taking a stand on whether this reading actually occurs.

- (15) Q farmer(s) who own(s) at least one donkey beat(s) every donkey he/she/they own(s).
- (16) Q farmer(s) who own(s) at least one donkey beat(s) at least one donkey he/she/they own(s).
- (17) Q farmer(s) who own(s) at least one donkey beat(s) Q donkey(s) he/she/they own(s).

Sentence (10) expresses (15) or equivalently (17), with Q = EVERY. Sentence (12) expresses (16), with Q = AT LEAST TWO. Sentence (13) expresses (16) or maybe (17), with Q = MOST.

As these examples illustrate, one or more of the three candidate meanings may be unavailable for a given ‘donkey’ sentence. The main point of Kanazawa’s paper was to present and justify a hypothesis explaining which reading is preferred and why. However, that hypothesis is not important for us here. We observe merely that no counterexamples have been adduced to the generalization that every meaning of a ‘donkey’ sentence of the form (14) is one of (15), (16), and (17).

Note that to assign these meanings one uses the ‘farmer’ quantifier monadically; it binds just the ‘farmer’ variable. The ‘donkey’ determiner *a* simply and straightforwardly expresses existential quantification.

Only the ‘donkey’ pronoun *it* requires special treatment. We treat the anaphoric dependency in this construction as introducing a quantifier to bind a variable ranging over donkeys owned by farmer *x*. The introduced quantifier is universal (15), existential (16), or may have the same force as the ‘farmer’ quantifier (17). Introduction of this quantifier is required not only when ‘donkey’ anaphora is expressed explicitly by a pronoun, but as Kanazawa noted also in sentences such as

- (18) Few politicians who run for an office win.

where it is implicit. Sentence (18) means (16), that is

- (19) Few politicians who run for an office win an office they run for.

The fact that the meanings thus produced are, in the case of some ‘donkey’ sentences, equivalent with the resumptive quantification Kamp and Heim postulate in those cases appears to be one explanation of why some linguists have thought (erroneously) that English D Quantification can bind more than one variable. But every case of resumptive quantification giving correct truth conditions is equivalent to one (or more) of (15), (16), and (17). Wherever resumptive quantification is

not equivalent to one (or more) of (15), (16), and (17), it gives wrong truth conditions for the ‘donkey’ sentence.

Kanazawa’s analysis of ‘donkey’ anaphora also gives correct truth conditions for sentence (11), avoiding the problems associated with resumptive quantification. Thus, we conclude, ‘donkey’ sentences do not express resumptive quantification (except ‘accidentally’ when that is equivalent to what they systematically express).

In fact, the ‘farmer’ quantifier in ‘donkey’ sentences binds only the ‘farmer’ variable, not also the ‘donkey’ variable, which is bound by the ‘donkey’ determiner *a*. The following examples show that the final quantifier in (15), (16) and (17) comes from the ‘donkey’ anaphoric dependency within the scope of the ‘farmer’ quantifier.

(20) Every farmer who owns exactly one donkey beats it.

(21) Few politicians who run for exactly one office win.

(22) Q farmer(s) who own(s) at least two donkeys beat(s) them.

It is inescapable in these cases that the ‘donkey’ quantifier EXACTLY ONE or AT LEAST TWO binds the ‘donkey’ variable inside the domain restriction on the ‘farmer’ quantifier EVERY or Q. The option of leaving the ‘donkey’ variable free to be bound by the ‘farmer’ quantifier does not exist. So the last quantifier in (23), (24), and (25)

(23) Every farmer who owns exactly one donkey beats every/at least one/the one donkey he owns.

(24) Few politicians who run for exactly one office win every/at least one/the one office they run for.

(25) Q farmer(s) who own(s) at least two donkeys beat(s) every/at least one/Q donkey(s) he/she/they own(s).

has to be introduced in association with the ‘donkey’ anaphoric dependency in the scope of the ‘farmer’ quantifier.

## 7 Doesn’t the ‘proportion problem’ arise for resumptive quantification in general?

It is noteworthy that the ‘proportion problem’ for a resumptive analysis of ‘donkey’ sentences, illustrated above with (11), (12), and (13), does not arise for proper resumptive quantification as in (4), (6), and (8).

The problem for ‘donkey’ sentences was that among the farmer-donkey pairs in which the farmer owns the donkey, the number or proportion of pairs where the farmer also beats the donkey may differ drastically from the number or proportion of farmers who beat donkeys that they own, among the donkey-owning farmers in general. This can happen when few farmers own many donkeys apiece, depending on how many of their own donkeys these farmers beat. This possibility is one important fact excluding a resumptive analysis of ‘donkey’ sentences. Why doesn’t it also rule out a resumptive analysis of (2), (4), (6), and (8)?

Consider a simple A Quantification sentence like (26),

(26) Cats usually dislike dogs.

taken to mean that most (more than half of the) cat-dog pairs are such that the cat dislikes the dog. Can this be true while at most half of the cats dislike at least one dog? Clearly not. To see the general principle at work here, we first need to introduce some notation.

Typically, a determiner denotes a quantifier  $Q$  (like EVERY, MOST, AT LEAST TWO, etc.), which on each universe of discourse is a binary relation between subsets of the universe. So the meaning of a simple quantified sentence like

(27) Most cats purr.

can be rendered in a relational format (with  $Q = \text{MOST}$ ) as (27'),

(27')  $\text{MOST}(A, C)$

where  $A$  is the set of cats and  $C$  the set of things that purr (we omit reference to the universe). The resumption of  $Q$  to pairs, written  $Q^2$ , is thus a binary relation between sets of pairs of individuals, i.e., between binary relations among individuals. In rendering English resumptive sentences like (26), the first argument usually has the form of a cartesian product  $A \times B$ . Thus, (26) becomes (26'),

(26')  $\text{MOST}^2(A \times B, R)$

with  $B$  as the set of dogs and  $R$  as the relation of disliking.

Finally, consider a typical English sentence with a transitive verb and quantified subject and object, like

(28) Most cats dislike some dogs.

This says that the MOST relation holds between the set of cats and the set of things that dislike some dogs, i.e., the set of things  $a$  such that the SOME relation holds between the set of dogs and the set of things that  $a$  dislikes. Using the notation  $R_a = \{b : R(a, b)\}$ , (28) thus becomes (28').

$$(28') \text{ MOST}(A, \{a : \text{SOME}(B, R_a)\})$$

In general, given any two determiner denotations  $Q_1$  and  $Q_2$ , there is in English a natural way of expressing their *iteration*, which is defined by

$$Q_1(A, \{a : Q_2(B, R_a)\})$$

for any sets  $A, B$  and any binary relation  $R$ . Notice that the inverse scope reading of (28), i.e., the reading which says that there are some dogs which most cats dislike, becomes in our relational notation

$$\text{SOME}(B, \{b : \text{MOST}(A, (R^{-1})_b)\})$$

(where for all  $a, b$ :  $R^{-1}(b, a)$  iff  $R(a, b)$ ).

Now we can state the following

**Fact A:** When  $Q$  is MOST, or more generally any proportional quantifier like MORE THAN  $m/n^{\text{THS}}$  OF THE, the resumptive sentence

$$(29) Q^2(A \times B, R)$$

logically implies each of the iterations

$$(30) Q(A, \{a : \text{SOME}(B, R_a)\})$$

and

$$(31) Q(B, \{b : \text{SOME}(A, (R^{-1})_b)\}).^6$$

So (26) logically implies that most cats dislike at least one dog, and that most dogs are disliked by at least one cat, which is why no ‘proportion problem’ can arise.

On the other hand, the truth condition for a resumptive analysis of a typical ‘donkey sentence’ is *not* (29) but (32),

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<sup>6</sup>Here is a proof of Fact A: Let  $|X|$  be the cardinality of the set  $X$ . Suppose  $|A| = p$ ,  $|B| = q$ , say,  $A = \{a_1, \dots, a_p\}$ . Without loss of generality we may assume  $R \subseteq A \times B$ . Then, clearly,

$$|R| = |R_{a_1}| + \dots + |R_{a_p}|.$$

Now suppose that (30) is false, for  $Q = \text{MORE THAN } m/n^{\text{THS}}$  OF THE. Then, for at most  $m/n^{\text{THS}}$  of the  $a_i$ ’s does it hold that  $|R_{a_i}| \neq 0$ . Let  $k$  be the largest natural number  $\leq m/n \cdot p$ . It follows, since  $|R_{a_i}| \leq q$  for each  $i$ , that

$$|R| \leq k \cdot q \leq m/n \cdot p \cdot q = m/n \cdot |A \times B|,$$

so (29) is false. Therefore, (30) follows from (29), and similarly for (31).

$$(32) Q^2((A \times B) \cap S, R)$$

where the relation  $S$  (OWNS) comes from the relative clause, and (33) does *not* follow logically from (32).

$$(33) Q(\{a \in A : \text{SOME}(B, S_a)\}, \{a : \text{SOME}(B, (S \cap R)_a)\})$$

That is, we can have a situation where (32) is true but (33) is false, exemplifying a ‘proportion problem’.<sup>7</sup>

Let us also remark that the observation that a resumptive analysis of ‘donkey’ sentences leads to the ‘proportion problem’ has nothing to do with whether D or A Quantification is used. For

$$(34) \text{Farmers who own donkeys/a donkey usually beat them/it.}$$

is also a ‘donkey’ sentence, and a resumptive analysis would give a truth condition of the form (32), leading to the ‘proportion problem’ again. Thus, (34) is not an instance of resumptive quantification, even though it is an instance of A Quantification. So although (34) can undoubtedly be construed as quantifying over pairs of individuals, that quantification is not resumption, i.e., it cannot be construed as using an ordinary monadic quantifier over a domain of pairs. Indeed, its truth condition appears to be the one expressed in (15), with  $Q = \text{MOST}$ .

We are, in effect, making the following claims:

**Claim I:** English D Quantification is monadic; in particular, it never expresses resumptive quantification.

**Claim II:** ‘Donkey’ sentences likewise do not express resumptive quantification in English.

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<sup>7</sup>The formal analogy between (30) and (33) can be brought out further as follows. (29) and (32) are both of the form

$$(*) Q^2(S', R),$$

where  $S' \subseteq A \times B$ . If we assume  $B \neq \emptyset$  it furthermore follows (using also the fact that  $Q$  is *conservative*, i.e., that  $Q(X, Y) \Leftrightarrow Q(X, X \cap Y)$  for all  $X, Y$ ) that (30) and (33) can then be written

$$(**) Q(\{a \in A : \text{SOME}(B, S'_a)\}, \{a \in A : \text{SOME}(B, (S' \cap R)_a)\}).$$

When  $S' = A \times B$  (so  $S'_a = B$  for  $a \in A$ ), (\*\*) follows from (\*) by Fact A. But (\*\*) does not follow from (\*) when  $S'$  is a proper subset of  $A \times B$ .

To put the same point less formally:

- (i) If  $R$  is ‘big’ relative to  $A \times B$  then  $\text{dom}(R)$  is ‘big’ relative to  $A$  (and  $\text{range}(R)$  relative to  $B$ ), by Fact A.
- (ii) But that  $R$  is ‘big’ relative to  $S \subseteq A \times B$  says next to nothing about the relative sizes of  $\text{dom}(R)$  and  $\text{dom}(S)$ .

**Claim III:** On the other hand, some simple forms of A Quantification in English, like (26), and presumably (2), (4), (6), and (8), do express resumptive quantification (and there is no ‘proportion problem’ in this case).

## 8 Are there other examples of resumptive D Quantification?

Enough has been said above about Claim II. Concerning Claim I, however, a putative counterinstance, which does not appeal to ‘donkey’ sentences, was put forward in (13) (and repeated in Keenan and Westerstähl (8)), with an example originating from Hans Kamp:

(35) Most lovers will eventually hate each other.

This has the reading that most pairs of people who love each other are pairs of people who will eventually hate each other, which, over a universe of individuals, is precisely a resumptive reading. Furthermore, since the same person may belong to different ‘loving pairs’, (35) is not equivalent to monadic versions like

(36) Most persons who love and are loved by someone will eventually hate someone/everyone they love.

However, in this and other similar cases, like (37),

(37) Few twins hate each other.

both the restricting (plural) noun and the (reciprocal) verb phrase indicate that we here have a *plural* predicate of sets or groups, as in

(38) Most twins agree about resumptive quantification.

Thus, allowing plural entities in the universe, as it seems we must anyway, (35) and (37) cease to be counterexamples, and Claim I stands.

## 9 How can we be certain that English has resumptive quantification?

Finally, we come back to Claim III in a little more detail. What does it mean that English sentences of a certain form *express* resumptive quantification? Presumably, the following two things:

- (i) There is a uniform syntactic and semantic analysis of sentences of this form in terms of resumptive quantification.

(ii) There is no plausible analysis without resumption.

We have indicated how (i) may be true (but note that such a uniform analysis must find a way to exclude adverbial ‘donkey’ sentences like (34)). However, this might still not be considered sufficient to establish Claim III if (ii) were false, namely if it were the case that the truth condition (29), repeated here as (39),

$$(39) \quad Q^2(A \times B, R)$$

of a typical simple English A Quantification sentence is logically equivalent to some simpler form of quantification. For example, suppose (counterfactually, as we shall see) that for each  $Q$  there exist quantifiers  $Q_1$  and  $Q_2$  such that (39) is logically equivalent to the iteration (40),

$$(40) \quad Q_1(A, \{a : Q_2(B, R_a)\})$$

which corresponds to a sentence of the form of

$$(41) \quad Q_1 \text{ cats dislike } Q_2 \text{ dogs.}$$

That is, suppose simple resumption were *reducible* in the sense of (7). Then one could argue that Claim III was too strong.

Claim III says something about *expressive power*, and facts about expressive power can sometimes be *proved* using only the logical tools of model-theoretic semantics. Such facts may then constitute evidence for linguistic claims. Here is an example:

**Proposition B:** The resumption of MOST is not reducible. That is, for  $Q = \text{MOST}$ , (39) is not equivalent to (40), for any  $Q_1$  and  $Q_2$ .<sup>8</sup>

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<sup>8</sup>This follows from a result in (14) in the following way: If

$$(\dagger) \quad \text{MOST}^2(A \times B, R) \Leftrightarrow Q_1(A, \{a : Q_2(B, R_a)\})$$

then, since  $\text{MOST}^2(A \times B, R) \Leftrightarrow \text{MOST}^2(B \times A, R^{-1})$ , it follows that

$$(\dagger\dagger) \quad Q_1(A, \{a : Q_2(B, R_a)\}) \Leftrightarrow Q_1(B, \{a : Q_2(A, (R^{-1})_a)\})$$

for all  $A, B, R$ . In particular, with  $B = A$ ,

$$(\dagger\dagger\dagger) \quad Q_1(A, \{a : Q_2(A, R_a)\}) \Leftrightarrow Q_1(A, \{a : Q_2(A, (R^{-1})_a)\}).$$

But by Theorem 4.12 in (14),  $(\dagger\dagger\dagger)$  holds for very few quantifiers pairs  $(Q_1, Q_2)$ , and certainly not for any that would make  $(\dagger)$  true. QED

Compare footnote 3 above: There we were *given*  $Q_1$  and  $Q_2$ , and our knowledge of their meaning told us that  $(\dagger\dagger)$  was false in that case. Here, we need the stronger result that  $(\dagger\dagger)$  is in fact only true for a few special cases (like  $Q_1 = Q_2 = \text{SOME}$ ), and this requires a proof.



Proposition B clearly goes some way to establish Claim III. It says that the resumption of a common quantifier is *not definable* in the form (40), which corresponds to a widespread type of quantified English sentence.

But couldn't it be the case that there were some *other* way to express (39), which also used monadic quantification only, and which corresponded to some other form of English sentences? Theorem C below gives us strong evidence that this is not so, thereby clinching, it would seem, the case for Claim III.

To formulate this theorem we need to observe that (39) and (40), expressing the form of truth conditions for certain English sentences, can themselves be recast as sentences in a formal language, more precisely, the language of *logics with generalized quantifiers*. We write  $\text{FO}(Q_1, \dots, Q_n)$  for first-order logic with added (generalized) quantifiers  $Q_1, \dots, Q_n$ . Then (39) would be written

$$(42) \quad Q^2xy(A(x) \wedge B(y), R(x, y))$$

and (40) becomes the following.

$$(43) \quad Q_1x(A(x), Q_2y(B(y), R(x, y)))$$

For details about the syntax and semantics of these formal languages (logics), see, for example, (13).

Now it is most certainly not the case that every sentence in such a logic is the truth condition of some form of English sentences. But the relevance of results like Theorem C to linguistic claims depends on some sort of converse of this being true. For then, if *no* sentence of such a logic defines (39), *a fortiori* no corresponding English sentence will define it. Of course, such a converse would have to be formulated with some care, since the expressive power of logics of the form  $\text{FO}(Q_1, \dots, Q_n)$  is very poor, *except* as regards quantification, so the claim would only be that English ‘quantificational structure’ can be captured (up to truth conditions) in such logics.

A final remark is that so far our only examples of monadic quantifiers have been of the kind denoted by determiners like *every*, *most*, *all but five*, etc., which (on each universe) can be taken as binary relations between sets of individuals. In general, a monadic quantifier is any  $n$ -place relation ( $n \geq 1$ ) between such sets. One can show that the expressive power increases with  $n$ , i.e., that for each  $n$  there are  $(n+1)$ -ary monadic quantifiers which are not definable in terms of any (finite number of) monadic quantifiers of arity at most  $n$ . Therefore, the claim, as in the next result, that a certain quantifier is not definable

in terms of *any* (finite number of) monadic quantifiers is a very strong claim indeed.

**Theorem C:** (Luosto (11)) For  $Q = \text{MOST}$ , (42) is not equivalent to any sentence in any logic of the form  $\text{FO}(Q_1, \dots, Q_n)$ , where  $Q_1, \dots, Q_n$  are monadic quantifiers.

Note that Proposition B is a special case of Theorem C. The proof of Theorem C, which is due to Kerkko Luosto, is too complicated to be even hinted at here.<sup>9</sup>

## 10 Conclusions

The generalization that English D Quantifiers really bind only the one variable associated with the determiner's own noun phrase is correct. In fact, it appears to be true across languages, not only for quantification expressed by determiners but also for quantification by agreement affixes of verbs and by focus particles. These also monoselectively bind just one variable.

In this note we have made some observations concerning ‘donkey’ sentences, A Quantification, and resumption, in English. It would be interesting to investigate the phenomenon of resumption across natural languages. Are Claims I–III valid for all of them? Furthermore, it is suggested in (1) that some languages, notably Straits Salish, do not use D Quantification at all but only have access to A Quantification. If so, what are the consequences for the expressive power of such languages?

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<sup>9</sup>Proofs of undefinability results can be quite complex. The proof of Proposition B is on the simpler side. Also, using standard model-theoretic tools it is not too hard to show that for  $Q = \text{MOST}$ , (42) is not equivalent to any sentence in  $\text{FO}(\text{MOST})$ . But that is just a special case of Theorem C, the full proof of which essentially uses advanced combinatorial mathematics (Ramsey theory), and is one of the most complex undefinability proofs that we are aware of.

We further note that what Luosto actually proves is that the resumption  $(Q_R)^2$  of the 1-place quantifier  $Q_R$  (sometimes called the Rescher quantifier), which, on a finite universe  $M$ , says of a set  $A \subseteq M$  that  $|A| > 1/2 \cdot |M|$ , is not definable in  $\text{FO}(Q_1, \dots, Q_n)$  for any monadic  $Q_1, \dots, Q_n$ . This yields Theorem C, since if the sentence  $\text{MOST}^2 xy(A(x) \wedge B(y), R(x, y))$  is expressible in some logic  $L$ , so is  $\text{MOST}^2 xy(x = y \wedge y = y, R(x, y))$ , and the latter is equivalent to  $(Q_R)^2 xyR(x, y)$ , i.e., to  $|R| > 1/2 \cdot |M^2|$ .

Is it really necessary to use Ramsey theory to obtain a linguistically significant undefinability result concerning resumption? We don't know, but it may be noted that (10) shows that Ramsey theory is indispensable for Theorem C. (More precisely, he shows that (a result similar to) Theorem C implies van der Waerden's theorem.)

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